

**Research Article****Selection of an Optimal Method for Calculation of  
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[Received-09/06/2016, Accepted-22/06/2016, Published-30/06/2016]

**ABSTRACT**

The article is dedicated to estimation of absolute errors in the methods of correlated color temperature (CCT) calculation, namely the methods of Robertson (1968), McCamy (1992), Yoshi Ohno (2013), Javier Hernandez (1999) by means of the program developed by the authors. The work represents graphs of the absolute error distribution across the range of CCT definition. The authors outlined recommendations on use of the above methods in the colorimetric measurement practice based on the obtained results. The work also contains comparison of the “standard” variant of the Robertson’s method using 31 isotherms and the variants using more isotherms. It was demonstrated that with reduction of the period between isotherms the error tends to decrease. The calculation program was implemented with use of Python language and Numpy library, the tables were also computed by means of Python language and Scipy library, the graphs were plotted with the help of Matplotlib library.

**Key words:** correlated color temperature, Planckian locus, line of correlated color temperature, chromaticity coordinates, absolute error.

**1.INTRODUCTION**

First methods of calculation of CCT appeared at the end of the 60’s in the XX century. During the following 50 years their number reached ten or so [1-3]. The authors of the methods surely specified method errors however sometimes omitted to mention the estimation method. The reference [2] contains an analytical method of CCT calculation and specifies absolute errors of the CCT calculation methods but the estimation technique is absent.

In other words there is hardly any common approach to estimation of the errors of CCT calculation by all of the existing methods. Therefore we’ve developed a program for

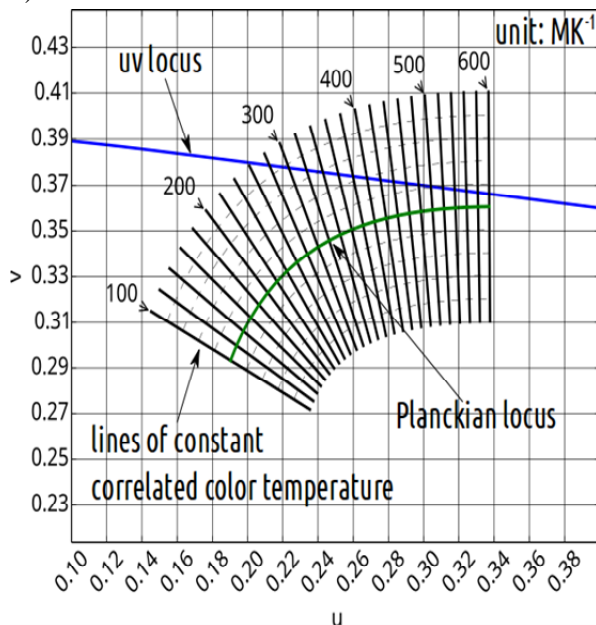
calculation of the absolute errors for the CCT calculation methods.

Prior to start the technique description we’d like to remind that according to [4 7] the correlated color temperature is the temperature of Planckian locus whose chromaticity most closely resembles the chromaticity of a given spectral distribution at the CIE 1960 uniform chromaticity scale diagram. The notion of correlated color temperature can not be applied if a distance between chromaticity of the source under test and the Plankian locus exceeds

$$\Delta C = [(u'_r - u'_p)^2 + 4/9(v'_r - v'_p)^2]^{1/2} = 5 \cdot 10^{-2},$$

where  $(u'_b, v'_b)$  and  $(u'_p, v'_p)$  are chromaticity coordinates of the source under test and Planckian locus correspondingly at the CIE 1974 uniform chromaticity scale diagram. Isotherm is a line all points of which correspond to one and the same CCT. Therefore if the temperature to which a certain isotherm corresponds is known then if we take a point at a given isotherm and calculate CCT for such point we'll be able to determine an error (both absolute and fractional) of a method at the given point, i.e. the temperature of an isotherm should be regarded as an actual value of CCT in the course of error estimation.

It is also essential to mention that based on the CCT definition a method error should not be determined at one point but within the band along the Planckian locus with the width of 0.10 (Fig. 1).



**Figure 1** - Planckian locus and isotherms at the CIE 1964 uniform chromaticity scale diagram

## 2.METHODOLOGY

The essence of methodology of the CCT calculation methods error estimation is as follows:  
 – the first stage involves forming of a table where chromaticity coordinates of isotherms are stored. At that the following equation is used (1):

$$u_l = u_0 \pm \frac{lu'}{\sqrt{u'^2 + v'^2}}, v_l = v_0 \pm \frac{lv'}{\sqrt{u'^2 + v'^2}}$$

where  $u' = du/dt, v' = dv/dt; u_0, v_0$  – the Planckian locus chromaticity coordinates;  $l$  – a distance between the Planckian locus and a curve parallel to it. Note that we used numerical differentiation for determination of  $u'$  and  $v'$ . Detailed isotherms calculation procedure is set forth in [8];

– after that another table is formed the cells of which contain the results of CCT calculation for chromaticity values from the first table;

– the third stage consists in formation of a table the cells of which contain absolute error values. In the course of the error calculation the isotherms temperature is designated as a real value.

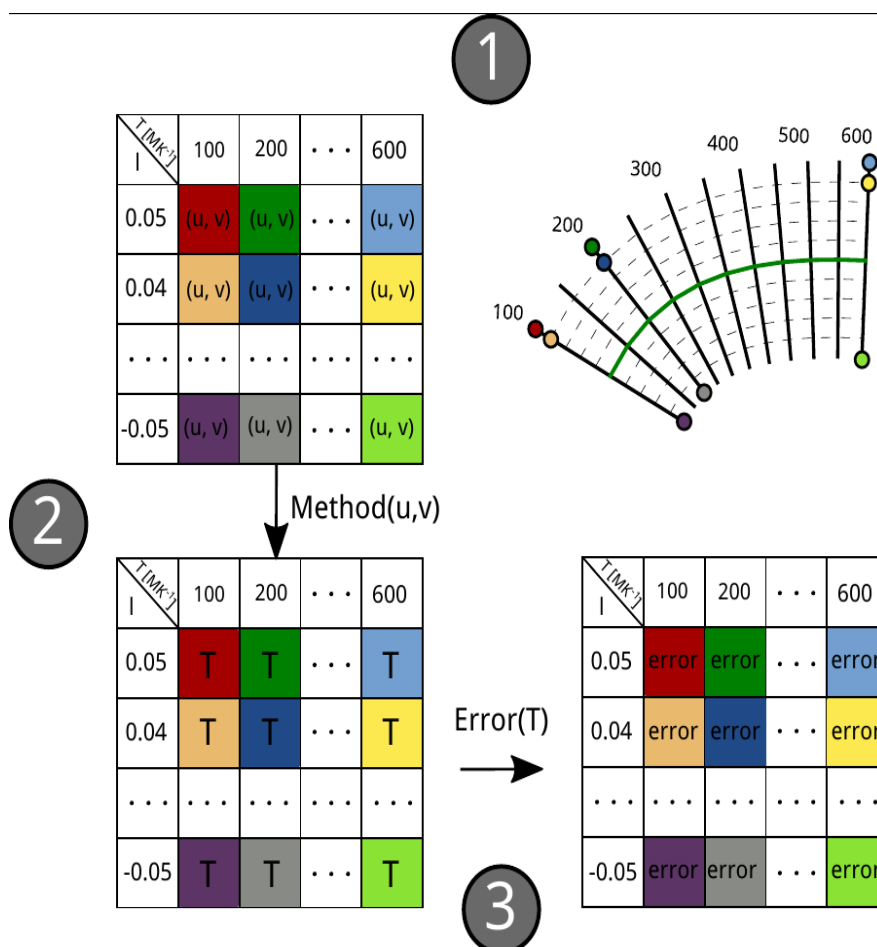
Therefore the methodology essence involves use of chromaticity coordinates of isotherms as input data at time of the CCT calculation and determination of the absolute error as a difference between the calculated CCT value and the temperature of an isotherm. The above algorithm is shown on Figure 2.

Use of reciprocal megakelvin  $MK^{-1}$  as a temperature measurement unit is conditioned by the fact that at the uniform chromaticity scale diagram the selected temperatures interval (for instance  $\Delta T = 20 MK^{-1}$ ) will correspond to approximately equal number of chromatic thresholds, i.e. the distance between isotherms will be equal.

## 3.MAIN PART

**The Robertson's method** for CCT calculation was presented for use in 1968 [3, 9]. It had widespread practical application and has preserved its applicability so far.

For example the software of modern spectroradiometers frequently involves this method which lies in linear interpolation of isotherms.



**Figure - 2.** For the methodology of error estimation of the CCT calculation methods:  $\text{Method}(u, v)$  – designation of a method under test,  $\text{Error}(T)$  – designation of a procedure of the absolute error determination

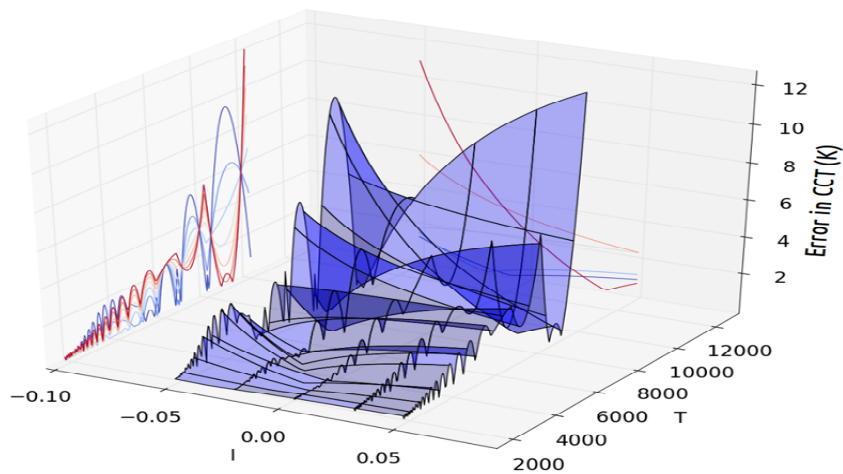
The original method is based on a calculation table describing properties of 31 isotherms (from 10 MK<sup>-1</sup> to 660 MK<sup>-1</sup> with 10 MK<sup>-1</sup> intervals). We've reduced the interval between the isotherms. As a result we've calculated tables for the intervals of 5 MK<sup>-1</sup>, 2.5 MK<sup>-1</sup> and 1 MK<sup>-1</sup> [10].

The results of the CCT calculation error estimate for this method are shown in Table 1, the analysis of the results demonstrated that the error decreased with the interval reduction. In case of the interval of 1 MK<sup>-1</sup> within the important practice-relevant temperature range from 1667 K to 10000 K the method error tends to zero.

**Table 1** –Errors of the Robertson's method

Errors in CCT (K)			
10 MK <sup>-1</sup>	5 MK <sup>-1</sup>	2.5 MK <sup>-1</sup>	1 MK <sup>-1</sup>
12.4	0.6	0.16	0.025

Figure 3 demonstrates distribution of the absolute error within the range from 1667 K to 10000 K for the table offered by G.Wyszecki. The X-axis of the graph specifies the temperature values, the Y-axis specifies the distance from the Planckian locus and the Z-axis specifies the absolute error values.



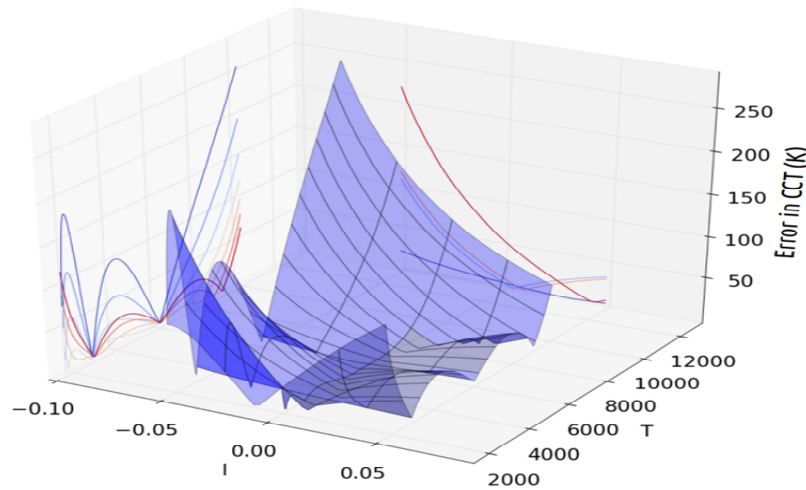
**Figure 3** - Errors in CCT (K) of the Robertson's method (with the table of G.Wyszecki)

It can be concluded from the graph (Figure 3) that the error value grows with the temperature rise.

**The McCamy's method** was presented for use in 1992. The method offers to use a third-degree polynomial for the CCT calculation. The polynomial was obtained based on the

assumption that all isotherms are intercrossed in a definite point at a XYZ chromaticity diagram.

Estimate of the error of the CCT calculation by this method showed (Fig. 4) that it was considerably inferior to the Robertson's method in the terms of accuracy. The maximum error in the range from 1700 to 10000 K made 285.4 K.

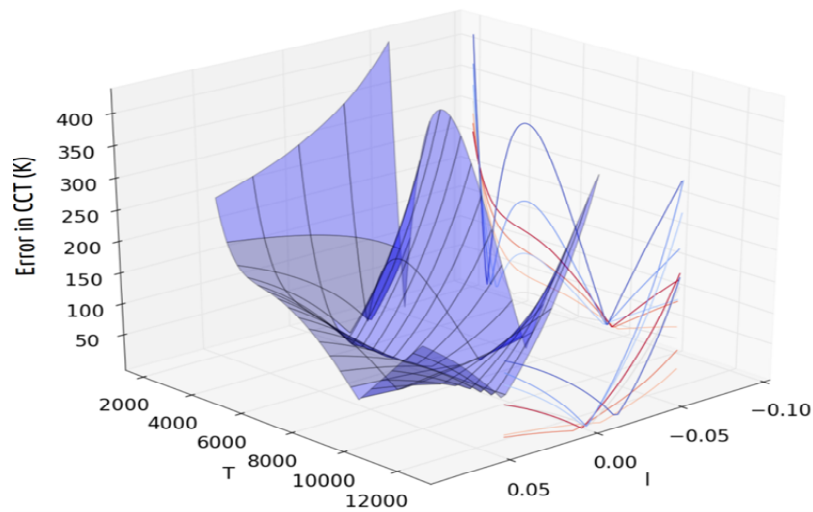


**Figure 4** - Errors in CCT (K) of McCamy's method in the range from 1700 to 10000 K

**The method of Javier Hernandez** was presented for use in 1999 [2]. Like the McCamy's method it involved an analytical solution. Estimation of the CCT calculation error for this method allowed stating (Figure 5) that it also ranks considerably below the Robertson's method accuracy. The

maximum error within the range from 1700 to 10000 K made 429.4 K.

In 2013 Yoshi Ohno offered three methods for the CCT calculation, i.e. **triangular solution, parabolic solution and combined solution** [1]. All of them are based on the same calculation table.



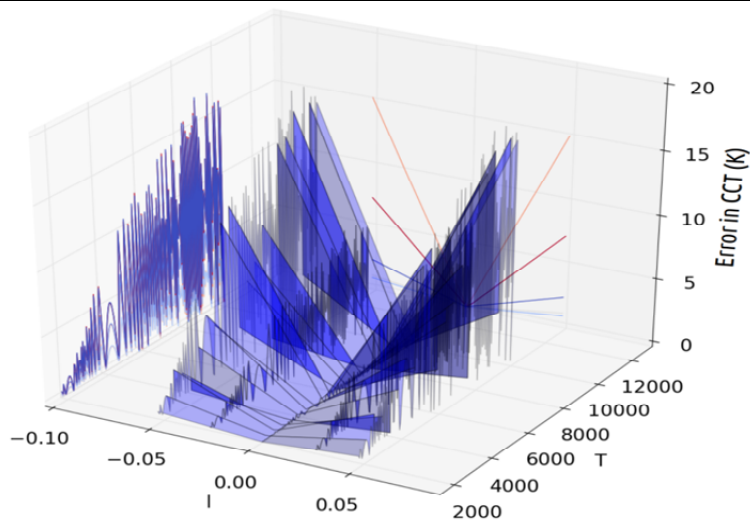
**Figure 5** - Errors in CCT (K) of Hernandez-Andres's method in the range from 1700 to 10000 K

Table 3 contains the absolute errors of the CCT calculation by the methods of Yoshi Ohno.

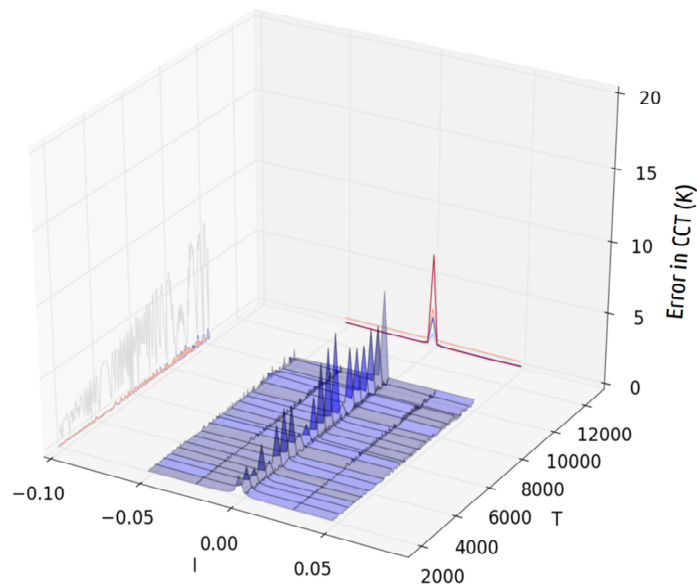
Figures 6-8 show the graphs of errors distribution for the corresponding CCT calculation methods.

**Table 3** – Errors of Y. Ohno's CCT calculation methods

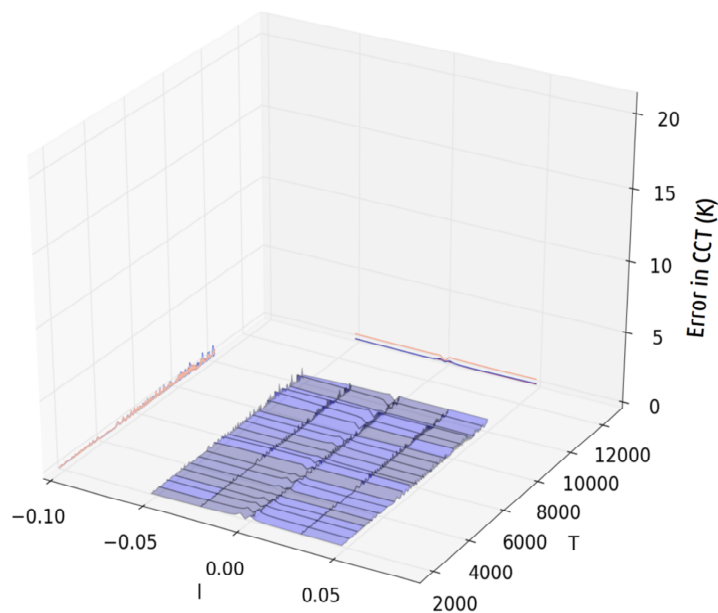
Method	Errors in CCT (K)
	1667 K – 10000 K
Triangular solution	19.1
Parabolic solution	8.5
Combined solution	0.9



**Figure 6** - Errors in CCT (K) of triangular solution in the range from 1700 to 10000 K



**Figure 7** - Errors in CCT (K) of parabolic solution in the range from 1700 to 10000 K



**Figure 8** - Errors in CCT (K) of combined solution in the range from 1700 to 10000 K

**4. CONCLUSION**

The article contains the analysis of the CCT calculation methods offered by such authors as Robertson (1968), McCamy (1992), Javier Hernandez (1999), Yoshi Ohno (2013) as well as the absolute error estimation for the mentioned methods made by means of the program developed by the authors of the article.

**5. FINDINGS**

The following findings were made as a result of the performed work:

- For the Robertson’s method CCT calculation error increases with the temperature increase. When the calculation table interval is equal to 1  $\mu$  within the range of temperatures from 1700 K to 10000 K the method error tends to zero;

- for the triangular solution of Yoshi Ohno within the interval of temperatures from 1667 K to 10000 K the absolute error does not exceed 19,1 K. The parabolic solution for the same interval is more accurate, its error does not exceed 8.5 K. The combined solution is characterized by the highest accuracy. The absolute error within the interval from 1667 K to 10000 K makes 0.9;

- for the McCamy's method the error reaches 250 K for the interval from 1700 to 10000 K. The McCamy's method error depends strongly on location within the band along the Planckian locus. As a rule the minimum error of the CCT calculation by the McCamy's method is observed in proximity of the Planckian locus;

- for the method of Javier Hernandez the error within the interval from 1700 to 10000 K comes up to 429.4 K. Like in the previous method the error value is strongly dependent on location within the band along the Planckian locus;

- we recommend to use the combined solution of Yoshi Ohno as the most optimal method of the CCT error calculation in the terms of accuracy.

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