Mathematical modeling of thermal energy storage tank

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ABSTRACT
In this study, a mathematical model based on effective heat capacity method to predict thermal behavior and performance of thermal energy storage using phase-change material was developed. The effect of various parameters including the temperature of the working fluid inlet, a fluid flow input, the phase-change material, the use of more than one material, phase-change and the melting process time as well as check the efficiency of thermal energy storage results were presented as graphs. It was also shown that increasing the fluid mass flow rate and inlet temperature working fluid other than a certain level will not be much impact on the melting process. Was shown in low mass flow for working fluid use two different PCM instead of one PCM to increase the efficiency of the thermal energy storage. It was also shown that increasing the temperature of the working fluid inlet because the difference in temperature between the melting temperature of the working fluid input and phase-change material increases. Thus, thermal energy storage efficiency increases as well as decreases the time required for the process of melting. It was also shown that increasing the fluid mass flow rate and inlet temperature working fluid other than a certain level will not be much impact on the melting process.

Key word: The phase change temperature, energy storage, thermal capacity effective, mathematical modeling, phase-change material

1- INTRODUCTION
Energy can be considered as the backbone for human life. Due to the growing population of the Earth as well as the industrial revolution took place in the 20th century, with significant growth in energy demand was met. First optimize fuel consumption and finding renewable energy sources that will replace fossil fuels. In recent decades much research has been done in these two areas that lead to significant progress as well. It should be noted that most of the world's fossil energy consumption related to heating and cooling systems are houses. So that, according to the Institute WBCSD 40% of global energy consumption is related to residential and buildings. In other words we can say that 40% of the carbon dioxide produced can be seen on buildings. A new study by scientists in the field of energy efficiency in buildings indicates that it is necessary to prevent editors from changing the global climate that by 2050, energy consumption in buildings by at least 60% lower. Sun can be used as a powerful and almost unlimited energy source used. Research shows that energy from the sun to the earth in one hour than the energy emitted more than one person during a year. But it should be noted that we can not have much control over this type of energy. In other words, one can say exactly when the sun intensity is the least energy demand rises. Obviously in this system have an auxiliary power source for emergency also be considered. The same motive led to the design of thermal energy storage (TES),
respectively. The operating principle of the system is based on the latent heat material. So that using phase-change material (PCM) thermal energy storage systems can be produced with high capacity. The most important feature of this material is that it can be a lot of energy without the need to increase the temperature of the material be stored. And only in this case will turn from solid to liquid phase material.

Another of the key points in energy storage, size and duration energy storage is the storage system. The size of the storage system is also due to the limited space as well as beauty tips structure is important. It should be noted that the size of the thermal energy storage system parameters such as: type of material used, temperature thermal energy storage, thermal storage mortality rate, average cost of energy storage, ambient temperature and so on.

Phase-change material (PCM) is a substance that requires a lot of heat for melting. When heat is stored in these materials are converted from solid to liquid and liquid to solid phase when the heat is released, got converted. In other words, absorption or release of heat in this type of material is associated with phase change. Phase-change material in terms of categories in the total thermal energy using latent heat storage materials are placed.

Phase-change material generally can be divided into two categories: organic and inorganic materials. The difference between these two types of phase-change material can be found in Table 1 was observed.

<table>
<thead>
<tr>
<th>Table (1). The advantages and disadvantages of organic and inorganic phase-change material</th>
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<tbody>
<tr>
<td><strong>non-organic</strong></td>
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<tr>
<td><strong>Benefits</strong></td>
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<tr>
<td>latent heat</td>
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<td>Sub-cooled temperature</td>
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<td>thermal stability</td>
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<td>flammability</td>
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2- literature review

Phase-change material has a phase change capability (for example from solid to liquid) are in a constant temperature range. This material is abundant in diverse industries including telecommunications, transportation, automobiles, satellites, medical, textile, greenhouses and other items used. The first reports on the application of these materials in the building was on 1940. The use of these materials in the building from 1980 was followed by more serious. They now have a special place in the construction industry. This material can be used for heating and cooling buildings and individual components used. Phase-change materials in buildings usually inside shutters, sun-facing walls, floor heating systems or building materials such as gypsum are used. Research and studies show that the use of phase-change material inside the building at about 19 percent can help to save energy consumption.

The first serious research on energy storage methods such as the use of phase-change material began in 1970. Peippo and et al [1] introduce a simple analytical relationship that the approximate melting simple optimum temperature and optimum thickness of the phase-change material used in building gains.

In 1983, Abhat [2] to select a series of phase-change material for thermal energy storage systems at low temperature (range 0 °C to 120 °C) was studied and analyzed. In his study of Arganmy the phase-change material such as paraffin and fatty acids and non-organic materials, such as phase-change hydrated salts were examined. She behavior of the material during melting and freezing using thermal analysis studied. Then the material terms of long-term stability to thermal energy storage for long periods examined. Tests were then carried out to determine the extent of corrosion of this material.

In 1995, Stoval and et al [3], the effect of the phase-change material used in the wall in delaying the peak cooling and heating load examined. Their earlier collaboration with Kedl [4] performance
phase-change material for wall mats to store solar energy into numerical method were analyzed. In 1998, Costa et al. [5] numerical modeling using latent heat energy storage system began. The main objective of the design of the system, its use in residential buildings and industrial facilities to reduce peak electric power demand. They enthalpy as well as for her modeling techniques finite difference method used implicitly.

In 2003, Cabeza and et al. [6] explores the advantages of hydrated salts as phase-change material presented.

In the year 2010, Agyenim and et al. [7] Experimental Study of a phase-change material's storage tanks. In this experiment they RT58 material as phase-change material used. The storage tank was examined for a horizontal cylinder. Length of 1.2 m and a copper cylinder that was filled with 93 kg of the RT58. The cylinder was equipped with Finn. They finally temperature profile for an hour, the temperature areas, the overall heat transfer coefficient and the amount of energy stored as test data had reported.

In 2013, Anisur and et al. [8] to investigate the use of phase-change material energy storage systems in order to prevent the ground from their heated.

In their research on the use of thermal energy storage systems to reduce their greenhouse gas.

In 2011, Arce et al. [9] investigated the feasibility of using thermal energy storage systems. They save on energy consumption in order to prevent their emissions.

In the year 2007, Arkar and colleagues [10] examined the latent heat cooling system based on heat load in a building with low-paid. They began modeling the cooling system using TRNSYS software.


In the year 2011, Dincer and et al. [12] examined the thermal energy storage systems and applications of it. They Ava storage systems Anrzhyrarty design methods and analysis, as well as optimize its Teramokonoli examined them.

3- governing equations

Various methods for studying the problem of freezing and thawing phase-change issues to be charged. Among these can be cited by Newman and heat balance integral method. The easiest problem to change the phase, a phase change next problem is that for the first time by Stefan [13] to be solved analytically. He solved his analysis with the assumption of constant thermo-physical properties do. Stefan number can be calculated from the following relationship:

\[ st = \left( \frac{C_1 (T_1 - T_m)}{H_f} \right)^{1/3} \]

\( C_1 \) is thermal capacity of the liquid, \( H_f \) heat of fusion, \( T_1 \) and \( T_m \), respectively, freezing temperatures are ambient temperature. Newman is assumed that the temperature of freezing and thawing are equal. As we know, the pure material melting and freezing temperatures are equal. It is called the melting temperature and show the whit \( T_m \). In non-pure material these temperatures are not equal. In this case the temperature at which the fluid is completely in the liquid phase with the symbol of show \( T_L \). The temperature at which the fluid is completely in the solid phase with the symbol \( T_s \) show.

Newman was the first to solve the Stephen problem of two-dimensional extended phase change [14]. But it's an issue that Newman solved in two-dimensional mode with infinite length was related to a narrow channel. He assumed that the initial conditions of temperatures in the points is equal to \( T_i \) and \( T_i < T_m \) is. He also thermo-physical properties during this process all fixed up. With these assumptions, the following steps have to be taken to solve this problem:

The relationship between heat transfer fluid guidance for the area can be written as follows:

\[ \alpha_1 \frac{\partial^2 T}{\partial x^2} + \alpha_2 \frac{\partial^2 T}{\partial x^2} = 0 \quad \text{for} \quad 0 < x < X(t), t > 0 \]

(2) Also in the area of heat transfer solid relationship can be written:
Borders also calls for liquid and solid surface temperature are:
\[
\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = \frac{1}{x} f(x,t), \quad x > 0, \quad t > 0
\tag{3}
\]
Stephen condition can also be written as follows:
\[
k_x \frac{\partial T}{\partial x} - k_l \frac{\partial T}{\partial x} = H_f \beta \frac{\partial X}{\partial t}
\tag{4}
\]
\[
x = X(t), \quad t > 0
\tag{4}
\]
As well as the need for basic condition:
\[
T(x,0) = T_a < T_m \quad \text{x} > 0 \quad X(0) = 0
\tag{5}
\]
The boundary conditions are:
\[
T(0,t) = T_1 \quad \text{t} > 0
\tag{6}
\]
\[
T(x,0) = T_s \quad \text{x} \rightarrow 0, \quad t > 0
\tag{7}
\]
It should be noted that in all high-order relationships of X (t) is the position of the common border between solid and liquid. Since this border is not fixed and moving it to other issues such moving boundary are known.

Stephen two-dimensional problem can be schematically and Figure (1) display.
Newman to solve this question bears some resemblance to a variable relationship defined as:
\[
- \frac{x}{2 \alpha t_1}
\tag{8}
\]
\[\text{figure(1). Schematics of two-dimensional Stefan problem with equations}\]
\[\text{After solving the problem of Newman for places where the contact area was presented the following relationship:}\]
\[
X(t) = 2x(\alpha_1 t)^{\frac{3}{2}}
\tag{9}
\]
\[\text{The temperature in the liquid phase can be calculated from the following relationship:}\]
\[\text{The purpose of the above erf is the error function.}\]
\[\text{The temperature distribution in the solid phase can be obtained from the following relationship:}\]
\[
T(x,t) = T_1 - (T_1 - T_m) \left[ \frac{1}{erfc} \left( \frac{x}{2 \sqrt{\alpha t}} \right) - 1 \right]
\tag{10}
\]
\[\text{In the above relation erf is complementary error function is called and is equal to 1-erf.}\]
\[\text{It should also be noted that the above equations, the equations were solved in non-deterministic parameter that achieved the following:}\]
\[
\frac{S_{t_1}}{S_{t_2}} = \frac{S_{t_1}}{S_{t_2}} \left( \frac{T_1}{T_2} \right) \left( \frac{T_1}{T_2} \right)
\tag{11}
\]
\[\text{In the above equation, we have:}\]
\[
S_{t_1} = \frac{S_{t_1} - S_{t_2}}{H_f} \quad S_{t_2} = \frac{S_{t_1} - S_{t_2}}{H_f}
\tag{12}
\]
\[\text{It should be noted that Newman solution only for moving boundary and in the Cartesian coordinate system can be used.}\]
\[\text{So for problem solving Newman phase change in the cylindrical coordinate system can not be used.}\]
\[\text{To solve these problems must be solved Peterson.}\]
\[\text{He is a phase change in the cylindrical coordinate system could provide an accurate analytical solution.}\]
\[\text{To this end, he can be an answer in the form of integral function } E_i (\sqrt{\frac{1}{4 \alpha t}}) \text{ defined. It also assumes that the level between the solid and liquid phases within } X (t) = r (t) \text{ BE. So we can say that for } r > X (t) \text{ liquid phase and for } r < X (t) \text{ will have the solid phase. He also assumed that the thermo-physical properties in both phases are fixed. He is a straight line in the center of the cylinder (r = 0) as the source of thermal energy to heat the equivalent of } Q \text{ (W / m) is entered into the system considered. So it can be energy conservation equation for a control volume around the drawing heat source is written as follows:}\]
\[
\lim_{\tau \to 0} \left[ 2\pi r k_2 \frac{d T_1}{d x} \right] = Q
\tag{13}
\]
In this issue can be solid or liquid temperature distribution in the region in order to obtain the following formula:

\[ T(r, t) - T_l = \frac{r^2}{4\alpha_t} \left[ T_m - T_l + \frac{T_r - T_m}{E_l} \right] - \frac{r^2}{4\alpha_t} \left[ \frac{T_r - T_m}{E_l} \right] \]

(15)

\[ T_s = T_m - \frac{Q}{4\pi k_s} \left[ E_l \left( \frac{r^2}{4\alpha_t} \right) - E_l \left( \frac{r^2}{4\alpha_t} \right) \right] \]

(14)

\[ -\frac{\partial}{\partial t} \left( \rho \cdot \frac{\partial f}{\partial x} \right) = \nabla \cdot \left( \alpha \frac{\partial f}{\partial t} \right) \]

Figure 2. Schematics of the problem of Patterson

The amount of non-algebraic equation in the equation below is obtained:

\[ \frac{d}{dt} \left( \rho \cdot \frac{\partial f}{\partial x} \right) + \frac{\partial}{\partial t} \left( \rho \cdot \frac{\partial f}{\partial x} \right) = \nabla \cdot \left( \alpha \frac{\partial f}{\partial t} \right) \]

(16)

As well as the interface between liquid and solid phases can be calculated from the following relationship:

\[ X(t) = 2(\alpha_s \cdot t)^{1/2} \]

(17)

As mentioned in the previous section, in this study, the number of latent thermal energy storage tank shell and tube type, we will. It is assumed that the layouts using phase-change material is filled. This research examined two types of phase-change material will be. Phase-change material has a melting temperature of \( \text{P}116 \) and \( n\text{-octadecane} \) respectively 50 °C and 27.7 °C are Satie. The numerical results obtained with available experimental data will be compared and validated. Parameters that their impact will be studied in this research include:

1. input temperature of the working fluid in the temperature range 50 to 60 °C is considered to be variable.
2. Mass Flow input to the tube
3. mass of the phase-change material in the shell

The trend mentioned parameters can be an excellent guide for the design and optimization of thermal energy storage systems.

Schematics of the geometry of the problem is that in this study we intend to model it can be in the form of (3) was observed.

Figure 3. Schematics of energy storage units of shell and tube heat
As can be seen in Figure 3 illustrates, storage unit includes an inner tube with a diameter $R_i$, which the working fluid flows inside the tube for forced displacement. It also includes an outer tube geometry with radius is $R_o$. The space between the two pipes that actually make up the circumference of the shell. Phase-change material in the circular shell to be filled.

It is assumed that the outer tube is fully insulated. In other words, it is assumed that no heat from the surface of the outer tube can not be exchanged with the surroundings. It is also assumed that the entire length of the working fluid that flows into the pipe Ydrvny water.

Mathematical modeling will be applied the following assumptions:

- operating fluid flow in the pipe is considered to be fully developed.
- thermal conductivity and viscous losses in the axial direction regardless.
- thermo-physical properties of the working fluid and phase-change material is assumed to be independent of time.
- thermo-physical properties of solid phase and liquid phase to the phase-change material is considered to be other than thermal conductivity. Regardless of the thermal resistance of the inner tube.

4- RESULTS

![Figure 4](image1)

**Figure (4).** Graphs comparing temperature changes over time for experimental and numerical data T1 and T2 in two locations for $k = 10k_3$

![Figure 5](image2)

**Figure (5).** Graphs comparing temperature changes over time for experimental and numerical data T1 and T2 in two places for $k = 20$
As Figure (4) and (5) also show that data from numerical simulations match experimental data are very affordable. In its review, the mass flow rate of fluid flow equal to \( \dot{m} = 5 \times 10^{-2} \) kg/s as well as the working fluid inlet temperature \( T_{in} = 60 \, ^\circ C \) consider.

\[ T_i = T_i = 25 \, ^\circ C(18) \]

The problem of the above it is assumed that the two different phase-change material to be used simultaneously. In other words, it is assumed that half of the shell with phase-change material of the P116 and the other half with phase-change material of n-octadecane is filled. It should be noted that the first half of the tank with the P116 phase-change material and the second half of the phase-change material of n-octadecane filled. The advantage of using two phase-change material with a melting temperature of the thermal capacity of the system as well as better control system.

REFERENCE