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Research Article

Formulation and Solution ofTwo-Level Capacitated Lot-Sizing Problem

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ABSTRACT

The integration of sequencing decisions in lot-sizing and scheduling problems has received an increased attention from the research community due to its inherent applicability to the real-world problems. In this paper, we develop and solve a synchronized and integrated two-level lot-sizing and scheduling problem motivated by a real-world problem arisen in the soft drink production. The soft drink production process has two main stages: flavor preparation (Stage1) and bottling (Stage2). In Stage 1, the liquid flavor (concentrated syrup plus some water) is prepared in tanks with various capacities. In Stage 2 in which the liquid flavors are bottled at the filling lines. we considered stock capacityconstraint, and shortages of the combination(shortages backlog and lost sales) added to the model. The aim is to determine the lot-sizing and scheduling raw materials and products so that the soft drink flavors and bottle types are assigned to the tanks and bottling lines, respectively, in order to meet a known weekly product demand. Finally, we implement LINGO solver to solve this model.

Keywords: Lot-Sizing, Mathematical Programming,Production Planning,Scheduling

1. INTRODUCTION

This paper considers a production lot-sizing and scheduling of the problem encountered in a soft drink bottling plant.The main challenge is how to determine the lot-sizing and scheduling of raw materials in the tanks of soft drinks in bottling lines simultaneously, with sequence-dependent setup costs and times and production capacity constraints at each level so that the total production costs are minimized. Chen and Thizy [1]proposed a multi-item capacitated lot-sizing problem consists of determining the magnitude and the timing of some operations of durable results for several items in a finite number of processing periods to fulfil a known demand in each period. The proposed model was solved by a Lagrangian relaxation. Fleischmann [2]presented discrete lot-sizing and scheduling problem consists in scheduling several products on a single machine in order to meet the known dynamic demand and minimize the sum of inventory and setup cost. Fleischmann's[3]paper deals with the problem of scheduling several products on a single machine to meet the known dynamic demand and minimize the sum of inventory costs and sequence-dependent setup costs. The planning interval is subdivided into many short periods, for example shifts or days, and any lot must last one or several full periods. He formulate this problem as a travelling salesman problem with time windows and presented a new procedure to determine lower bounds using Lagrange, an relaxation as well as a heuristic. Fleischmann and Meyr [4]presented general lot-sizing and scheduling problem. A new model was presented that permits the integrated lot-sizing and scheduling of multiple

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items on a single, and capacitated production line when inventory holding and sequence dependent setup costs are presented. Model formulations are introduced for both situations when the setup state is lost or preserved after idle periods. Haase and Kimms [5]proposedlot-sizing and scheduling with sequence dependent setup costs and times and efficient rescheduling opportunities. The large-bucket mixed integer programming (MIP) model is formulated, which considers only efficient sequences. A tailor-made enumeration method of the branch-and-bound type solves problem instances optimally and efficiently. Toledo and Armentano [6] presented a Lagrangian-based heuristic for the capacitated lot-sizing problem in the parallel machines. A production plan should be determined in order to meet the forecast demand for the items, without exceeding the capacity of the machines and minimizing the sum of production, setup and inventory costs. Araujo et al. [7] proposed lot-sizing and furnace scheduling in small foundries. Two related decision levels exist: (1) the furnace scheduling of metal alloy production, and (2) moulding machine planning, which specifies the type and size of production lots, and a faster relax-and-fix (RF) approach is developed that can be also used on a rolling horizon basis where only immediate-term schedules are implemented. Kovacs et al. [8] presented a novel mathematical programming approach to the single-machine capacitated lot-sizing and scheduling problem with sequence-dependent setup times and setup costs. They introduced a new mixed-integer programming model in which binary variables indicate whether individual items are produced in a period, and parameters for this program are generated by a heuristic procedure in order to establish a tight formulation. Ferreira et al. [9] presented a mixed integer programming model that integrates the production lot-sizing and scheduling decisions of beverage plants with sequence-dependent setup costs and times. The model regards that the fact the industrial process produces soft drink bottles in different flavors and sizes, and it is carried out in two production stages: liquid preparation (Stage I) and bottling (Stage II). The model also takes account of the fact that the production bottleneck may alternate between Stages I and II, and a synchronization of the production between these stages is required. A relaxation approach and several strategies of the relax-and-fix heuristic are proposed to solve the model. Menezes et al. [10] presented the first linear mixed-integer programming extension for the capacitated lot-sizing and scheduling problem incorporating all the necessary features of sequence sub-tours and setup crossovers. This formulation is more efficient than other well-known lot-sizing and scheduling models. The study conducted by Ferreira et al. [11] deals with industrial processes that produce soft drink bottles in different flavours and sizes, in two synchronised production stages: liquid preparation and bottling. Four single-stage formulations are proposed to solve the synchronised two-stage lot-sizing and scheduling problem in soft drink production synchronising the first stage's syrup lots in tanks with the second stage's soft drink lots on bottling lines. The first two formulations are the variants of the general lot-sizing and scheduling problem with sequence-dependent setup times and costs, while the other two are based on the Asymmetric Travelling Salesman Problem with different subtour elimination constraints. Guimarães et al. [12]proposed modeling lotsizing and scheduling problems with sequence dependent setups, and presented a new formulation for the problem using subtour elimination constraints based on commodity flow. Sel and Bilgen [13] presented a hybrid solution methodology combining simulation and mixed integer programming based on Fixed and Optimized heuristic to solve the considered problem. The study carried out by Toledo et al. [14] applies a genetic algorithm embedded with mathematical programming techniques to solve a synchronized and integrated two-level lot-sizing and scheduling problem motivated by a real-world problem arisen in soft drink production. The problem considers a production process compounded by raw material preparation/storage and soft drink bottling. The lot-sizing and scheduling decisions should be made simultaneously for raw material preparation/storage in tanks and soft drink bottling in several production lines minimizing inventory, shortage and setup costs. Almeder et al. [15] presented two models, one considering batch production and the other one allowing lot-streaming. They used CPLEX solver in order to solve the model. We develop the model proposed byToledo et al. [14]. The objectiveof this paper is to consider stock capacityconstraint, and shortages of the combination(shortages backlog and lost sales) added to the model.

The remaining part of this paper is organized as follows: Section 2 describes two-level lot-sizing model and Section3 presents computational experiments. The conclusion and suggestions for future studies are included in Section4.

2. Mathematical Models

In this section, we presenttwo-level lot-sizing model.

2.1. Model Parameters

J= Number of final products (soft drinks);
M= Number of machines (production lines and tanks)
F= Number of raw materials (liquid flavors);
T= Number of macro-periods;
N= Number of micro-periods (setups in each macro-period);

2.2. Model Sets

St= Set of micro-periods in each macro-period t;
Pdt= First micro-period of macro-period t;
λj= Set of lines that can produce product j;
αm= Set of products that can be produced in line m;
βm= Set of raw materials that can be produced in tank m;
γml= Set of products that can be produced in line m and needs raw material l;

Superscript below is related to the parameters and variables at Level I (tanks), while Superscript II is related to the ones at Level II (bottling lines).

2.3. Model Data

djt = Demand of product j in macro-period t;
hj= Non-negative inventory cost of product j;
gj = Non-negative backorder cost of product j;
sIkl = Changeover cost from raw material k to l;
sIIij = Changeover cost from product i to j;
bIkl = Changeover time from raw material k to l;
bIIij = Changeover time from product i to j;
aIImj= Production time of one unit of product j in line m;
kIm = Total capacity of tank m, in liters of raw material;
kIImt = Total time capacity available for line m in micro-period t;
rjl = Quantity of raw material l necessary for the production of one unit of product j;
qIlm =Minimum quantity of raw material l necessary to fill tank m;
I+j0=Initial inventory of product j;
I-j0=Initial backorder of product j;
yIml0=1, If there is an initial setup of tank m for raw material l; 0 otherwise;
yIImj0=1, If there is an initial setup of line m for product j; 0 otherwise;
o= Amount of space occupied by each unit of product
φ_t = Total space available in period t
π_j = Cost of lost sales each product j
W= Risk of backlog

2.4. Model Variables:

I+jt = Inventory of product j at the end of macro-period t;
I-jt = Backorder of product j at the end of macro-period t;
xIImjs = Production quantity of product j in line m in micro-period s;
vIIms = Waiting time of line m in micro-period s;
yImls =1 If tank m is setup for raw material l in micro period s; 0 otherwise;
yIImjs =1 If line m is setup for product j in micro period s; 0 otherwise;
zImkls =1 If there is a changeover in tank m from raw material k to l in micro period s; 0 otherwise;
zIImijs =1 If there is a changeover in line m from item i to j in micro period s; 0 otherwise;

2.5. Proposed Model

Equation (1) is the objective function that minimizes inventory, setup costs, shortages backlog and lost sales. Constraints (2) and (3) determine the maximum and minimum tank capacities, if raw material is assigned to tank(yImls=1). Constraints (4) enforce idle micro-periods to occur at the end of each time period. The setup of raw material in tanks and products in lines is established by Constraints (5), (6) and (13). Constraint(6) is necessary to control raw material changeovers at the beginning of each period. Constraints(7) and (14) ensure only one raw material and product setup in the tank and line, respectively, for each micro period. Constraint(8) is the inventory balance equation for products in lines and Constraint (9) define the production capacity for each line in each period. Idle time is determined by Constraint (10) and it equals the difference between the raw material setup time and the line setup time.Constraint (11) define setup for products in lines, which means that nothing is produced if a product is not set up in a line (yIImjs=0). Constraint (12) ensure that a line is always ready to produce exactly one product in each micro-period. Constraint (15) is stock capacity. The variable domains are summarized by Constraint (16).

$$MinZ = \sum_{j=1}^{J}\sum_{t=1}^{T}(h_j I_{jt}^{+} + g_j W I_{jt}^{-} + \pi_j (1-W) I_{jt}^{-}) + \sum_{s=1}^{N}\sum_{m=1}^{M}\sum_{k\in B_m}\sum_{l\in B_m} S_{kl}^{I} Z_{mkls}^{I} + \sum_{s=1}^{N}\sum_{m=1}^{M}\sum_{i\in\alpha_m}\sum_{j\in\alpha_m} S_{ij}^{II} Z_{mijs}^{II} \tag{1}$$

Subject to:

$$\sum_{j\in\gamma_{ml}} r_{jl} x_{mjs}^{II} \le K_m^I y_{mls}^I \quad m=1,\dots,M, l\in\beta_{ml}, s=1,\dots,N; \tag{2}$$

$$\sum_{j\in\gamma_{ml}} r_{jl} x_{mjs}^{II} \ge q_{lm}^I y_{mls}^I \quad m=1,\dots,M, l\in\beta_{ml}, s=1,\dots,N; \tag{3}$$

$$\sum_{l\in\beta_{ml}} y_{ml(s-1)}^I \ge \sum_{l\in\beta_m} y_{mls}^I \quad m=1,\dots,M, t=1,\dots,T, s\in S_t - \{Pd_t\}; \tag{4}$$

$$z_{mkls}^I \ge y_{mk(s-1)}^I + y_{mks}^I - 1 \quad m=1,\dots,M, k,l\in\beta_m, s=1,\dots,N; \tag{5}$$

$$z_{mkls}^I \ge \sum_{j\in\gamma_{mk}} y_{mj(s-1)}^{II} + y_{mls}^I - 1 \quad m=1,\dots,M, k,l\in\beta_m, t=2,\dots,T, s=Pd_t; \tag{6}$$

$$\sum_{k\in\beta_m}\sum_{l\in\beta_m} z_{mkls}^I \le 1 \quad m=1,\dots,M, t=1,\dots,T, s\in S_t; \tag{7}$$

$$I_{j(t-1)}^{+} + WI_{jt}^{-} + \sum_{m\in\lambda_j}\sum_{s\in S_t} x_{mjs}^{II} = I_{jt}^{+} + WI_{j(t-1)}^{-} + d_{it} \qquad j=1,\dots,J, t=1,\dots,T; \tag{8}$$

$$\sum_{j\in\alpha_m}\sum_{s\in S_t} a_{mj}^{II} x_{mjs}^{II} + \sum_{i\in\alpha_m}\sum_{j\in\alpha_m}\sum_{s\in S_t} b_{ij}^{II} z_{mijs}^{II} + \sum_{s\in S_t} v_{ms}^{II} \le \mathrm{K}_{mt}^{II} \quad m=1,\dots,M, t=1,\dots,T; \tag{9}$$

$$v^{II}_{ms} \geq \sum_{k\in\beta_m}\sum_{l\in\beta_m} b^{I}_{kl} z^{I}_{mkls} - \sum_{i\in\alpha_m}\sum_{j\in\alpha_m} b^{II}_{ij} z^{II}_{mijs} \quad m=1,\dots,M, s=1,\dots,N; \tag{10}$$

$$x^{II}_{mjs} \leq \frac{K^{II}_{mt}}{a^{II}_{mj}} y^{II}_{mjs}, m=1,\dots,M, j\in\alpha_m, t=1,\dots,T, s\in S_t; \tag{11}$$

$$\sum_{j\in\alpha_m} y^{II}_{mjs} = 1 \quad m=1,\dots,M \qquad s=1,\dots,N; \tag{12}$$

$$z^{II}_{mijs} \geq y^{II}_{mi(s-1)} + y^{II}_{mjs} - 1 \quad m=1,\dots,M, i,j\in\alpha_m, s=1,\dots,N; \tag{13}$$

$$\sum_{i\in\alpha_m}\sum_{j\in\alpha_m} z^{II}_{mijs} \leq 1 \quad m=1,\dots,M, s=1,\dots,N \tag{14}$$

$$o(\sum_{j=1}^{J}\sum_{s\in S_t} x^{II}_{mjs}) \leq \varphi_t \quad t=1,\dots,T, m=1,\dots,M; \tag{15}$$

$$I^{+}_{jt}, I^{-}_{jt} \geq 0,\ j=1,\dots,J, t=1,\dots,T; x^{II}_{mjs}, v^{II}_{ms}, z^{II}_{mijs}, z^{I}_{mkls} \geq 0;$$
$$y^{II}_{mjs}, y^{I}_{mls} \in \{0,1\} \quad m=1,\dots M, k,l\in\beta_m, i,j\in\alpha_m, \tag{16}$$
$$t=1,\dots,T, s\in S_t$$

4. RESULTS

In order to evaluate the performance of the model, 10 problems with different sizes are randomly generated. The proposed model coded with LINGO 8 Software was used to solve the instances. All tests are conducted on a not book at Intel Core 2 Duo Processor 2.00 GHz and 2 GB of RAM.Table 1 shows the details of computational results obtained by solution method for all test problems.

Table 1. Details of computational results

NO	Macro period	Micro period	line	Product I	Product J	Raw material k	Raw material l	machine	Lingo	Time(s)
1	3	4	2	2	2	2	2	2	2127270	1
2	6	4	2	2	2	2	2	2	10902270	1
3	6	8	2	2	2	2	2	2	10902630	1
4	6	8	2	2	2	3	3	2	10710130	2
5	6	10	2	2	2	4	4	2	11860810	2
6	8	10	2	2	2	4	4	2	21852310	3
7	6	8	3	2	2	2	2	3	10902940	18
8	6	8	2	3	3	2	2	2	17144530	58
9	6	8	4	2	2	2	2	4	10903260	571
10	6	8	2	4	4	2	2	2	20876230	4870

5. CONCLUSION

The objective of this paper is to formulate and solve the two-level capacitated lot-sizing problem with backlogging, lost sale and safety stocks, for two-level production systems in which the objective function minimizes inventory, shortage and setup costs.We develop a mixed integer programming model that can be used to compute optimal solution for the problems by an operation research solver. Table 1 shows that increasing the number of machines, production lines, product i and j have a significant impact on the increase of the CPU time. One straightforward opportunity for future research is extending the assumption of the proposed model to includethe real conditions of production systems such as fuzzy demands.Moreover, developing a new heuristic or metaheuristic to construct feasible solutions can be among other cases for future resaerch works.

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