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BLOOD FLOW IN PRESENCE OF MAGNETIC FIELD THROUGH POROUS MEDIUM AND ITS EFFECT ON HEAT TRANSFER RATE

***Ruchi Chaturvedi, R. K. Shrivastav[1], Vinay Kumar Jadon[2]**

*Department of Mathematics, FET-Agra College, Agra, India Email: ruchiaec3@gmail.com,
[1]Department of Mathematics,Agra College, Agra, India Email: dr.srivastavark@yahoo.com,
[2]Department of Mathematics, Anand Engineering College, Agra, India Email: vinayjadon@gmail.com
*Corresponding Author

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ABSTRACT:

This paper deals with a Mathematical model of blood flow describing the heat transfer through porous medium in presence of Magnetic field. Blood flow plays an important role in determining the effectiveness of thermal therapy, in this paper we have analyzed magnetic effect on it. Transportation of particles and interconnected voids that contains either arterial or venous blood through porous media has significant applications of biomedical systems such as biological tissues which include flow, heat and mass transfer through porous media with effect of magnetic field. In this paper we have investigated the effect of porosity on Nusselt number and skin friction stress we observed that skin friction τ decreases due to increase in M (Hartman number) and skin friction increase due to increase in pososity and Grashoff number. We also investigated the effect of Magnetic field on Nu and skin friction effect of shear stress on Nu. In this paper we have considered that the blood flows in a exactly cylindrical veins or artery. The results and effect of parameter are discussed with the help of graphs.

Key Words: Nusselt number, Grashoff number Magnetic field, Hartman number, Shear stress skin friction, Porosity.

BACKGROUND :

The subject of Magneto-hydrodynamic attracted the attention of many research workers in view of its applications to Astrophysics, Geophysics and Engineering In recent years; there has been some theoretical and experimental work on the stratified laminar flow of two immiscible liquids in a horizontal pipe. The interest in these types of problems stems from the possibility of reducing the power required to pump oil in a pipe line by suitable addition of water. Magnetic fields of moderate to high intensity can be experienced by human body. Many medical diagnostics tools being used in diagnosing cardiovascular diseases and make use of magnetic fields. When stationary transverse magnetic field is applied externally to a moving electrically conducting fluid, electrical currents are induced in the fluid. The interaction between these induced currents and applied magnetic field produces force known as Lorentz force which tends to retard the flow of conducting fluid. Low intensity magnetic energy has been employed for treating chronic pain, secondary to tissue ischemia and slow healing and non healing ulcers. During the exposure to low frequency electromagnetic field current flow appears to be necessary to measure cardiovascular effects to occur. The interaction of magnetic field with electric current to produce the pressure of the body force in Stokes problem for the discussion of the motion of the fluids in classical hydrodynamics. In arc welding and medical application such as MRI are examples of stronger fields exposures while using such magnetic instrument devices are sold to patients for general use static magnetic field generated by permanent magnet. Magnetic fields have been shown to have positive effects on numerous human systems.

The study of magneto hydrodynamic flows through porous medium has been studied by several authors. Liquid carriers as magnetic particles suspended in flowing blood serve as drug carriers of serious diseases to the diseased site. This type of energy affects biological processes not through heat production but through electrically induced charges in the environment of cells within the organisms.

It is known from magneto hydrodynamics that when magnetic field is applied to a moving electrically conducting fluid, electrical currents are induced in the fluid. The interaction between induced currents and applied magnetic field produces Lorentz force as an external force which retards the blood flow. During Thermal therapy a Magnetic field generated according as Lorentz force.

In recent years, the flow of fluids through porous media has become an important topic. The study of flow of an electrically conducting fluid has many applications in engineering problems such as Magneto Hydrodynamics (MHD), plasma studies and the boundary layer control in the field of aerodynamics [1]. In the past few years, several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of Magneto Hydrodynamics (MHD).

The study of the motion of Newtonian fluids in the presence of a magnetic field has applications in many areas including the handling of biological fluids, plasma and blood [14,2,3]. Raptis *et al*. (1982) [4] have analyzed hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss *et al*. (1995) [5] have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field.

Vinay Jadon et.al. (2007) [6] have studied the effects of mass transfer on unsteady MHD flow through porous medium and heat transfer past a porous vertical moving plate. In this study we consider the problem of Bimal Kumar et.al. (2010) [7]. The aim of present investigation. Is to study the effect of M on velocity distribution and, the effect of M on skin friction and heat transfer rate.

A porous medium basically consists of a bed of many relatively closely packed particles or some other form of solid matrix which remains at rest and through which a fluid flows. Porous media can be characterized by their specific surface (s) and porosity (ε), respectively defined as:

$$S = \frac{\text{Total interface area}}{\text{Total volume}}$$

$$\varepsilon = \frac{\text{Void volume}}{\text{Total volume}}$$

Different examples of pore structure in tissues are depicted. It is well known from flow studies in porous media, where the pore network shows strong similarities with the capillary network that spatial correlation plays an important role for the macroscopic flow behavior. Therefore, in analogy with flow in porous media, the blood flow in the thin vessels is described by macroscopic equations, i.e. by Darcy's law and mass conservation [8].

The development of transport models in porous media have a bearing in the progress of several applications such as transport of macromolecules in aortic media, blood flow through contracting muscles, interstitial fluid flow in axisymmetric soft connective tissue, heat transfer in muscle and skin tissues, thermal therapy applications and others. Biological tissues contain dispersed cells separated by voids. Blood enter these tissues through vessels referred to as arteries and perfuse to the tissue cells via blood capillaries.

Returned blood from the capillaries is accumulated in veins where the blood is pumped back to the heart. Energy transport in tissues is due to thermal conduction, blood perfusion and heat generation, (e.g. metabolic heat generation).

Darcy Flow Model:

The flow model has been initialized to interact between solid and a liquid phase in porous media was first quantified by Darcy (1856) [9]. When the fluid flows through a porous media, the solid particles exert a force on the fluid equal and opposite to the drag force on the solid particles. This force must be balanced by the pressure gradient in the flow i.e., for flow through a control volume for any chosen direction.

In the Darcy model of flow through a porous media, it is assumed that the flow velocities are low and that momentum changes and viscous forces into the fluid are consequently negligible compared to the drag force on the particles. In such flows, the drag force on a body is proportional to the velocity over the body and to the viscosity of the fluid.

$$u = \frac{\mathrm{KG}}{\eta}$$

u	=	Velocity of blood
G	=	Pressure gradient
η	=	Viscosity of blood
K	=	Porous parameter

This derivation reveals that Darcy's law neglects the friction within the fluid and exchange of momentum between the fluid and solid phases. Therefore, Darcy's law has been widely used in the analysis of interstitial fluid flow.

The real application of the porous media models and Bio-heat transfer in human tissues is relatively recent. Xuan and Roetzel (1997, 1998) [10,11] used the transport through porous media concepts to model the tissue blood system composed mainly of solid particles (tissue cells) and interconnected voids that contain either arterial or venous blood.

Energy Transport and Biological Systems(The Pennes Equation):

The energy transport in a biological system is usually expressed by the Bio-heat equation. The Bio-heat equation developed by Pennes (1948) [12] is one of the earliest models for energy transport in tissues. Pennes assumed that the arterial blood temperature TB is uniform throughout the tissue (Fig. 1), while he considered the vein temperature to be equal to the tissue temperature which is denoted by T at the same point. The equation that Pennes utilized is summarized as follows in its simplest form:

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + C_{pb}W_b(T_b - T) + q_m$$

x =Space coordinate
ρ =Tissue density
C_p = Tissue specific heat
C_{pb} = Blood specific heat
W_b = Blood volumetric perfusion rate
k = Tissue thermal conductivity
q_m = Heat generation within the tissue
T_b = Arterial blood temperature
T = Tissue temperature, respectively

Mathematical formulation: We assume the flow of blood in the porous medium to be Newtonian and its viscosity and density to be constant. The flow of blood in an artery with saturated porous medium is depicted in Fig. 1.

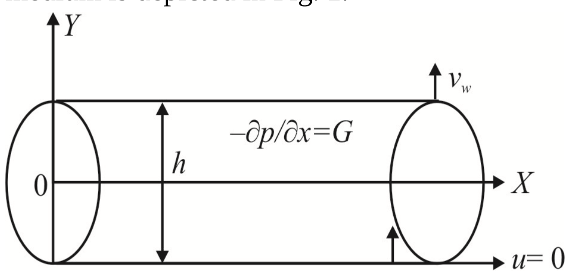


Fig. 1 : Geometry of the problem

We further assume the pressure gradient in x-direction to be constant. We external equation of motion given by Spurk (1997) [13] as:

$$\nu\frac{d^2u}{dy^2}-V_w\frac{du}{dy}=-\frac{G}{\rho}+\frac{\sigma B_0^2u}{\rho} \qquad \ldots (1)$$

Where,

$$u = 0;\ y = 0,\ u = 0;\ y = h \qquad \ldots (2)$$

Effect of Magnetic Field and Spurk Equation:

In order to solve the Eq. 1, we consider the complementary function and particular solution.

Let, $\frac{G}{\rho}=\lambda, \frac{\sigma B_0^2}{\rho}=M$

∴ Equation (1) becomes

$$\nu D^2u-V_wDu=-\lambda+Mu \qquad \ldots(3)$$

$$\Rightarrow \qquad D^2u-\frac{V_w}{\nu}Du-M_1u=-\lambda_1$$

$$\Rightarrow \qquad D^2u - A_1Du - M_1u = -\lambda_1$$

Where $A_1=\frac{V_w}{\nu}, \lambda_1=\frac{-\lambda}{\nu}, M_1=\frac{M}{\nu}$

$$\Rightarrow \qquad (D^2 - A_1D - M_1)u = -\lambda_1 \qquad (4)$$

∴ Solution of Equation (4) is —

$$u = C_1e^{A_2y} + C_2e^{A_3y} + \frac{\lambda_1}{\rho M_1} \qquad (5)$$

Where $A_2 = \frac{A_1 + \sqrt{A_1^2 + 4M_1}}{2}$

$$A_3 = \frac{A_1 - \sqrt{A_1^2 + 4M_1}}{2}$$

Now using boundary conditions

∵ $u = 0;\ y = 0,\ u = 0;\ y = h$

And applying the limit

$$\text{Lim } V_w \to 0$$

$$\Rightarrow A_2 \to \sqrt{M_1}, A_3 \to -\sqrt{M_1}$$

Thus the expression for velocity is given by,

$$\boxed{u(y) = A_4e^{\sqrt{M_1}(y-h)} + A_4e^{-\sqrt{M_1}y} + \frac{\lambda_1}{M_1}}$$

$$A_4 = \frac{\lambda_1(1-e^{\sqrt{M_1}h}}{M_1(e^{\sqrt{M_1}h}-e^{-\sqrt{M_1}h})}$$

Where $\lambda_1 = \frac{\lambda}{\upsilon}$ and $M_1 = \frac{M}{\upsilon}$

∵ The momentum

$$u = -\frac{kG}{\eta}$$

For Newtonian fluid the shear stress is

$$\tau = \eta\left(\frac{du}{dy}\right)$$

$$\Rightarrow \qquad \tau = \frac{KG}{u}\left(\frac{du}{dy}\right)$$

$\Rightarrow$

$$\tau_n=\frac{M_1KG}{(4+2D)}\left\{\begin{matrix}4y+4\sqrt{M_1}y^2-8Ey^2+Gy^2-4\sqrt{M_1}yh+8Eyh-4Gyh-4h^2E \\ +h^2+\sqrt{M_1}h^2-yh\end{matrix}\right\}$$

The expression for stress for the Artery is given as,

at $h = 2r$, $\qquad \tau_{art} = \int(\tau_n)dr$

$\Rightarrow$

$\tau_{art} =$

$$\frac{M_1KG}{(4+2D)}\left\{\begin{matrix}4yr + 4\sqrt{M_1}y^2r - 8Ey^2r + Gy^2r - 4\sqrt{M_1}yr^2 + 8Eyr^2 - 4Gyh - 4Gyr^2 \\ -\frac{16Er^3}{3} + \frac{4r^3}{3} + \frac{4\sqrt{M_1}r^3}{3} - 4r^2\end{matrix}\right\}$$

Where:

$C=\frac{\lambda_1}{M_1}$, $\qquad D=\frac{C}{A_4}$, $\qquad E=\frac{\sqrt{M_1}}{(2+D)}$,

$F=\frac{M_1}{(4+2D)}$, $G=\frac{D\sqrt{M_1}}{(4+2D)}$,

The rate of heat transfer across the artery's wall is given as:

$$Nu = -\frac{\partial\theta}{\partial y}$$

$$\theta = \frac{T - T_0}{T_w - T_0}, \quad Gr = \frac{g\beta\alpha^2(T_w - T_0)}{v_u}, Da = \frac{k}{\lambda^2}$$

∴ From eqn. (12) and (13), we get the expression for the heat transfer rate as:

$$Nu = -\frac{g\beta(T - T_0)}{GrDaGv}\left(\frac{\tau_{art}}{u}\right)$$

Gr (Grashoff number) = 1
ρ (Density of blood) = 1.056 g cm^{-3}
η (Viscosity of blood) = 0.04 dyne cm^{-2}
h (Diameter of artery) = 4 cm

Porous media are usually characterized on the macroscopic level by the introduction of macro parameters like the porosity and the permeability. So we aim to find the relation between microscopic pore structures and the macroscopic pore parameters (porosity, viscosity and shear stress). This study investigates the effect of porous parameter on shear stress and heat transfer rate of the blood flow in an artery filled with porous medium. From the study, we observe that as we increase porous parameter the wall shear stress and heat transfer rate also increases. Also as we increase the wall shear stress heat transfer rate also increases.

Nomenclature:

u = Velocity of blood
N = Pressure gradient
V_W = Normal velocity component at the wall of artery
h = Diameter of the artery
r = Radius of the artery
Nu = Heat transfer rate
F = Fluid temperature
g = Gravitational force
T_0 = Fluid temperature at the inner wall
T_w = Fluid temperature at the upper wall
Gr = Grashoff number
Da = Darcy number
e = ($= -du\, dy^{-1}$) Strain rate
v = Kinematics viscosity of blood
η = Viscosity of blood
ρ = Density of blood
τ = Shear stress
β = Coefficient of volume expansion due to temperature
θ = Non dimensional temperature

The Analysis:

The velocity distribution of boundary layer flow plotted against *y* in figure-(1) for Hartmann number M and It is observed that the fluid velocity decreases due to increasing Hartmann number *M*.

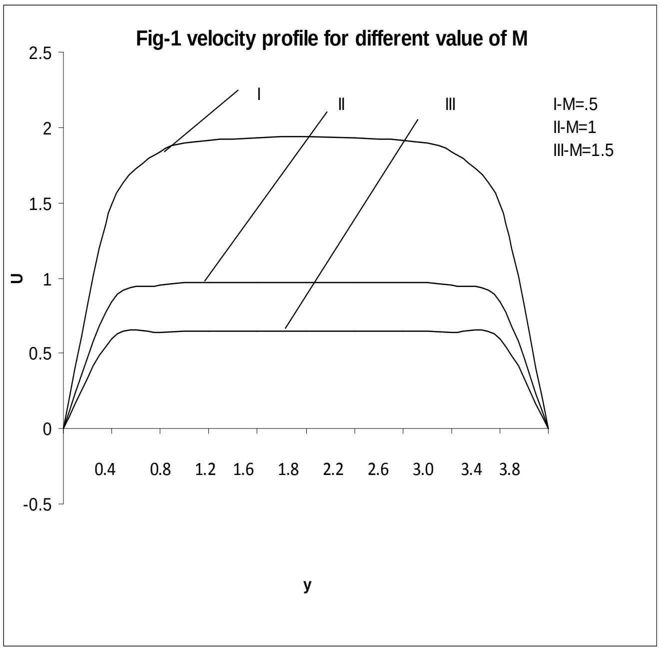
The shear stress plotted against *y* in figure-(2) for Hartmann number *M* and It is observed that the shear stress decreases due to increasing Hartmann number *M*,

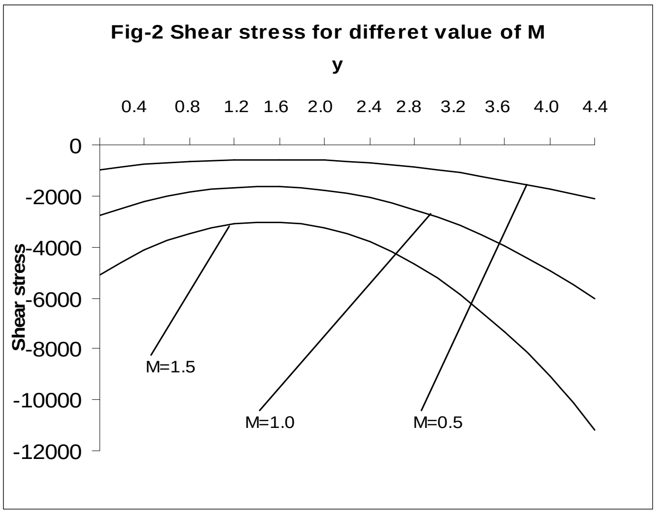
The rate of heat transfer plotted in figure-(3) for Hartman number M and It is observed that the heat transfer decreases due to increasing Hartmann number M,

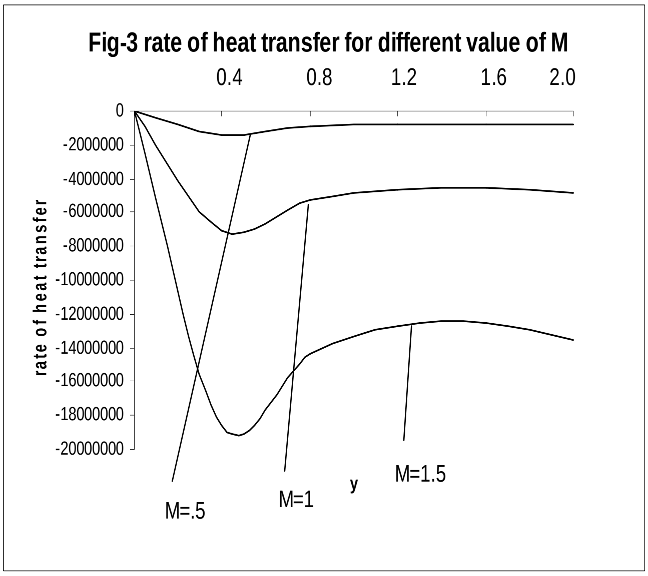
CONCLUSION :

In this study, the results can be used in wide range of application such as thermal simulations within the brain, hypothermic sessions, heat transfer in muscle and skin tissues and thermal therapy applications. The human body is made up of cells and organs that are very sensitive to changes in temperature. While the outer surface of human skin can adapt and tolerate rather large changes in skin temperature, the human internal organs cannot. That is why, the human body has a very sophisticated heat regulating system that allows human internal body temperature to stay at a fined definite. 98.6°F. Once human internal temperature begins to changes from 98.6°F, the body reacts to counter the heat gain or loss.

Figures:







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