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EFFECTS OF FLOW BEHAVIOR INDICES ON BLOOD FLOW RESISTANCE FOR AN ATHEROSCLEROTIC ARTERY BY ABNORMAL SEGMENTS

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ABSTRACT:

In this paper, the influence of flow behaviour indices on non –Newtonian blood flow in small arteries having multiple stenoses and post-stenotic dilatation has been discussed. The result of varying the flow and geometric properties, three surface response curvilinear maps for the flow resistance ratio are generated for different arterial configurations. Here, the rheology of the flowing blood is characterized by a generalized power law model. We have shown that as flow index behaviour (n) increases, rate of flow resistance ratio ($\bar{\lambda}$) increases. The results also show that variations of stenosis have the same effect on the flow resistance for bend and straight artery.

Keywords: Resistance-to-flow, flow index behavior, stenosis, Atherosclerosis

[I] INTRODUCTION

Arteries are blood vessels that carry oxygen and nutrients from your heart to the rest of your body. Healthy arteries are flexible and elastic. Over time, however, too much pressure in your arteries can make the walls thick and stiff — sometimes restricting blood flow to your organs and tissues. This process is called arteriosclerosis, or hardening of the arteries.

Atherosclerosis is a specific type of arteriosclerosis, but the terms are sometimes used interchangeably. Atherosclerosis refers to the buildup of fats and cholesterol in and on our artery walls (plaques), which can restrict blood flow. These plaques can also rupture, triggering a blood clot. Although atherosclerosis is often considered a heart problem, it can affect arteries anywhere in your body. Atherosclerosis is a preventable and treatable condition.

Arora [1] investigated that the velocity and flow rate increase with the increase of the peripheral layer thickness and decrease with the angle of tapering and depth of the stenosis. They also showed that the flow rate decreases non-linearly with the increase of the viscosity ratio and yield stress. Back et al. [2] measured pressure distributions along a hollow vascular axisymmetric replica of a segment of the left circumflex coronary artery of man with mildly atherosclerotic diffuse disease. Ikbal [3] studied a mathematical model of unsteady non-Newtonian flow of blood through a stenosed artery. The flowing blood is considered as a viscoelastic fluid having shear-thinning rheology and characterized by generalized Oldroyd-B model. The arterial wall is considered to be rigid having cosine shaped stenosis in its lumen. Johnston et al. [4] showed that to study the wall shear stress distribution for transient blood flow in arteries, the use of a Newtonian blood model is a reasonably good approximation. However, to study the flow within the artery in greater detail, a non-Newtonian model is more appropriate. Ley and Kim [5] calculated the arterial wall temperature distribution of arteries affected by plaque. It is shown that the plaque temperature correlates positively to inflammatory cell density and layer thickness, whereas the plaque temperature varies inversely with the depth of the inflammatory cell layer or fibrous cap. Mishra et al. [6] presented an analytical study on the behaviour of blood flow through an arterial segment having a mild stenosis. The artery has been treated as a thin-walled initially stressed orthotropic non-linear viscoelastic cylindrical tube filled with a non-Newtonian fluid representing blood. Mukhopadhyay and Layek [7] presented a mathematical model of arterial flow through differently shaped stenosis. They studied that flow resistance decreases as the shape of a smooth stenosis changes and maximum resistance is attained in case of a symmetric stenosis. Sapna Ratan Shah [8] investigated the non-Newtonian behaviour on blood flow through a stenosed artery using power law fluid model. They have shown resistance to flow and wall shear stress increases as the size of stenosis increases. Vassilevski et al. [9] developed two mathematical models of elastic walls of healthy and atherosclerotic blood vessels. The models are including in numerical model global blood circulation via recovery of the vessel wall state equation. Wong et al. [10] studied the effects of flow behavour indices and wall ratio on the blood flow through an axially non-symmetric but radially symmetric atherosclerotic artery with multiple stenoses and post stenotic-dilatations.

In this paper, we have investigated the effects of flow behavour indices on the non –Newtonian blood flow through straight, rigid, axisymmetric, atherosclerotic artery having multiple stenoses and post stenotic-dilatations.

[II] MATHEMATICAL FORMULATION

Let us consider the flow of blood through a straight, rigid, axisymmetric artery containing multiple abnormal segments. The Geometry of the wall of artery is given by

$$\frac{R}{R_0} = 1 - \frac{\delta_j}{2R_0}\left[1 + COS\,\frac{2\pi}{l_j}\left(z - \alpha_j - \frac{l_j}{2}\right)\right]; \alpha_j \le z \le \beta_j$$

$$= 1;\ \text{Otherwise} \qquad (1)$$

The distance from the origin to the start of the j^{th} abnormal segment is given by

$$\alpha_j = \left[\sum_{i=1}^{j}(d_i + l_i) - l_j\right] \qquad (2)$$

The distance from the origin to the end of j^{th} abnormal segment is

$$\beta_j = \sum_{i=1}^{j}(d_j + l_j) \qquad (3)$$

where

δ_j = Maximum distance of j^{th} abnormal segment
R =Radius of abnormal artery
R_0 =Radius of normal artery
l_j =Length of the j^{th} abnormal segments
α_j = Distance from the origin to the start of the j^{th} abnormal segment
β_j =Distance from the origin to the end of j^{th} abnormal segment
d_j = Distance separating the start of j^{th} abnormal segment from the end of the $(j-1)^{th}$

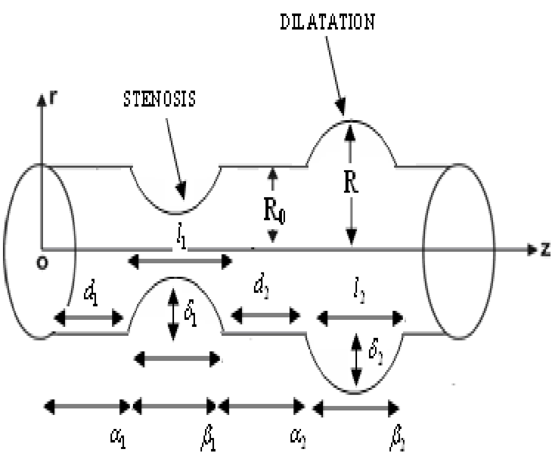

Fig.1: Geometry of Abnormal Segments

The constitutive equation for power law fluid is $\tau = \mu e^n, n < 1$

$$\tau = \frac{1}{2}\Pr \tag{4}$$

$$e = -\frac{du}{dr} \tag{5}$$

From equation (2), (3) and (4), we have

$$\frac{du}{dr} = -\left(\frac{1}{2}\frac{P}{\mu}r\right)^{\frac{1}{n}} \tag{6}$$

Integrating (5) to obtain

$$u = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(R^{\frac{1}{n+1}} - r^{\frac{1}{n+1}}\right) \tag{7}$$

$$Q = \int_0^R 2\pi r u dr = \left(\frac{1}{2}\frac{P}{\mu}\right)^{\frac{1}{n}} \frac{n\pi}{3n+1} R^{\frac{3n+1}{n}} \tag{8}$$

Pressure gradient from equation can be obtained from eq. (7)

$$\frac{dp}{dz} = 2\mu\left(\frac{(3n+1)}{n\pi}Q\right)^n \frac{1}{R^{3n+1}} \tag{9}$$

$p = p_0$ at $z = 0$ and $p = p_1$ at $z = l$, we have

$$p_1 - p_0 = \frac{2\mu}{R_0^{3n+1}}\left[\frac{(3n+1)}{n\pi}Q\right]^n \int_0^l \frac{1}{(R/R_0)^{3n+1}}dz \tag{10}$$

$$\lambda = \frac{p_1 - p_0}{Q} = \frac{2\mu}{QR_0^{3n+1}}\left[\frac{(3n+1)}{n\pi}Q\right]^n \int_0^l \frac{1}{(R/R_0)^{3n+1}}dz \tag{11}$$

$$\lambda = \frac{p_1 - p_0}{Q} = \frac{2\mu}{QR_0^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n [\int_0^{\alpha_j} dz + \sum_{j=1}^{k}\int_{\alpha_j}^{\beta_j} \frac{1}{(R/R_0)^{3n+1}}dz + \sum_{j=1}^{k-1}\int_{\beta_j}^{\alpha_{j+1}} dz + \int_{\beta_k}^{l} dz] \tag{12}$$

Where k is the number of diseased segments.

$$= \frac{2\mu}{QR_0^{3n+1}}\left(\frac{(3n+1)Q}{n\pi}\right)^n \left[\sum_{j=1}^{k+1} dj + \sum_{j=1}^{k}\int_{\alpha_j}^{\beta_j} \frac{1}{(R/R_0)^{3n+1}}dz\right] \tag{13}$$

In the normal condition,

$$\lambda_N = \frac{2\mu}{QR_0^{3n+1}}\left(\frac{(3n+1)}{n\pi}Q\right)^n l \tag{14}$$

$$\lambda_N = \frac{2\mu}{QR_0^{3n+1}}\left(\frac{(3n+1)}{n\pi}Q\right)^n l$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = \frac{\sum_{j=1}^{k+1} dj}{l} + \frac{1}{l} \sum_{j=1}^{k} \int_{\alpha_j}^{\beta_j} \frac{1}{(R/R_0)^{3n+1}} dz \quad (15)$$

$$a_j = 1 - \frac{\delta_j}{2R_0}, \; b_j = \frac{\delta_j}{2R_0}$$

$$\theta = \pi - \frac{2\pi}{l_j}\left(z - \alpha_j - \frac{l_j}{2} \right)$$

$$\frac{R}{R_0} = 1 - b_j (1 - \cos\theta)$$

(16)

As $z = \alpha_j$ implies that $\theta = 2\pi$ and $z = \beta_j$ implies that $\theta = 0$

The substitution of eqn. (15) in (14) gives

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = \frac{\sum_{j=1}^{k+1} dj}{l} + \frac{l_j}{2\pi l} \sum_{j=1}^{k} \int_{0}^{2\pi} \frac{1}{(a_j + b_j \cos\theta)^{3n+1}} d\theta$$

(17)

[III] NUMERICAL RESULTS

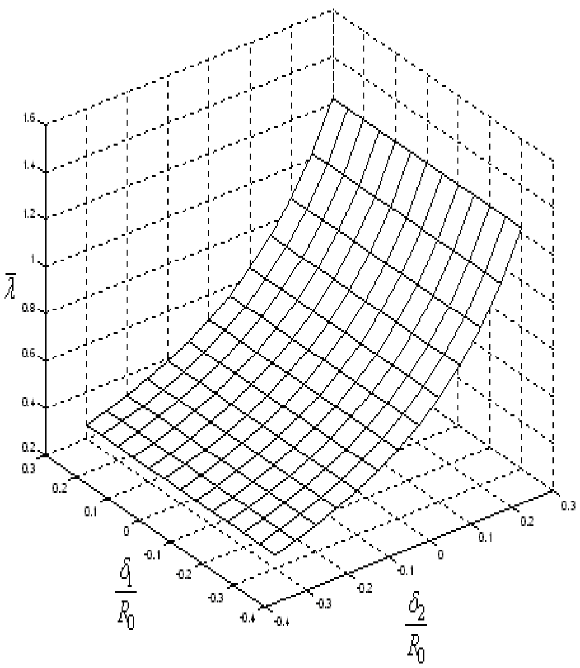

Fig.2: Variation in resistance to flow for $n = 1/3$

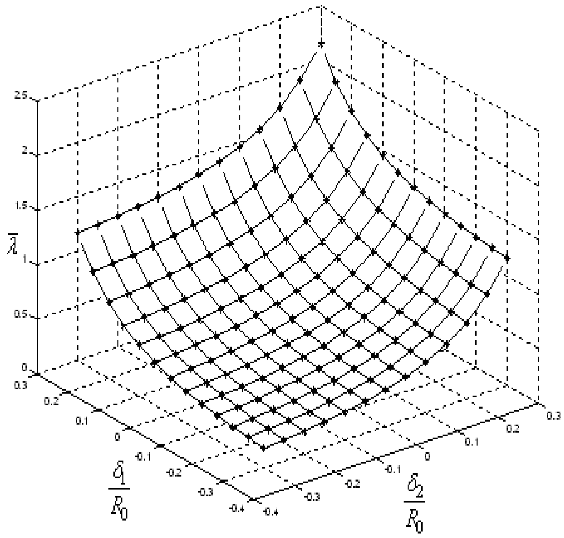

Fig.3: Variation in resistance to flow $n = 2/3$

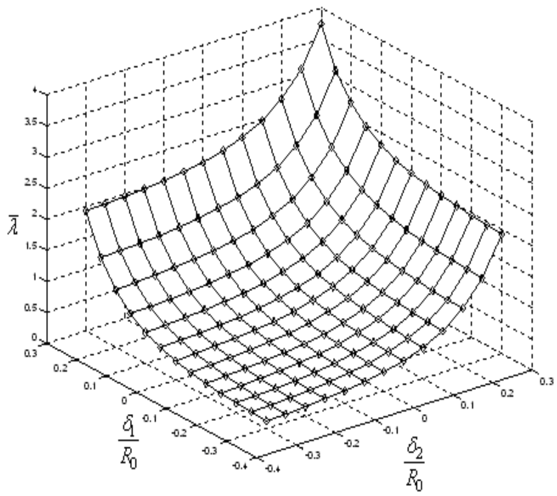

Fig.4: Variation in resistance to flow for $n = 1$

[VI] CONCLUSION

In this study, we have examined the power law model of blood flow through the abnormal segment of the straight, rigid, axisymmetric artery for the flow resistance ratio with different flow behaviour indices. Blood flow through an artery mainly depends on pressure drop and resistance to flow. Pressure drop in a atherosclerotic artery may be of use as reference data for the estimation of flow resistance through inconsistent arterial configuration. The degree of flow resistance for arteries may help in a clinical diagnosis.

REFERNCES:

[1]. Arora, J.K. (2011): "Artificial Neural Network modeling for the system of blood flow through tapered artery with mild stenosis", International Journal of Mathematics Trends & Technology, 2(1):1-4

[2]. Back, L.H.; Radbill, J.R.; Cho, Y.I. (1986): "Measurement and prediction of flow through a replica segment of a mildly atherosclerotic coronary artery of man", Journal of Biomechanics, 19(1):1-17

[3]. Ikbal, M.A. (2012): "Viscoelastic blood flow through arterial stenosis-Effect variable Viscosity", 47(8): 888-894

[4]. Johnston, B.M.; Johnston, P.R.; Corney, S.; Kilpatrick, D. (2006): "Non-Newtonian blood in human right coronary arteries: Transient simulation", Journal of Biomechanics, 39(6):1116-1128

[5]. Ley, O; Kim, T. (2008): "Determination of atherosclerotic plaque temperature in large arteries", International Journal of Thermal Sciences, 47(2):147-156

[6]. Mishra, J.C.; Patra, M.K.; Mishra, S.C. (1996): "A non-Newtonian fluid model for blood flow through arteries under stenotic condition", Journal of Biomechanics, 26(9):1129-1141

[7]. Mukhopadhayay, S.; Layek, G.C. (2008): "Numerical Modeling of a stenosed artery using mathematical model of variable shape", Applications and Applied Mathematics, 3(2):308-328

[8]. Shah, S.R. (2011): "Non-Newtonian flow of blood through an atherosclerotic Artery", Research Journals of Applied Sciences, 6(1): 76-80

[9]. Vessilevski, Y.; Simakov, S.; Salamatova, V.; Ivanov, Y.; Dobroserdova, T. (2011): "Vessel wall Models for simulation of Atherosclerotic vascular network", Math. Model. Nat. Phenom., 6(7): 1-18

[10]. Wong, K.; Tu, J.; Mazumdar, J.; Abbott, D. (2003): "Modelling of blood flow resistance for an atherosclerotic artery with multiple stenoses and post dilatations", ANZIAM J., 51:C66-C82