International Journal of Advanced Computer and Mathematical Sciences
ISSN 2230-9624. Vol 3, Issue 4, 2012, pp 482-486
http://bipublication.com

A MARKOV CHAIN MODELLING OF DAILY PRECIPITATION OCCURRENCES OF ODISHA

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[Received-07/07/2012, Accepted-01/08/2012]

ABSTRACT

In this study, a stochastic daily Precipitation generation model was adapted for the state of Odisha. The model simulates the sequence of Precipitation occurrence using the method of transitional probability matrices, while daily Precipitation amount was generated using a two parameter Gamma distribution. Daily average Precipitation data from Odisha from the year 2001 to 2010 were used for this model. First order Markov chains can adequately represent the Precipitation occurrences in all the months. Four states are used for representing Precipitation in a wet day as a more good fit can be obtained for the distribution representing the Precipitation amount in each class.

Key Words : Markov Chain, transition probability matrix, Two parameter Gamma Distribution

1. INTRODUCTION

The natural systems are so complex that no exact laws have yet been developed that can explain completely and precisely the natural hydrological phenomena. Precipitation affects weather variables affecting the growth and development of crops, and the spread of diseases and pests. Hence Precipitation forms the principal input to all agronomic models. The future probability of occurrence of Precipitation can be used for crop planning and management and water management decisions, as the risk due to weather uncertainty can be reduced.

The Markov models are frequently proposed to quickly obtain forecasts of the weather “States” at some future time using information given by the current state. One of the applications of the Markov chain models is the daily precipitation occurrence forecast.

One of the statistical techniques is the Markov chain used to predict precipitation on short term, at meteorological stations. The Markov chain models have two advantages:

- The forecasts are available immediately after the observations are done because the use as predictors only the local information on the weather and
- They need minimal computation after the climatologically data have been processed.

A first-order Markov chain is one in which knowing one variable (like cloudiness, precipitation amount, temperature, fog, frost, wind) at time t is sufficient to forecast it at some later time. The objective of this study is to simulate daily Precipitation sequences for Odisha to use as inputs to crop, hydrologic and water resources models.

Markov chains specify the state of each day as ‘wet’ or ‘dry’ and develop a relation between the state of the current day and the states of the preceding days. The order of the Markov chain is the number of preceding days taken into account. Most Markov chain models referred in the literature are first order. Many authors have used Markov chains to model the daily occurrence of precipitation. The low order chains are mostly preferable for two reasons. The number of parameters to be estimated is kept to be a minimum, so that better estimates are obtained. Second, the subsequent use of the fitted model to calculate other quantities, such as the probabilities of long dry spells, is simpler. The distribution of the amounts of Precipitation on wet days is usually modelled by gamma distributions.

2 . MODEL STRUCTURE

A first-order Markov chain and skewed normal distribution method requires daily weather records for many years in order to estimate the model parameters. Thus the availability of the weather data limits the applicability of the simulation method. Daily Precipitation data for all the thirty districts are taken and the daily average Precipitation for the state is calculated for the period 2001 to 2010. The stations within the districts were chosen due to their spatial representations as well as availability of adequate data for the study. The model consists of

(I) Precipitation Occurrence Model and

(II) Precipitation Magnitude Model.

3.PRECIPITATION OCCURRENCE MODEL

Modelling precipitation data at useful time for different applications has been an important problem in hydrology for the last 30 years. Cox and Isham [3] presented an interesting classification of precipitation models in three types:

- Empirical statistical models,
- Dynamic models and
- Intermediate stochastic models.

The idea behind this classification is the amount of physical realism incorporated into the model structure. A Russian mathematician, Markov, introduced the concept of a process (later named after him 'a Markov process') in which a sequence or chain of discrete states in time for which the probability of transition from one state to any given state in the next step in the chain depends on the condition during the previous step. Daily rainfall includes the occurrence of rain and the amount of rain. Markov chain process is used to find out rainfall occurrence. Once precipitation occurrence has been specified, Precipitation amount is then generated using a Gamma or mixed Exponential distribution [7,19,5,6,10,16,17,11,14,15]. Models of second or higher orders have been studied by Chin [2], Gates and Tong [8], Eidsvik [4].

A first order Markov chain is a stochastic process having the property that the value of the process at time t, X_t , depends only on its value at time $t-1, X_{t-1}$, and not on the sequence of values that the process passed through in arriving at X_{t-1}.

In general, the number of states at each time instant can be assumed as n. Hence, there will be $n \times n$ transitions between two successive time instances. It is then possible to find the number of transition probabilities, p_{ij} from a state at time t to another state at time $(t+1)$, and accordingly, the following transition probability matrix, $P_{t,t+1}$ can be prepared from observed precipitation data. The structure of the transition probability matrix would be

$$P = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1j} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2j} & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{i1} & P_{i2} & \cdots & P_{ij} & \cdots & P_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{nj} & \cdots & P_{nn} \end{pmatrix}$$

And it can be estimated by (Siriwardena *et al.*, [12]):

$$P_{ij} = \frac{n_{ij}}{\sum_{j=1}^{n} n_{ij}}$$

where n_{ij} = historical frequency of transition from state i to state j and n = the maximum number of states. With the yearly extreme maximum precipitation data this matrix shows the transition probabilities, P_{ij} of yearly precipitation in state i at time t to state j at time $(t+1)$ given n Precipitation state the following properties of the transition matrix are valid by definition. Any state probability varies between zero and one. Notationally, $0 \le P_{ij} \le 1$ where $i,j = 1,2\ldots,n$. Also, $\sum_{j=1}^{n} P_{ij} = 1$, where $i = 1,2,\ldots\ldots n$.

Once P is known, all that is required to determine the probabilistic behavior of the Markov Chain is the initial state of the chain. In the following, $P_j^{(m)}$ denotes the probability that the chain is in state j at step or time m. The $1 \times n$ vector $P^{(m)}$ has elements $P_j^{(m)}$. Thus

$$P^{(m)} = [P_1^{(m)}, P_2^{(m)}, \ldots, P_n^{(m)}] \text{ and } P^{(1)} = P^{(0)} P \tag{3}$$

where, $P^{(0)}$ is the initial probability vector. In general, $P^{(m+C)} = P^{(m)} P^{(C)}$ (4)

4. MODEL FOR PRECIPITATION MAGNITUDES ON WET DAYS

The Precipitation amounts on wet days are modelled by a two parameter gamma distribution with the density function is given by

$$P_X(x) = \frac{1}{\Gamma(\eta)} \lambda^{\eta} x^{\eta-1} e^{-\lambda x} \quad , \quad x > 0 \tag{5}$$

where, $\Gamma(\eta) = (\eta - 1)!$ for $\eta = 1,2,3 \ldots\ldots$ (6)

and $\eta = t^{\eta-1}\, e^{-1}$ (7)

in which η and λ are the shape and scale parameters respectively.

The parameters of the gamma distribution η and λ were estimated using Greenwood and Durand (1960) method as given below

$$\eta^* = \begin{cases} \dfrac{(0.5000876 + 0.1648852y - 0.0544274y^2)}{y}, & for \quad 0 \le y \le 0.5772 \\ \dfrac{8.898919 + 9.05995y - 0.09775373y^2}{y(17.79728 + 11.968477y + y^2)}, & for \quad 0.5772 < y \le 17.0 \end{cases} \tag{8}$$

where, $y = \frac{ln\bar{u}}{\overline{ln\,u}}$ in which, u is the Precipitation amount on wet days. The estimate η* was corrected for small sample bias using Bowman and Shenton equation,

$$\eta = \left(\frac{n-3}{n}\right)\eta^* \tag{9}$$

where, n is the sample size the estimate for λ is $\lambda = {}^{\eta}/_{\bar{u}}$ (10)

5. RESULTS AND CONCLUSIONS

MODEL CALCULATION

To analyze the Precipitation data using Markov Chain Model, we consider four states and the state boundaries are as given in the following table (Table – 1).

TABLE- 1: STATES AND THEIR BOUNDARIES

State	Limits (mm)
1	0.0 - 0.0
2	0.1 - 3.9
3	4.0 - 27.9
4	28.0 - ∞

Here, the first state (State – 1) indicates that, there is no Precipitation in a particular day, so the day is a dry day, whereas the other states are rainy days. Since, there are four states, the transition probability matrix has order 4×4. These transition probabilities are calculated month-wise. These transition probabilities for different months are given in Table – 2.

TABLE – 2: MONTHLY TRANSITION PROBABILITY MATRIX

April

$$\begin{bmatrix} 0.662162 & 0.324324 & 0.013514 & 0.0000000 \\ 0.205357 & 0.526786 & 0.250000 & 0.0178857 \\ 0.011494 & 0.287356 & 0.586207 & 0.1149443 \\ 0.00000 & 0.294118 & 0.352941 & 0.3529441 \end{bmatrix}$$

May

$$\begin{bmatrix} 0.500000 & 0.400000 & 0.100000 & 0.00000 \\ 0.041667 & 0.583333 & 0.333333 & 0.041667 \\ 0.014493 & 0.137681 & 0.521739 & 0.326087 \\ 0.00000 & 0.087500 & 0.550000 & 0.362500 \end{bmatrix}$$

June

$$\begin{bmatrix} 0.666667 & 0.250000 & 0.083333 & 0.00000 \\ 0.090909 & 0.424242 & 0.303030 & 0.181818 \\ 0.000000 & 0.139241 & 0.531646 & 0.329114 \\ 0.00000 & 0.018072 & 0.138554 & 0.843373 \end{bmatrix}$$

July

$$\begin{bmatrix} 0.00000 & 1.000000 & 0.00000 & 0.00000 \\ 0.058824 & 0.352941 & 0.529412 & 0.058824 \\ 0.000000 & 0.152174 & 0.478261 & 0.369565 \\ 0.00000 & 0.008475 & 0.067797 & 0.923729 \end{bmatrix}$$

August

$$\begin{bmatrix} 0.000000 & 1.000000 & 0.000000 & 0.0000000 \\ 0.058824 & 0.352941 & 0.529412 & 0.0588824 \\ 0.000000 & 0.152174 & 0.478261 & 0.3695665 \\ 0.000000 & 0.008475 & 0.067797 & 0.9237229 \end{bmatrix}$$

September

$$\begin{bmatrix} 0.000000 & 0.000000 & 0.300000 & 0.700000 \\ 0.250000 & 0.526316 & 0.131579 & 0.092105 \\ 0.013158 & 0.223684 & 0.526316 & 0.236842 \\ 0.000000 & 0.083333 & 0.218750 & 0.697917 \end{bmatrix}$$

October

$$\begin{bmatrix} 0.692308 & 0.269231 & 0.038462 & 0.0000000 \\ 0.250000 & 0.526316 & 0.131579 & 0.0921055 \\ 0.013158 & 0.223684 & 0.526316 & 0.2368442 \\ 0.000000 & 0.083333 & 0.218750 & 0.6979177 \end{bmatrix}$$

November

$$\begin{bmatrix} 0.863354 & 0.136646 & 0.000000 & 0.000000 \\ 0.376812 & 0.434783 & 0.15942 & 0.028986 \\ 0.086957 & 0.239130 & 0.521739 & 0.152174 \\ 0.000000 & 0.214286 & 0.571429 & 0.214286 \end{bmatrix}$$

December

$$\begin{bmatrix} 0.974453 & 0.021898 & 0.000000 & 0.00365 \\ 0.357143 & 0.357143 & 0.214286 & 0.071429 \\ 0.400000 & 0.2000000 & 0.2000000 & 0.200000 \\ 0.000000 & 0.285714 & 0.142857 & 0.571429 \end{bmatrix}$$

After calculating the transition probabilities for each month, we now calculate the parameters of the Gamma distribution, which are represented in Table – 3.

TABLE- 3: PARAMETERS η AND λ FOR GAMMA DISTRIBUTION

Months	η			λ		
	State 2	**State 3**	**State 4**	**State 2**	**State 3**	**State 4**
April	0.08013	0.30112	0.10112	0.02511	0.06213	0.00076
May	0.16011	0.24971	0.00097	0.09231	0.01402	0.00045
June	0.05896	0.30119	0.25432	0.04934	0.03302	0.00110
July	0.12117	0.16312	0.33102	0.10313	0.02003	0.00386
August	0.43215	0.27962	0.26120	0.40456	0.01232	0.00201
September	0.21003	0.05946	0.04986	0.17325	0.06983	0.00098
October	0.49875	0.05102	0.02352	0.04279	0.00512	0.00389
November	0.03583	0.03112	0.00056	0.09284	0.01986	0.00524
December	0.19898	0.04813	0.01989	0.05983	0.01042	0.00045

6. CONCLUSION

Stochastic Precipitation models are concerned with the time of occurrence and depth of Precipitation. Various Precipitation models have been using different time scales. Daily Precipitation models have gained wide applicability as being appropriate for use in detailed water balance and agricultural and environmental models. In this study a stochastic daily Precipitation generation model was adapted for the state of Odisha. The model simulates the sequence of Precipitation occurrence using the method of transitional probability matrices, while daily Precipitation amount was generated using a two parameter Gamma distribution. Daily average Precipitation data from Odisha from the year 2001 to 2010 were used for this model. The model parameters were estimated from historical Precipitation records.

Markov chains indicated that first order Markov chains can adequately represent the Precipitation occurrences in all the months. A number of states are used for representing Precipitation in a wet day as a more good fit can be obtained for the distribution representing the Precipitation amount in each class. The states and their boundary limits are given in [Table -1]. The monthly transition probabilities are given in [Table-2]. The two parameters, η and λ of the gamma distributions are given in [Table- 3].

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