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E-CORDIAL LABELING IN THE CONTEXT OF SWITCHING OF A VERTEX

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ABSTRACT:

Let G=(V(G),E(G)) be a graph and $f: E(G) \rightarrow \{0,1\}$ be a binary edge labeling. Define $f^*: V(G) \rightarrow \{0,1\}$ by $f^*(v) = \sum_{uv \in E(G)} f(uv) (mod\, 2)$. The function f is called E-cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. In the present work we discuss E-cordial labeling in the context of switching of a vertex in cycle, wheel, helm and closed helm.

Keywords: E-cordial labeling , Cycle, Wheel, Vertex switching

[I] INTRODUCTION

We begin with finite, connected and undirected graph $G = (V(G), E(G))$ without loops and multiple edges. For standard terminology and notations we follow Harary[4]. For extensive survey of graph labeling as well as bibliographic references we refer Gallian[2].

1.1. Definition If the vertices of the graph are assigned values subject to certain condition(s) then it is known as *graph labeling*.

Most of the graph labeling techniques trace their origin to graceful labeling introduced independently by Rosa[7] and Golomb[3] which is defined as follows.

1.2. Definition

A function $f: V(G) \rightarrow \{0,1,\ldots,|E(G)|\}$ is called *graceful labeling* of graph G if f is injective and the induced function $f^*: E(G) \rightarrow \{1,2,\ldots,|E(G)|\}$ defined by

$f^*(e=uv)=|f(u)-f(v)|$ is bijective. A graph which admits graceful labeling is called a *graceful graph*.

The famous Ringel-Kotzig conjecture [6] and illustrious work on it brought a tide of labeling problems with graceful theme.

1.3. Definition

A graph G is said to be *edge-graceful* if there exists a bijection $f: E(G) \to \{1, 2, \ldots, |E(G)|\}$ such that the induced function $f^*: V(G) \to \{0, 1, 2, \ldots, |V(G)|-1\}$ defined by $f^*(x) = \sum(f(xy))(mod|V(G)|)$, taken over all edges xy is a bijective.

The notion of edge gracefulness was introduced by Lo[5].

1.4. Definition

A mapping $f: V(G) \to \{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the *label* of vertex v of G under f.

1.5. Notation

For an edge $e=uv$, the induced edge labeling $f^*: E(G) \to \{0,1\}$ is given by $f^*(e=uv)=|f(u)-f(v)|$ then $v_f(i)$ = the number of vertices of G having label i under f and let $e_f(i)$ = the number of edges of G having label i under f^* for $i=0,1$.

1.6. Definition

A binary vertex labeling of graph G is called a *cordial labeling* if $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$. A graph G is called *cordial* if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit[1]. He also investigated several results on this newly defined concept.

1.7. Definition

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$ and let $f: E(G) \to \{0,1\}$. Define f^* on $V(G)$ by $f^*(v) = \sum\{f(uv) * uv \in E(G)\}(mod\,2)$. The function f is called an *E-cordial labeling* of G if $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$. A graph is called *E-cordial* if it admits E-cordial labeling.

In 1997 Yilmaz and Cahit[12] have introduced E-cordial labeling as a weaker version of edge-graceful labeling having the blend of cordial labeling. They proved that the trees with n vertices, K_n, C_n are E-cordial if and only if $n \not\equiv 2(mod\,4)$ while $K_{m,n}$ admits E-cordial labeling if and only if $m+n \not\equiv 2(mod\,4)$.

Vaidya and Lekha[8] have proved that the graphs obtained by duplication of an arbitrary vertex as well as an arbitrary edge in cycle C_n admit E-cordial labeling. In addition to this they also derived that the joint sum of two copies of cycle C_n, the split graph of even cycle C_n and the shadow graph of path P_n for even n are E-cordial graphs. The same authors in [9] proved that the middle graph, total graph and split graph of P_n and the composition of P_n with P_2 admit E-cordial labeling.

Vaidya and Vyas[10] proved that the mirror graphs of even cycle C_n, even path P_n and hypercube Q_k are E-cordial graphs. The same authors in [11] proved that $K_n \times P_2$ and $P_n \times P_2$ are E-cordial graphs for even n while $W_n \times P_2$ and $K_{1,n} \times P_2$ are E-cordial graphs for odd n.

1.8. Definition

A *vertex switching* G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

1.9. Definition

The *wheel graph* W_n is defined to be the join K_1+C_n. The vertex corresponding to K_1 is known as *apex vertex* and vertices corresponding to cycle are known as *rim vertices* while the edges corresponding to cycle are known as *rim edges*. We continue to recognize apex of wheel as the apex of respective graphs obtained from wheel.

1.10. Definition

The *helm* H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.

1.11. Definition

The *closed helm* CH_n is the graph obtained from a helm H_n by joining each pendant vertex to form a cycle.

We continue to recognize terminology used in Definition 1.9 for Definitions 1.10 and 1.11 also.

In the following section we will investigate some new results on E-cordial labeling of graphs.

[II] MAIN RESULTS

2.1. Theorem

The graph obtained by switching of an arbitrary vertex in cycle C_n admits E-cordial labeling except for $n \equiv 2(mod\,4)$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the successive vertices of C_n and G_v denotes graph obtained by switching of vertex v of G. Without loss of generality let the switched vertex be v_1 and we initiate the labeling from v_1. To define $f : E(G_{v_1}) \to \{0,1\}$ we consider following cases.

Case 1: When n is odd

For $3 \le i \le n-1$:

$$f(v_1v_i) = \begin{cases} 1, & i \equiv 1(\bmod 2) \\ 0, & otherwise; \end{cases}$$

Subcase 1: $n \equiv 3(mod\,4)$

For $2 \le i \le n-1$:

$$f(v_iv_{i+1}) = \begin{cases} 1, & i \equiv 0(\bmod 2) \\ 0, & otherwise; \end{cases}$$

Subcase 2: $n \equiv 1(mod\,4)$

For $2 \le i \le n-1$:

$$f(v_iv_{i+1}) = \begin{cases} 0, & i \equiv 0(\bmod 2) \\ 1, & otherwise; \end{cases}$$

Case 2: When n is even

Subcase 1: $n \equiv 0(mod\,4)$

$f(v_2v_3) = 1;$

For $3 \le i \le n-1$:

$$f(v_1v_i) = \begin{cases} 0, & i \equiv 1(\text{mod } 2) \\ 1, & otherwise; \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} 1, & i \equiv 1(\text{mod } 2) \\ 0, & otherwise; \end{cases}$$

Subcase 2: $n \equiv 2(mod\,4)$

A graph with n vertices is not E-cordial when $n \equiv 2(mod\,4)$ as observed by Yilmaz and Cahit [12].

In view of the labeling pattern defined above f satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ as shown in *Table 1*.

Hence G_{v_1} admits E-cordial labeling.

2.2. Illustration

Consider the graph obtained by switching of a vertex v_1 in cycle C_7. The E-cordial labeling is as shown in *Figure 1*.

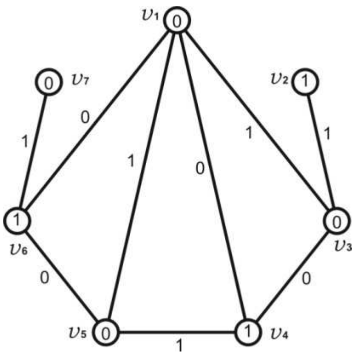

Figure 1

2.3. Theorem

The graph obtained by switching of a rim vertex in wheel W_n admits E-cordial labeling except for $n \equiv 1(mod\,4)$.

Proof: Let v as the apex vertex and $v_1, v_2, .., v_n$ be the rim vertices of wheel W_n. Let G_{v_1} denotes graph obtained by switching of a rim vertex v_1 of $G=W_n$. We define $f : E(G_{v_1}) \to \{0,1\}$ as follows.

<u>Case 1: When *n* is even</u>

For $2 \le i \le n$:

$$f(vv_i) = \begin{cases} 1, & i \equiv 0(\bmod 2) \\ 0, & otherwise. \end{cases}$$

For $2 \le i \le n$:

$$f(vv_i) = \begin{cases} 1, & i \equiv 0(\bmod 2) \\ 0, & otherwise. \end{cases}$$

For $2 \le i \le n-1$:

$$f(v_i v_{i+1}) = \begin{cases} 1, & i \equiv 0(\bmod 2) \\ 0, & otherwise. \end{cases}$$

For $3 \le i \le n-1$:

$$f(v_1 v_i) = \begin{cases} 0, & 3,, i,, \frac{n}{2}+1 \\ 1, & otherwise. \end{cases}$$

<u>Case 2: When *n* is odd</u>

<u>Subcase 1:</u> $n \equiv 3(mod\,4)$

For $2 \le i \le n$:

$f(vv_i) = 1;$

For $2 \le i \le n-1$:

$f(v_i v_{i+1}) = 0;$

For $3 \le i \le n-1$:

$$f(v_1 v_i) = \begin{cases} 0, & 3,, i,, \frac{n+1}{2} \\ 1, & otherwise. \end{cases}$$

<u>Subcase 2:</u> $n \equiv 1(mod\,4)$

In this case $|V(W_n)| = n+1 \equiv 2(mod\,4)$ equivalently $n \equiv 1(mod\,4)$. This graph is not E-cordial because the graph *G* with number of vertices congruent to 2(mod 4) does not admits E-cordial labeling as observed by Yilmaz and Cahit [12].

In view of the labeling pattern defined above f satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ as shown in *Table 2*.

Hence G_{v_1} admits E-cordial labeling.

2.4. Illustration

Consider the graph obtained by switching of a rim vertex in wheel W_8. The E-cordial labeling is as shown in *Figure 2*.

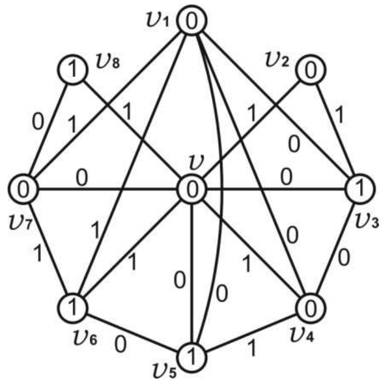

Figure 2

2.5. Theorem

The graph obtained by switching of an apex vertex in helm H_n admits E-cordial labeling.

Proof: Let H_n be a helm with v as the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of cycle and $u_1, u_2, \ldots, u_n$ be the pendant vertices. Let G_v denotes graph obtained by switching of an apex vertex v of $G=H_n$. We define $f: E(G_v) \rightarrow \{0,1\}$ as follows.

For $1 \le i \le n$:

$f(vu_i) = 1;$

$f(v_i v_{i+1}) = 0; \quad (v_{n+1} = v_1)$

$$f(v_i u_i) = \begin{cases} 0, & 1,, i,, \left\lceil \frac{n}{2} \right\rceil \\ 1, & \textit{otherwise.} \end{cases}$$

In view of the labeling pattern defined above f satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ as shown in *Table 3*.

Hence G_v admits E-cordial labeling.

2.6. Illustration

Consider the graph obtained by switching of an apex vertex in Helm H_6. The E-cordial labeling is as shown in *Figure 3*.

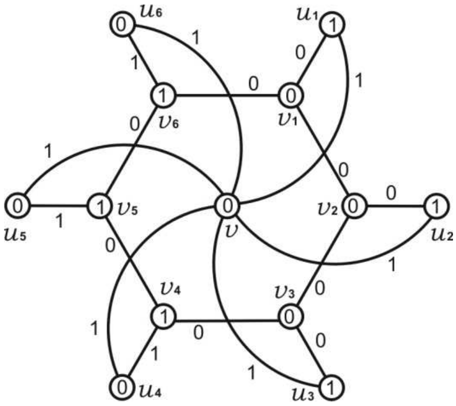

Figure 3

2.7. Theorem

The graph obtained by switching of an apex vertex in closed helm CH_n admits E-cordial labeling.

Proof: Let v as the apex vertex, $v_1, v_2, \ldots, v_n$ be the vertices of inner cycle and $u_1, u_2, \ldots, u_n$ be the vertices of outer cycle CH_n. Let G_v denotes graph obtained by switching of an apex vertex v of $G=CH_n$. We define $f: E(G_v) \rightarrow \{0,1\}$ as follows.

For $1 \le i \le n$:

$f(vu_i) = 1;$

$f(v_i v_{i+1}) = 0; \quad (v_{n+1} = v_1)$

$f(u_i u_{i+1}) = 0; \quad (u_{n+1} = u_1)$

$f(v_i u_i) = 1.$

In view of the labeling pattern defined above f satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ as shown in *Table 4*.

Hence G_v admits E-cordial labeling.

2.8. Illustration

Consider the graph obtained by switching of an apex vertex in closed helm CH_5. The E-cordial labeling is as shown in *Figure 4*.

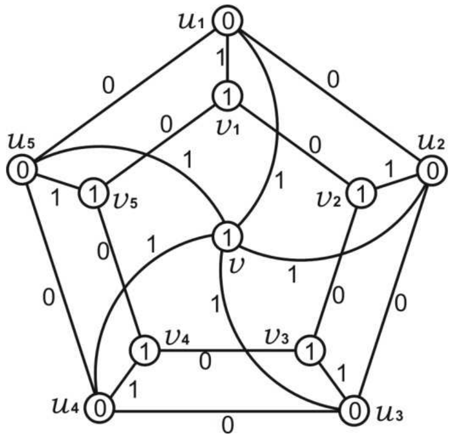

Figure 4

[III] CONCLUDING REMARKS

Here we investigate E-cordial labeling in the context of switching of a vertex of some graphs. To investigate similar results for other graph families and in the context of different graph labeling problems is an open area of research.

	vertex condition	*edge condition*
$n \equiv 0(mod4)$	$\upsilon_f(0) = \upsilon_f(1) = \frac{n}{2}$	$e_f(0)+1 = e_f(1) = \left\lceil \frac{2n-5}{2} \right\rceil$
$n \equiv 1(mod4)$	$\upsilon_f(0) = \upsilon_f(1)+1 = \left\lceil \frac{n}{2} \right\rceil$	$e_f(0) = e_f(1)+1 = \left\lceil \frac{2n-5}{2} \right\rceil$
$n \equiv 3(mod4)$	$\upsilon_f(0)+1 = \upsilon_f(1)+1 = \left\lceil \frac{n}{2} \right\rceil$	$e_f(0)+1 = e_f(1) = \left\lceil \frac{2n-5}{2} \right\rceil$

Table 1: vertex and edge conditions corresponding to Theorem 2.1

	vertex condition	*edge condition*
$n \equiv 0(\mathrm{mod}\,4)$	$\upsilon_f(0)-1 = \upsilon_f(1) = \frac{n}{2}$	$e_f(0) = e_f(1) = \frac{3(n-2)}{2}$
$n \equiv 2(\mathrm{mod}\,4)$	$\upsilon_f(0) = \upsilon_f(1)-1 = \frac{n}{2}$	$e_f(0) = e_f(1) = \frac{3(n-2)}{2}$
$n \equiv 3(\mathrm{mod}\,4)$	$\upsilon_f(0) = \upsilon_f(1) = \frac{n+1}{2}$	$e_f(0)+1 = e_f(1) = \frac{3(n-2)+1}{2}$

Table 2: vertex and edge conditions corresponding to Theorem 2.3

	vertex condition	*edge condition*
$n \equiv 0(mod\,2)$	$\upsilon_f(0)-1 = \upsilon_f(1) = n$	$e_f(0) = e_f(1) = n + \frac{n}{2}$
$n \equiv 1(mod\,2)$	$\upsilon_f(0) = \upsilon_f(1)-1 = n$	$e_f(0) = e_f(1)+1 = n + \left\lceil \frac{n}{2} \right\rceil$

Table 3: vertex and edge conditions corresponding to Theorem 2.5

	vertex condition	*edge condition*
$n \equiv 0(mod2)$	$\upsilon_f(0)-1 = \upsilon_f(1) = n$	$e_f(0) = e_f(1) = 2n$
$n \equiv 1(mod2)$	$\upsilon_f(0) = \upsilon_f(1)-1 = n$	$e_f(0) = e_f(1) = 2n$

Table 4: vertex and edge conditions corresponding to Theorem 2.7

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