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ON SOFT MULTISETS THEORY

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ABSTRACT:

As a generalization of Molodtsov's soft set theory, Alkhazaleh, Salleh and Hassan introduced Soft Multiset Theory. But the complement of a soft multiset initiated by Alkhazaleh, Salleh and Hassan do not satisfy the axioms of contradiction and exclusion, whereas Neog and Sut have already proved that soft sets follow the axioms of contradiction and exclusion. In what follows, soft multisets with the notion of complement initiated by Alkhazaleh, Salleh and Hassan cannot be considered as generalization of soft sets. In this paper, we reintroduce the notion of complement of a soft multiset and come to the conclusion that the laws of exclusion and contradiction, Involution and De Morgan laws are satisfied by soft multisets if we define complement in our way. We justify our claim with proof, examples and counter examples. Our work is an endeavor for realization of soft multisets as generalized soft sets in actual practice.

Keywords: Soft set, Soft multiset, Complement of a soft multiset.

[I] NTRODUCTION

Many complicated problems in the fields of engineering, social science, economics, medical science etc. which involve uncertainties, cannot be dealt with the available important theories such as Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. Molodtsov [5] pointed out that these theories have certain limitations. While trying to remove those limitations, he introduced the notion of Soft Sets and established the fundamental results of the new theory. In 2003, P.K.Maji, R.Biswas and A.R.Roy [4] studied further the theory of soft sets and put forward some results. However, the axioms of exclusion and contradiction are not valid under the definition of complement of a soft set initiated by Maji et al. [4]. In this regard, Neog and Sut [6] reintroduced the concept of complement of a soft set and showed that the laws of exclusion and contradiction, Involution, De Morgan Inclusions and De Morgan laws are valid for soft sets with respect to the new definition of complement. As a generalization of Molodtsov's soft set [5], Alkhazaleh et al. [3] introduced Soft Multiset but with the same limitation of not satisfying the laws of contradiction and exclusion by its complement. In the present work, we have reintroduced the concept of complement of a soft multiset and showed that the laws of exclusion and contradiction, Involution and De Morgan laws are valid for soft multiset in our way. We justify our claim with proof, examples and counter examples.

[II] PRELIMINARIES

We first recall some basic notions related to soft set and soft multiset. Let *U* be an initial universe, and *E* be the set of all possible parameters under consideration with respect to *U*. The set of all subsets of *U*, *i.e.* the power set of *U* is denoted by *P*(*U*) and *A* is a subset of *E*.

Definition 2.1 [5]

A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.

In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε- approximate elements of the soft set.

Definition 2.2 [4]

Union of two soft sets (*F*, *A*) and (*G*, *B*) over a common universe *U*, is the soft set (*H*, *C*), where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F,A) \tilde{\cup} (G,B) = (H,C)$.

Definition 2.3 [4]

Intersection of two soft sets (*F*, *A*) and (*G*, *B*) over a common universe *U*, is the soft set (*H*, *C*), where

$C = A \cap B$ and $\forall \varepsilon \in C$,

$H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$

(as both are same set) and is written as $(F,A) \tilde{\cap} (G,B) = (H,C)$.

Pei and Miao [7] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, Ahmad and Kharal [1] proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows.

Definition 2.4 [1]

Let (*F*, *A*) and (*G*, *B*) be two soft sets over a common universe *U* with $A \cap B \neq \phi$. Then Intersection of two soft sets (*F*, *A*) and (*G*, *B*) is a soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$,

$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.

We write

$(F,A) \tilde{\cap} (G,B) = (H,C)$.

In [2], Ali et al. put forward a new operation "extended intersection" of two soft sets in the following way –

Definition 2.5 [2]

The extended intersection of two soft sets (*F*, *A*) and (*G*, *B*) over a common universe *U* is the soft set (*H*, *C*) where *C* = *A*∪*B*, and ∀ε ∈ *C*,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cap G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

We write $(F,A)\ \Pi_g (G,B) = (H,C)$.

Definition 2.6 [4]

The complement of a soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F,A)^c = (F^c, \rceil A)$, where $F^c: \rceil A \to P(U)$ is a mapping given by $F^c(\sigma) = U - F(\neg\sigma)$ for all $\sigma \in \rceil A$.

Definition 2.7 [6]

The complement of a soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F,A)^c = (F^c, A)$, where $F^c: A \to P(U)$ is a mapping given by $F^c(\varepsilon) = [F(\varepsilon)]^c$ for all $\varepsilon \in A$.

Alkhazaleh et al. [3] defined soft multisets in the following way -

Definition 2.8 [3]

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \varphi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i , $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (*F*, *A*) is called a soft multiset over U , where *F* is a mapping given by

$F : A \to U$.

In other words, a soft multiset over *U* is a parameterized family of subsets of *U*. For $\varepsilon \subset E,\ F(\varepsilon)$ may be considered as the set of ε - approximate elements of the soft multiset (*F*, *A*).

Definition 2.9 [3]

For any soft multiset (*F*, *A*) , a pair $\left(e_{U_i,j}, F_{e_{U_i,j}}\right)$ is called a $U_i -$ soft multiset part $\forall\, e_{U_i,j} \in a_k$ and $F_{e_{U_i,j}} \subseteq F(A)$ is an approximate value set, where $a_k \in A$, $k = \{1, 2, 3, ..., n\}$, $i \in \{1, 2, 3, ...,m\}$ and $j \in \{1, 2, 3, ..., r\}$.

Definition 2.10 [3]

For two soft multisets (*F*, *A*) and (*G*, *B*) over *U*, (*F*, *A*) is called a soft multi subset of (*G*, *B*) if

1. $A \subseteq B$ and
2. $\forall\, e_{U_i,j} \in a_k$, $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) \tilde{\subseteq} \left(e_{U_i,j}, G_{e_{U_i,j}}\right)$

where $a_k \in A$, $k = \{1, 2, 3, ..., n\}$, $i \in \{1, 2, 3, ...,m\}$ and $j \in \{1, 2, 3, ..., r\}$. This relationship is denoted by $(F, A) \tilde{\subseteq} (G, B)$. In this case (*G*, *B*) is called a soft multi superset of (*F*, *A*).

Definition 2.11 [3]

A soft multiset (*F*, *A*) over *U* is called a semi-null soft multiset denoted by $(F, A)_{\approx\varphi_i}$, if at least one of a soft multiset parts of (*F*, *A*) equals φ.

Definition 2.12 [3]

A soft multiset (*F*, *A*) over *U* is called a null soft multiset denoted by $(F, A)_\varphi$, if all the soft multiset parts of (*F*, *A*) equals φ.

Definition 2.13 [3]

A soft multiset (*F*, *A*) over *U* is called a semi-absolute soft multiset denoted by $(F, A)_{\approx A_i}$ if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = U_i$ for at least one *i*, $a_k \in A$, $k = \{1, 2, 3, ..., n\}$, $i \in \{1, 2, 3, ...,m\}$ and $j \in \{1, 2, 3, ..., r\}$.

Definition 2.14 [3]

A soft multiset (*F*, *A*) over *U* is called an absolute soft multiset denoted by $(F, A)_A$ if $\left(e_{U_i,j}, F_{e_{U_i,j}}\right) = U_i\ \forall\, i$.

Definition 2.15 [3]

The union of two soft multisets (*F*, *A*) and (*G*, *B*) over *U* denoted by $(F, A) \tilde{\cup} (G, B)$ is the soft multiset (*H*, *C*) where

$C = A \cup B$ and

$$\forall \varepsilon \subset C, H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \subset \text{A - B} \\ G(\varepsilon) & \text{if } \varepsilon \subset \text{B - A} \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \subset \text{A} \cap \text{B} \end{cases}$$

Definition 2.16 [3]

The intersection of two soft multisets (*F*, *A*) and (*G*, *B*) over *U* denoted by $(F, A) \tilde{\cap} (G, B)$ is the soft multiset (*H*, *C*) where $C = A \cup B$ and

$$\forall \varepsilon \subset C, H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \subset \text{A - B} \\ G(\varepsilon) & \text{if } \varepsilon \subset \text{B - A} \\ F(\varepsilon) \cap G(\varepsilon) & \text{if } \varepsilon \subset \text{A} \cap \text{B} \end{cases}$$

Definition 2.17 [3]

Let $E = \prod_{i=1}^{m} E_{U_i}$, where E_{U_i} is a set of parameters. The NOT set of *E* denoted by $\neg E$ is defined by $\neg E = \prod_{i=1}^{m} \neg\, E_{U_i}$ where

$\neg\, E_{U_i} = \{\ \neg\, e_{U_i,j} = \text{not}\, e_{U_i,j} \forall i, j\}$

Definition 2.18 [3]

The complement of a soft multiset (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c$ = (*F*

^{c}, $\neg A$), where F^{c}: $\neg A \rightarrow U$ is a mapping given by $F^{c}(\alpha) = U - F(\neg\alpha)$ for all $\alpha \in \neg A$.

[III] ILLUSTRATING EXAMPLE

We consider **Example 3.2 of [3]**.

Example 3.1 [3]:

Suppose that there are three universes U_1, U_2 and U_3. Let us consider a soft multiset (F, A) which describes the "attractiveness of houses", "cars" and "hotels" that Mr. *X* is considering for accommodation purchase, transportation purchase and venue to hold a wedding celebration respectively.

Let $U_1 = \{h_1, h_2, h_3, h_4, h_5, h_6\}$,
$U_2 = \{c_1, c_2, c_3, c_4, c_5\}$ and $U_3 = \{v_1, v_2, v_3, v_4\}$.

Let $E_U = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$E_{U_1} = \{e_{U_1,1} = \text{expensive}, e_{U_1,2} = \text{cheap}$,
$e_{U_1,3} = \text{beautiful}, e_{U_1,4} = \text{wooden}$,
$e_{U_1,5} = \text{in green surroundings}\}$

$E_{U_2} = \{e_{U_2,1} = \text{expensive}, e_{U_2,2} = \text{cheap}$,
$e_{U_2,3} = \text{Model 2000 and above}$,
$e_{U_2,4} = \text{Black}$,
$e_{U_2,5} = \text{Made in Japan}$,
$e_{U_2,5} = \text{Made in Malaysia}\}$

$E_{U_3} = \{e_{U_3,1} = \text{expensive}, e_{U_3,2} = \text{cheap}$,
$e_{U_3,3} = \text{majestic}$,
$e_{U_3,4} = \text{in Kuala Lumpur}$,
$e_{U_3,4} = \text{in Kajang}\}$

Let $U = \prod_{i=1}^{3} P(U_i), E = \prod_{i=1}^{3} E_U$ and $A \subseteq E$ such that

$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1})$,
$a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1})$,
$a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1})$,
$a_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2})$, $a_5 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3})$,
$a_6 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2})$,
$a_7 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})$, $a_8 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}$

Suppose that

$F(a_1) = (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})$,
$F(a_2) = (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\})$
$F(a_3) = (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \varphi)$,
$F(a_4) = (\{h_1, h_4, h_6\}, \varphi, \{v_1, v_4\})$
$F(a_5) = (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})$,
$F(a_6) = (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_3)$
$F(a_7) = (\{h_1, h_4\}, \varphi, \{v_3\})$,
$F(a_8) = (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\})$

Then we can view the soft multiset (F, A) as consisting of the following collection of approximations.

$(F, A) = \{(a_1, (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\}))$,
$(a_2, (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\}))$,
$(a_3, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \varphi))$,
$(a_4, (\{h_1, h_4, h_6\}, \varphi, \{v_1, v_4\}))$,
$(a_5, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\}))$,
$(a_6, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_3))$,
$(a_7, (\{h_1, h_4\}, \varphi, \{v_3\}))$,
$(a_8, (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\}))\}$

The complement of the soft multiset (F, A) is given by

$(F, A)^c$
$= (F^c, \neg A)$
$= \{(\neg a_1, (\{h_1, h_4, h_5\}, \{c_1, c_3, c_4, c_5\}, \{v_1, v_4\}))$,
$(\neg a_2, (\{h_1, h_4, h_5\}, \{c_2\}, \{v_1, v_3, v_4\}))$,
$(\neg a_3, (\{h_2, h_3, h_6\}, \{c_2, c_4, c_5\}, U_3))$,
$(\neg a_4, (\{h_2, h_3, h_5\}, U_2, \{v_2, v_3\}))$,

$(\neg a_5,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(\neg a_6,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi)),$
$(\neg a_7,(\{h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\})),$
$(\neg a_8,(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}))\}$

Let us now try to find out $(F,A)\,\tilde{\cap}\,(F,A)^c$

We have

$(F,A)\,\tilde{\cap}\,(F,A)^c$
$=(F,A)\,\tilde{\cap}\,(F^c,\neg A)$
$=(H,C)$ where $C=A\cup\neg A$
$=\{(a_1,(\{h_2,h_3,h_6\},\{c_2\},\{v_2,v_3\})),$
$(a_2,(\{h_2,h_3,h_6\},\{c_1,c_3,c_4,c_5\},\{v_2\})),$
$(a_3,(\{h_1,h_4,h_5\},\{c_1,c_3\},\varphi)),$
$(a_4,(\{h_1,h_4,h_6\},\varphi,\{v_1,v_4\})),$
$(a_5,(\{h_1,h_4\},\{c_1,c_3\},\{v_1\})),$
$(a_6,(\{h_1,h_4,h_5\},\{c_1,c_3\},U_3)),$
$(a_7,(\{h_1,h_4\},\varphi,\{v_3\})),$
$(a_8,(\{h_2,h_3,h_6\},\{c_1,c_3\},\{v_1,v_4\})),$
$(\neg a_1,(\{h_1,h_4,h_5\},\{c_1,c_3,c_4,c_5\},\{v_1,v_4\})),$
$(\neg a_2,(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\})),$
$(\neg a_3,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},U_3)),$
$(\neg a_4,(\{h_2,h_3,h_5\},U_2,\{v_2,v_3\})),$
$(\neg a_5,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(\neg a_6,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi)),$
$(\neg a_7,(\{h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\})),$
$(\neg a_8,(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}))\}$

Also

$(F,A)\,\tilde{\cup}\,(F,A)^c$
$=(F,A)\,\tilde{\cup}\,(F^c,\neg A)$
$=(H,C)$ where $C=A\cup\neg A$
$=\{(a_1,(\{h_2,h_3,h_6\},\{c_2\},\{v_2,v_3\})),$
$(a_2,(\{h_2,h_3,h_6\},\{c_1,c_3,c_4,c_5\},\{v_2\})),$
$(a_3,(\{h_1,h_4,h_5\},\{c_1,c_3\},\varphi)),$
$(a_4,(\{h_1,h_4,h_6\},\varphi,\{v_1,v_4\})),$
$(a_5,(\{h_1,h_4\},\{c_1,c_3\},\{v_1\})),$
$(a_6,(\{h_1,h_4,h_5\},\{c_1,c_3\},U_3)),$
$(a_7,(\{h_1,h_4\},\varphi,\{v_3\})),$
$(a_8,(\{h_2,h_3,h_6\},\{c_1,c_3\},\{v_1,v_4\})),$
$(\neg a_1,(\{h_1,h_4,h_5\},\{c_1,c_3,c_4,c_5\},\{v_1,v_4\})),$
$(\neg a_2,(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\})),$
$(\neg a_3,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},U_3)),$
$(\neg a_4,(\{h_2,h_3,h_5\},U_2,\{v_2,v_3\})),$
$(\neg a_5,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(\neg a_6,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi)),$
$(\neg a_7,(\{h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\})),$
$(\neg a_8,(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}))\}$

We see that

$(F,A)\,\tilde{\cup}\,(F,A)^c=(F,A)\,\tilde{\cap}\,(F,A)^c$ and

neither $(F,A)\,\tilde{\cup}\,(F,A)^c=(F,A)_A$

nor $(F,A)\,\tilde{\cap}\,(F,A)^c=(F,A)_\varphi$

[IV] COMPLEMENT OF A SOFT MULTISET REDEFINED

Definition 4.1

The complement of a soft multiset (F, A) is denoted by $(F, A)^c$ and is defined by $(F,A)^c=(F^c, A)$, where $F^c: A\to U$ is a mapping given by

$F^c(\alpha)=U-F(\alpha)=[F(\alpha)]^c$

for all $\alpha\in A$.

Example 4.1

We consider the soft multiset (F,A) given in **Example 3.1**. If we use the definition of complement in our way, the complement of (F,A) is given by

$(F,A)^c$
$=(F^c,A)$
$=\{(a_1,(\{h_1,h_4,h_5\},\{c_1,c_3,c_4,c_5\},\{v_1,v_4\})),$
$(a_2,(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\})),$
$(a_3,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},U_3)),$

$(a_4, (\{h_2, h_3, h_5\}, U_2, \{v_2, v_3\})),$
$(a_5, (\{h_2, h_3, h_5, h_6\}, \{c_2, c_4, c_5\}, \{v_2, v_3, v_4\})),$
$(a_6, (\{h_2, h_3, h_6\}, \{c_2, c_4, c_5\}, \varphi)),$
$(a_7, (\{h_2, h_3, h_5, h_6\}, U_2, \{v_1, v_2, v_4\})),$
$(a_8, (\{h_1, h_4, h_5\}, \{c_2, c_4, c_5\}, \{v_2, v_3\}))\}$

Let us now find out $(F, A) \tilde{\cap} (F, A)^c$

We have

$$
\begin{aligned}
&(F, A) \tilde{\cap} (F, A)^c \\
&= (F, A) \tilde{\cap} (F^c, A) \\
&= (H, A) \\
&= \{(a_1, (\varphi, \varphi, \varphi)), (a_2, (\varphi, \varphi, \varphi)), (a_3, (\varphi, \varphi, \varphi)), \\
&(a_4, (\varphi, \varphi, \varphi)), (a_5, (\varphi, \varphi, \varphi)), (a_6, (\varphi, \varphi, \varphi)), \\
&(a_7, (\varphi, \varphi, \varphi)), (a_8, (\varphi, \varphi, \varphi))\} \\
&= (F, A)_\varphi
\end{aligned}
$$

Also

$$
\begin{aligned}
&(F, A) \tilde{\cup} (F, A)^c \\
&= (F, A) \tilde{\cup} (F^c, A) \\
&= (H, A) \\
&= \{(a_1, (U_1, U_2, U_3)), (a_2, (U_1, U_2, U_3)), \\
&(a_3, (U_1, U_2, U_3)), (a_4, (U_1, U_2, U_3)), \\
&(a_5, (U_1, U_2, U_3)), (a_6, (U_1, U_2, U_3)), \\
&(a_7, (U_1, U_2, U_3)), (a_8, (U_1, U_2, U_3))\} \\
&= (F, A)_A
\end{aligned}
$$

We thus arrive at the following two axioms of contradiction and exclusion of soft multisets which are not satisfied under the definition of complement initiated in [3].

Proposition 4.1

(*i*) $(F, A) \tilde{\cup} (F, A)^c = (F, A)_A$
(Exclusion)

(*ii*) $(F, A) \tilde{\cap} (F, A)^c = (F, A)_\varphi$
(Contradiction)

Proof: The proof is straight forward and follows from definition.

One can verify that the following properties initiated in [3] are also satisfied by our definition of complement of a soft multiset.

(*i*) $((F, A)^c)^c = (F, A)$

(*ii*) $(F, A)^c{}_{\approx\varphi_i} = (F, A)_{\approx A_i}$

(*iii*) $(F, A)^c{}_\varphi = (F, A)_A$

(*iv*) $(F, A)^c{}_{\approx A_i} = (F, A)_{\approx\varphi_i}$

(*v*) $(F, A)^c{}_A = (F, A)_\varphi$

Proposition 4.2 (De Morgan Laws)

Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \varphi$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i, $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$. Let (*F*, *A*) and (*G*, *B*) be two soft multisets over *U*. Then we have the following -

(*i*) $(F, A)^c \tilde{\cap} (G, B)^c = ((F, A) \tilde{\cup} (G, B))^c$

(*ii*) $((F, A) \tilde{\cap} (G, B))^c = (F, A)^c \tilde{\cup} (G, B)^c$

Proof

(*i*) Let $(F, A) \tilde{\cup} (G, B) = (H, C)$,

where $C = A \cup B$ and

$$
\forall \varepsilon \subset C, H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \subset \text{A - B} \\ G(\varepsilon) & \text{if } \varepsilon \subset \text{B - A} \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \subset \text{A} \cap \text{B} \end{cases}
$$

Thus

$((F, A) \tilde{\cup} (G, B))^c = (H, C)^c = (H^c, C)$,

where $C = A \cup B$ and

$$
\begin{aligned}
\forall \varepsilon \subset C, H^c(\varepsilon) &= (H(\varepsilon))^c \\
&= \begin{cases} (F(\varepsilon))^c & \text{if } \varepsilon \subset \text{A - B} \\ (G(\varepsilon))^c & \text{if } \varepsilon \subset \text{B - A} \\ (F(\varepsilon) \cup G(\varepsilon))^c & \text{if } \varepsilon \subset \text{A} \cap \text{B} \end{cases}
\end{aligned}
$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A-B} \\ G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{B-A} \\ (F(\varepsilon))^c \cap (G(\varepsilon))^c & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A-B} \\ G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{B-A} \\ F^c(\varepsilon) \cap G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

Again,

$(F, A)^c \tilde{\cap} (G, B)^c = (F^c, A) \tilde{\cap} (G^c, B)$

$= (I, J)$, say

Where $J = A \cup B$ and $\forall \varepsilon \subset J, I(\varepsilon)$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A-B} \\ G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{B-A} \\ F^c(\varepsilon) \cap G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

$\forall \varepsilon \subset J, I(\varepsilon) \quad = F^c(\varepsilon) \cap G^c(\varepsilon)$

We see that

$J = C$ and $\forall \varepsilon \subset J, I(\varepsilon) = H^c(\varepsilon)$

Thus

$(F, A)^c \tilde{\cap} (G, B)^c = ((F, A) \tilde{\cup} (G, B))^c$

(*ii*) Let $(F, A) \tilde{\cap} (G, B) = (H, C)$, where $C = A \cup B$ and

and

$$\forall \varepsilon \subset C, H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A-B} \\ G(\varepsilon) & \text{if } \varepsilon \subset \mathrm{B-A} \\ F(\varepsilon) \cap G(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

Thus $((F, A) \tilde{\cap} (G, B))^c = (H, C)^c = (H^c, C)$,

Where $C = A \cup B$ and

$\forall \varepsilon \subset C, H^c(\varepsilon) \quad = (H(\varepsilon))^c$

$$= \begin{cases} (F(\varepsilon))^c & \text{if } \varepsilon \subset \mathrm{A-B} \\ (G(\varepsilon))^c & \text{if } \varepsilon \subset \mathrm{B-A} \\ (F(\varepsilon) \cap G(\varepsilon))^c & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A-B} \\ G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{B-A} \\ (F(\varepsilon))^c \cup (G(\varepsilon))^c & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A-B} \\ G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{B-A} \\ F^c(\varepsilon) \cup G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

Again,

$(F, A)^c \tilde{\cup} (G, B)^c = (F^c, A) \tilde{\cup} (G^c, B)$

$= (I, J)$, say

Where $J = A \cup B$ and $\forall \varepsilon \subset J, I(\varepsilon)$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A-B} \\ G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{B-A} \\ F^c(\varepsilon) \cup G^c(\varepsilon) & \text{if } \varepsilon \subset \mathrm{A \cap B} \end{cases}$$

We see that

$C = J$ and $\forall \varepsilon \subset C, H^c(\varepsilon) = I(\varepsilon)$

Thus $((F, A) \tilde{\cap} (G, B))^c = (F, A)^c \tilde{\cup} (G, B)^c$

Remark 4.1

For two soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*, we have the following De Morgan inclusions when we use the definition of intersection of two soft sets put forward in [1].

(*i*) $(F, A)^c \tilde{\cap} (G, B)^c \tilde{\subseteq} ((F, A) \tilde{\cup} (G, B))^c$

(*ii*) $((F, A) \tilde{\cap} (G, B))^c \tilde{\subseteq} (F, A)^c \tilde{\cup} (G, B)^c$

One can verify that the reverse inclusions are valid if we use the notion of extended intersection initiated by Ali et al. in [2].

If we take *A* = *B* and use the definition of intersection of soft sets initiated in [1] then the reverse inclusions are valid and we get the following De Morgan Laws for soft sets (*F*, *A*) and (*G*, *A*) over *U*.

(*i*) $((F, A) \tilde{\cup} (G, A))^c = (F, A)^c \tilde{\cap} (G, A)^c$

(*ii*) $((F, A) \tilde{\cap} (G, A))^c = (F, A)^c \tilde{\cup} (G, A)^c$

Example 4.2
We consider **Example 3.7 considered in [3]**. Suppose that there are three universes U_1, U_2 and U_3. Let us consider a soft multiset (F, A) which describes the "attractiveness of houses", "cars" and "hotels" that Mr. *X* is considering for accommodation purchase, transportation purchase and venue to hold a wedding celebration respectively and another soft multiset (G, B) which describes the "attractiveness of houses", "cars" and "hotels" that Mr. *Y* is considering for accommodation purchase, transportation purchase and venue to hold a wedding celebration respectively.

Let $U_1 = \{h_1, h_2, h_3, h_4, h_5, h_6\}$,
$U_2 = \{c_1, c_2, c_3, c_4, c_5\}$ and $U_3 = \{v_1, v_2, v_3, v_4\}$.
Let $E_U = \{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$E_{U_1} = \{e_{U_1,1} = \text{expensive}, \ e_{U_1,2} = \text{cheap},$
$e_{U_1,3} = \text{beautiful}, \ e_{U_1,4} = \text{wooden},$
$e_{U_1,5} = \text{in green surroundings}\}$
$E_{U_2} = \{e_{U_2,1} = \text{expensive}, \ e_{U_2,2} = \text{cheap},$
$e_{U_2,3} = \text{Model 2000 and above},$
$e_{U_2,4} = \text{Black}, e_{U_2,5} = \text{Made in Japan},$
$e_{U_2,5} = \text{Made in Malaysia}\}$
$E_{U_3} = \{e_{U_3,1} = \text{expensive}, \ e_{U_3,2} = \text{cheap},$
$e_{U_3,3} = \text{majestic},$
$e_{U_3,4} = \text{in Kuala Lumpur},$
$e_{U_3,4} = \text{in Kajang}\}$

Let $U = \prod_{i=1}^{3} P(U_i), E = \prod_{i=1}^{3} E_U$ and $A, B \subseteq E$ such that

$A = \{a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),$
$a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),$
$a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}),$
$a_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})\}$

and

$B = \{b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),$
$b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),$
$b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),$
$b_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}),$
$b_5 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}),$
$b_6 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}),$
$b_7 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}),$
$b_8 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}$

Here

$K = A \cup B$
$= \{k_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), k_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),$
$k_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}), \ k_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}),$
$k_5 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), k_6 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}),$
$k_7 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}), k_8 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}$

We consider the soft multisets (F, A) and (G, B) as follows -

(F, A)
$= \{(a_1, (\{h_2, h_3\}, \{c_2\}, \{v_2\})), (a_2, (\{h_1, h_5\}, \{c_1, c_3\}, \varphi)),$
$(a_3, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})), (a_4, (\{h_4\}, \varphi, \{v_3\}))$

and

$(G, B) = \{(b_1, (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})),$
$(b_2, (\{h_2, h_3, h_6\}, \{c_1, c_3, c_4, c_5\}, \{v_2\})),$
$(b_3, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, \varphi)),$
$(b_4, (\{h_1, h_4, h_6\}, \varphi, \{v_1, v_4\})),$
$(b_5, (\{h_1, h_4\}, \{c_1, c_3\}, \{v_1\})),$
$(b_6, (\{h_1, h_4, h_5\}, \{c_1, c_3\}, U_3)),$
$(b_7, (\{h_1, h_4\}, \varphi, \{v_3\})),$
$(b_8, (\{h_2, h_3, h_6\}, \{c_1, c_3\}, \{v_1, v_4\}))\}$

$(F, A) \breve{\cup} (G, B) = (H, K)$
$= \{(k_1, (\{h_2, h_3, h_6\}, \{c_2\}, \{v_2, v_3\})),$

$(k_2,(\{h_1,h_4,h_5\},\{c_1,c_3\},\varphi)),$
$(k_3,(\{h_1,h_4\},\{c_1,c_3\},\{v_1\})),$
$(k_4,(\{h_1,h_4\},\varphi,\{v_3\})),$
$(k_5,(\{h_2,h_3,h_6\},\{c_1,c_3,c_4,c_5\},\{v_2\})),$
$(k_6,(\{h_1,h_4,h_6\},\varphi,\{v_1,v_4\})),$
$(k_7,(\{h_1,h_4,h_5\},\{c_1,c_3\},U_3)),$
$(k_8,(\{h_2,h_3,h_6\},\{c_1,c_3\},\{v_1,v_4\}))\}$
--------- (4.2.1)

$((F,A)\tilde{\cup}(G,B))^c=(H,K)^c$
$=(H^c,K)$
$=\{(k_1,(\{h_1,h_4,h_5\},\{c_1,c_3,c_4,c_5\},\{v_1,v_4\})),$
$(k_2,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},U_3)),$
$(k_3,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(k_4,(\{h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\})),$
$(k_5,(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\})),$
$(k_6,(\{h_2,h_3,h_5\},U_2,\{v_2,v_3\})),$
$(k_7,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi)),$
$(k_8,(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}))\}$
--------- (4.2.2)

$(F,A)\tilde{\cap}(G,B)=(H,K)$
$=\{(k_1,(\{h_2,h_3\},\{c_2\},\{v_2\})),(k_2,(\{h_1,h_5\},\{c_1,c_3\},\varphi)),$
$(k_3,(\{h_1,h_4\},\{c_1,c_3\},\{v_1\})),$
$(k_4,(\{h_4\},\varphi,\{v_3\})),$
$(k_5,(\{h_2,h_3,h_6\},\{c_1,c_3,c_4,c_5\},\{v_2\})),$
$(k_6,(\{h_1,h_4,h_6\},\varphi,\{v_1,v_4\})),$
$(k_7,(\{h_1,h_4,h_5\},\{c_1,c_3\},U_3)),$
$(k_8,(\{h_2,h_3,h_6\},\{c_1,c_3\},\{v_1,v_4\}))\}$
--------- (4.2.3)

$((F,A)\tilde{\cap}(G,B))^c=(H,K)^c$
$=(H^c,K)$
$=\{(k_1,(\{h_1,h_4,h_5,h_6\},\{c_1,c_3,c_4,c_5\},\{v_1,v_3,v_4\})),$
$(k_2,(\{h_2,h_3,h_4,h_6\},\{c_2,c_4,c_5\},U_3)),$
$(k_3,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(k_4,(\{h_1,h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\})),$
$(k_5,(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\})),$
$(k_6,(\{h_2,h_3,h_5\},U_2,\{v_2,v_3\})),$
$(k_7,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi)),$
$(k_8,(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}))\}$
--------- (4.2.4)

$(F,A)^c$
$=(F^c,A)$
$=\{(a_1,(\{h_1,h_4,h_5,h_6\},\{c_1,c_3,c_4,c_5\},\{v_1,v_3,v_4\})),$
$(a_2,(\{h_2,h_3,h_4,h_6\},\{c_2,c_4,c_5\},U_3)),$
$(a_3,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(a_4,(\{h_1,h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\}))\}$
--------- (4.2.5)

$(G,B)^c$
$=(G^c,B)$
$=\{(b_1,(\{h_1,h_4,h_5\},\{c_1,c_3,c_4,c_5\},\{v_1,v_4\})),$
$(b_2,(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\})),$
$(b_3,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},U_3)),$
$(b_4,(\{h_2,h_3,h_5\},U_2,\{v_2,v_3\})),$
$(b_5,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(b_6,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi)),$
$(b_7,(\{h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\})),$
$(b_8,(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}))\}$
--------- (4.2.6)

$(F,A)^c\tilde{\cup}(G,B)^c=(F^c,A)\tilde{\cup}(G^c,B)$
$=(H,K)$
$=\{(k_1,(\{h_1,h_4,h_5,h_6\},\{c_1,c_3,c_4,c_5\},\{v_1,v_3,v_4\})),$
$(k_2,(\{h_2,h_3,h_4,h_6\},\{c_2,c_4,c_5\},U_3)),$
$(k_3,(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\})),$
$(k_4,(\{h_1,h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\})),$
$(k_5,(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\})),$
$(k_6,(\{h_2,h_3,h_5\},U_2,\{v_2,v_3\})),$
$(k_7,(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi)),$

$\left(k_8,\left(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}\right)\right)\}$

--------- (4.2.7)

$(F,A)^c \tilde{\cap} (G,B)^c = \left(F^c,A\right)\tilde{\cap}(G^c,B)$

$=(H,K)$

$=\{\left(k_1,\left(\{h_1,h_4,h_5\},\{c_1,c_3,c_4,c_5\},\{v_1,v_4\}\right)\right),$

$\left(k_2,\left(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},U_3\right)\right),$

$\left(k_3,\left(\{h_2,h_3,h_5,h_6\},\{c_2,c_4,c_5\},\{v_2,v_3,v_4\}\right)\right),$

$\left(k_4,\left(\{h_2,h_3,h_5,h_6\},U_2,\{v_1,v_2,v_4\}\right)\right),$

$\left(k_5,\left(\{h_1,h_4,h_5\},\{c_2\},\{v_1,v_3,v_4\}\right)\right),$

$\left(k_6,\left(\{h_2,h_3,h_5\},U_2,\{v_2,v_3\}\right)\right),$

$\left(k_7,\left(\{h_2,h_3,h_6\},\{c_2,c_4,c_5\},\varphi\right)\right),$

$\left(k_8,\left(\{h_1,h_4,h_5\},\{c_2,c_4,c_5\},\{v_2,v_3\}\right)\right)\}$

--------- (4.2.8)

From 4.2.4. and 4.2.7 it is obvious that $\left((F,A)\tilde{\cap}(G,B)\right)^c = (F,A)^c \tilde{\cup} (G,B)^c$ and from 4.2.2. and 4.2.8 it is obvious that $(F,A)^c \tilde{\cap} (G,B)^c = \left((F,A)\tilde{\cup}(G,B)\right)^c$

[V] CONCLUSION

In view of our discussion in the preceding sections, we come to the conclusion that soft multisets follow the set theoretic axioms of contradiction and exclusion in addition to all the properties that complement of a set in classical sense does. The notion of complement initiated in [3] does not give us the axioms of contradiction and exclusion. In what follows, soft multisets initiated in [3] cannot be considered as a generalization of soft set theory. It would be logical if we discard the notion of complement initiated in [3] and proceed further in our way so as to realize soft multisets as generalization of soft sets in actual practice.

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