International Journal of Advanced Computer and Mathematical Sciences
ISSN 2230-9624. Vol 4, Issue 1, 2013, pp50-57
http://bipublication.com

A COMMON UNIQUE RANDOM FIXED POINT THEOREM FOR SIX RANDOM OPERATORS IN HILBERT SPACE

Neetu Vishwakarma

Sagar Institute of Research and Technology - Excellence, Bhopal, M.P., India,
Email: neetu.vishu@gmail.com

[Received-05/11/2012, Accepted-3/01/2013]

ABSTRACT

Using the notion of weak compatibility, semi-compatibility of random operators a common random fixed point theorem for six random operators defined on a non-empty closed subset of a separable Hilbert space has been proved.

Keywords: Semi-compatible, weak compatible, separable Hilbert space, Random operator, measurable mapping.

AMS Mathematics Subject Classification (2000): 47H10, 54H25.

INTRODUCTION

In the recent years, the study of random fixed point theory have attached much attention, some of its recent literature noted in [4,5,6,8,9,10]. Choudhary [7] construct a sequence of measurable function and consider its convergence to find a common unique fixed point of two random operators in Hilbert spaces. Badshah and Sayyed [2], Badshah and Gagrani [1] studied the structure of common random fixed point and proved some common random fixed theorem in Polish spaces. Recently Badshah and Shrivastava [3] introduced the concept of semi-compatibility in Polish spaces. We first review the following concepts which are essentials for our study in this paper.

Throughout this paper, $(\Omega, \sum)$ denotes a measurable space, H stands for a separable Hilbert space and C is a non-empty subset of H.

A mapping $\xi : \Omega \to C$ is measurable if $\xi^{-1}(B \cap C) \in \sum$ for every Borel subset B of H. A function $f : \Omega \to C$ is said to be a random operator, if $f(., x) : \Omega \to C$ is measurable for every $x \in C$. A measurable function $\xi : \Omega \to C$ is said to be random fixed point of the random operator $f: \Omega \times C \to C$, if $f(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$. A random operator $f: \Omega \times C \to C$ is said to be continuous if for fixed $\omega \in \Omega$, $f(\omega, .) : C \to C$ is continuous.

Let X be a Polish space that is separable complete metric space. Mapping $f, g : X \to X$ are compatible if $\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, provided that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n)$ exists in X.

Random operators S, T: $\Omega \times X \to X$ are compatible if $S(\omega, .)$ and $T(\omega, .)$ are compatible for each $\omega \in \Omega$.

Random operators S, T: $\Omega \times X \to X$ are weakly compatible if $T(\omega, \xi(\omega)) = S(\omega, \xi(\omega))$ for some measurable mapping $\xi : \Omega \to X$, then

$T(\omega, S(\omega, \xi(\omega)) = S(\omega, T(\omega, \xi(\omega)))$ for every $\omega \in \Omega$.

Random operators S, T: $\Omega \times X \to X$ are semi-compatible if

$d(S(\omega, T(\omega, \xi_n(\omega))), T(\omega, \xi(\omega))) \to 0$ as $n \to \infty$

whenever $\xi_n : \Omega \to X$, $n > 0$ is a measurable mapping such that

$T(\omega, \xi_n(\omega)), S(\omega, \xi_n(\omega)) \to \xi(\omega)$ as $n \to \infty$ for some measurable mapping $\xi : \Omega \to X$.

Main Result.

Theorem. Let C be a non-empty closed subset of a separable complete Hilbert space H. Let A, B, S, T, I and J be random operators from $\Omega \times C \to C$ satisfying condition

$AB(\omega, X) \subset J(\omega, X)$ and $ST(\omega, X) \subset I(\omega, X)$

and

$$\|AB(\omega,x) - ST(\omega,y)\|^2 \leq \alpha(\omega)\frac{\|J(\omega,y) - ST(\omega,y)\|^2 . \|I(\omega,x) - AB(\omega,x)\|^2}{\|I(\omega,x) - J(\omega,y)\|^2}$$

$$+ \beta(\omega) \|I(\omega, x) - J(\omega, y)\|^2$$

for each $x, y \in C$ and $\omega \in \Omega$ where α, β are measurable mapping from $\Omega \to (0, 1)$ such that $\alpha(\omega) + \beta(\omega) < 1$.

If either [AB, I] are semi-compatible, I or AB is continuous and [ST, J] are weakly compatible or [ST, J] are semi-compatible, J or ST is continuous and [AB, I] are weakly compatible.

Then AB, ST, I and J have a unique common random fixed point.

Proof. Let $\xi_0: \Omega \to C$ be an arbitrary measurable mapping.

We define a sequence of measurable mapping $\xi_n: \Omega \to C$ by

$AB(\omega, \xi_{2n}(\omega)) = J(\omega, \xi_{2n+1}(\omega)) = f_{2n}(\omega)$ (say)

and $ST(\omega, \xi_{2n+1}(\omega)) = I(\omega, \xi_{2n+2}(\omega)) = f_{2n+1}(\omega)$ (say)

for all n = 0, 1, 2, ... and $\omega \in \Omega$.

Then for each $\omega \in \Omega$ and n = 0, 1, 2,...

$$\|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2 \leq \|AB(\omega, \xi_{2n}(\omega)) - ST(\omega, \xi_{2n+1}(\omega))\|^2$$

$$\leq \alpha(\omega)\frac{\|J(\omega,\xi_{2n+1}(\omega)) - ST(\omega,\xi_{2n+1}(\omega))\|^2 . \|I(\omega,\xi_{2n}(\omega)) - AB(\omega,\xi_{2n}(\omega))\|^2}{\|I(\omega,\xi_{2n}(\omega)) - J(\omega,\xi_{2n+1}(\omega))\|^2}$$

$$+ \beta(\omega) \|I(\omega, \xi_{2n}(\omega)) - J(\omega, \xi_{2n+1}(\omega))\|^2$$

$$\leq \alpha(\omega)\frac{\|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2 . \|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2}{\|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2}$$

$$+ \beta(\omega) \|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2$$

$$\leq \alpha(\omega) \|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2 + \beta(\omega) \|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2$$

Therefore,

$$\|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2 \le \left(\frac{\beta(\omega)}{1-\alpha(\omega)}\right) \|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2$$

$$\|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2 \le k(\omega)\ \|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2$$

where $k(\omega) = \left(\frac{\beta(\omega)}{1-\alpha(\omega)}\right) < 1$.

Similarly,

$$\|f_{2n+1}(\omega) - f_{2n+2}(\omega)\|^2 \le \|AB(\omega, \xi_{2n+1}(\omega)) - ST(\omega, \xi_{2n+2}(\omega))\|^2$$

$$\le \alpha(\omega)\frac{\left\|J(\omega,\xi_{2n+2}(\omega)) - ST(\omega,\xi_{2n+2}(\omega))\right\|^2 . \left\|I(\omega,\xi_{2n+1}(\omega)) - AB(\omega,\xi_{2n+1}(\omega))\right\|^2}{\left\|I(\omega,\xi_{2n+1}(\omega)) - J(\omega,\xi_{2n+2}(\omega))\right\|^2}$$

$$+ \beta(\omega)\ \|I(\omega, \xi_{2n+1}(\omega)) - J(\omega, \xi_{2n+2}(\omega))\|^2$$

$$\le \alpha(\omega)\frac{\left\|f_{2n+1}(\omega) - f_{2n+2}(\omega)\right\|^2 . \left\|f_{2n}(\omega) - f_{2n+1}(\omega)\right\|^2}{\left\|f_{2n}(\omega) - f_{2n+1}(\omega)\right\|^2}$$

$$+ \beta(\omega)\ \|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2$$

This implies that

$$\|f_{2n+1}(\omega) - f_{2n+2}(\omega)\|^2 \le \left(\frac{\beta(\omega)}{1-\alpha(\omega)}\right) \|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2$$

$$\|f_{2n+1}(\omega) - f_{2n+2}(\omega)\|^2 \le k(\omega)\ \|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2$$

Therefore,

$$\|f_{2n+1}(\omega) - f_{2n+2}(\omega)\|^2 \le k^2(\omega)\ \|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2.$$

Similarly proceeding in the same way, by induction we get a measurable mapping $f_{2n} : \Omega \to X$ such that for all $n = 1, 2, \ldots$ and $\omega \in \Omega$, we have

$$\|f_{2n}(\omega) - f_{2n+1}(\omega)\|^2 \le k^{2n}(\omega)\ \|f_{2n-1}(\omega) - f_{2n}(\omega)\|^2.$$

Thus, it follows that for $\omega \in \Omega$, $\{f_{2n}(\omega)\}$ is a Cauchy sequence and hence is convergent in the separable complete Hilbert space H.

For $\omega \in \Omega$, let $\{f_{2n}(\omega)\}$ and its subsequences converges to some measurable mapping $\xi : \Omega \to C$

i.e. $\{f_{2n}(\omega)\} \to \xi(\omega)$ as $n \to \infty$

so $AB(\omega, \xi_{2n}(\omega)) \to \xi(\omega)$, $J(\omega, \xi_{2n+1}(\omega)) \to \xi(\omega)$,

$ST(\omega, \xi_{2n+1}(\omega)) \to \xi(\omega)$, $I(\omega, \xi_{2n+2}(\omega)) \to \xi(\omega)$,

for every $\omega \in \Omega$.

Case I (a). If I is continuous.

In this case, we have

$I(\omega, AB(\omega, \xi_{2n}(\omega))) \to I(\omega, \xi(\omega))$,

and $I(\omega, I(\omega, \xi_{2n+2}(\omega))) \to I(\omega, \xi(\omega))$.

Since pair (AB, I) is semi-compatible, hence

$AB(\omega, I(\omega, \xi_{2n}(\omega))) \to I(\omega, \xi(\omega))$, for each $\omega \in \Omega$.

Step I. For each $\omega \in \Omega$, we have

$$\|AB(\omega, I(\omega, \xi_{2n}(\omega))) - ST(\omega, \xi_{2n+1}(\omega))\|^2$$

$$\leq \alpha(\omega)\frac{\left\|J\left(\omega,\xi_{2n+1}(\omega)\right)-ST\left(\omega,\xi_{2n+1}(\omega)\right)\right\|^2.\left\|I\left(\omega,I(\omega,\xi_{2n}(\omega))\right)-AB\left(\omega,I(\omega,\xi_{2n}(\omega)\right)\right\|^2}{\left\|I\left(\omega,I(\omega,\xi_{2n}(\omega))\right)-J\left(\omega,\xi_{2n+1}(\omega)\right)\right\|^2}$$

$$+ \beta(\omega) \|I(\omega, I(\omega,\xi_{2n}(\omega))) - J(\omega, \xi_{2n+1}(\omega))\|^2.$$

Letting $n \to \infty$ and using above results, we get

$$\|I(\omega, \xi(\omega)) - \xi(\omega)\|^2$$

$$\leq \alpha(\omega)\frac{\left\|\xi(\omega)-\xi(\omega)\right\|^2.\left\|I\left(\omega,\xi(\omega)\right)-I\left(\omega,\xi(\omega)\right)\right\|^2}{\left\|I\left(\omega,\xi(\omega)\right)-\xi(\omega)\right\|^2}$$

$$+ \beta(\omega) \|I(\omega, \xi(\omega)) -\xi(\omega)\|^2$$

$(1 - \beta(\omega)) \|I(\omega, \xi(\omega)) - \xi(\omega)\|^2 \leq 0$

so that $I(\omega, \xi(\omega)) = \xi(\omega)$.

Step II. Now, we also obtain

$$\|AB(\omega, \xi(\omega)) - ST(\omega, \xi_{2n+1}(\omega))\|^2$$

$$\leq \alpha(\omega)\frac{\left\|J\left(\omega,\xi_{2n+1}(\omega)\right)-ST\left(\omega,\xi_{2n+1}(\omega)\right)\right\|^2.\left\|I\left(\omega,\xi(\omega)\right)-AB\left(\omega,\xi(\omega)\right)\right\|^2}{\left\|I\left(\omega,\xi(\omega)\right)-J\left(\omega,\xi_{2n+1}(\omega)\right)\right\|^2}$$

$$+ \beta(\omega) \|I(\omega, \xi(\omega))) - J(\omega, \xi_{2n+1}(\omega))\|^2.$$

Letting $n \to \infty$ and using the results of Step I of Case I, we have

$$\|AB(\omega, \xi(\omega)) - \xi(\omega)\|^2$$

$$\leq \alpha(\omega)\frac{\left\|\xi(\omega)-\xi(\omega)\right\|^2.\left\|\xi(\omega)-AB\left(\omega,\xi(\omega))\right)\right\|^2}{\left\|\xi(\omega)-\xi(\omega)\right\|^2}$$

$$+ \beta(\omega) \|\xi(\omega) -\xi(\omega)\|^2$$

$\|AB(\omega, \xi(\omega)) - \xi(\omega)\|^2 \leq 0$

so that $AB(\omega, \xi(\omega)) = \xi(\omega) = I(\omega, \xi(\omega))$ for each $\omega \in \Omega$.

Since $AB(\omega, X) \subset J(\omega, X)$ and hence there exists a measurable mapping $g : \Omega \to C$ such that

$AB(\omega, \xi(\omega)) = J(\omega, g(\omega))$.

Therefore, $\xi(\omega) = AB(\omega, \xi(\omega)) = I(\omega, \xi(\omega)) = J(\omega, g(\omega))$.

Step III.

$$\|AB(\omega, \xi_{2n}(\omega)) - ST(\omega, g(\omega))\|^2$$

$$\leq \alpha(\omega)\frac{\left\|J\left(\omega,g(\omega)\right)-ST\left(\omega,g(\omega)\right)\right\|^2.\left\|I\left(\omega,\xi_{2n}(\omega)\right)-AB\left(\omega,\xi_{2n}(\omega)\right)\right\|^2}{\left\|I\left(\omega,\xi_{2n}(\omega)\right)-J\left(\omega,g(\omega)\right)\right\|^2}$$

$$+ \beta(\omega) \|I(\omega, \xi_{2n}(\omega))) - J(\omega, g(\omega))\|^2$$

$$\leq \alpha(\omega)\frac{\|\xi(\omega)-ST(\omega,g(\omega))\|^2.\|\xi(\omega)-\xi(\omega)\|^2}{\|\xi(\omega)-\xi(\omega)\|^2}$$

$+ \beta(\omega)\ \|\xi(\omega)-\xi(\omega)\|^2$

$\|\xi(\omega) - ST(\omega, g(\omega))\|^2 \leq 0$

so that $ST(\omega, g(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Therefore,

$ST(\omega, g(\omega)) = J(\omega, g(\omega)) = \xi(\omega)$.

Since (ST, J) are weakly compatible then

$ST(\omega, J(\omega, g(\omega)) = J(\omega, ST(\omega, g(\omega)))$,

$J(\omega, \xi(\omega)) = ST(\omega, \xi(\omega))$ for each $\omega \in \Omega$.

Step IV.

$\|AB(\omega, \xi(\omega)) - ST(\omega, \xi(\omega))\|^2$

$$\leq \alpha(\omega)\frac{\|J(\omega,\xi(\omega))-ST(\omega,\xi(\omega))\|^2.\|I(\omega,\xi(\omega))-AB(\omega,\xi(\omega))\|^2}{\|I(\omega,\xi(\omega))-J(\omega,\xi(\omega))\|^2}$$

$+ \beta(\omega)\ \|I(\omega, \xi(\omega))) - J(\omega, \xi(\omega))\|^2$

$\|\xi(\omega) - J(\omega, \xi(\omega))\|^2$

$$\leq \alpha(\omega)\frac{\|J(\omega,\xi(\omega))-J(\omega,\xi(\omega))\|^2.\|\xi(\omega)-\xi(\omega)\|^2}{\|\xi(\omega)-J(\omega,\xi(\omega))\|^2}$$

$+ \beta(\omega)\ \|\xi(\omega) - J(\omega, \xi(\omega)\|^2$

$\|\xi(\omega) - J(\omega, \xi(\omega))\|^2 \leq \beta(\omega)\|\xi(\omega) - J(\omega, \xi(\omega))\|^2$

$(1 - \beta(\omega))\ \|\xi(\omega) - J(\omega, \xi(\omega))\|^2 \leq 0$

so that $J(\omega, \xi(\omega)) = \xi(\omega)$.

Thus,

$AB(\omega, \xi(\omega)) = ST(\omega, \xi(\omega)) = I(\omega, \xi(\omega)) = J(\omega, \xi(\omega))$ for each $\omega \in \Omega$.

Therefore, $\xi(\omega)$ is a common random fixed point of AB, ST, I and J.

Similarly, we can also complete the proof when J is continuous.

Case II. Suppose that AB is continuous. In this case, we have

$AB(\omega, AB(\omega, \xi_{2n}(\omega))) \to AB(\omega, \xi(\omega))$.

Also by the semi-compatibility of pair (AB, I), we have

$AB(\omega, I(\omega, \xi_{2n}(\omega))) \to I(\omega, \xi(\omega))$, for each $\omega \in \Omega$.

Step I. For each $\omega \in \Omega$, we have

$\|AB(\omega, I(\omega, \xi_{2n}(\omega))) - ST(\omega, \xi_{2n+1}(\omega))\|^2$

$$\leq \alpha(\omega)\frac{\|J(\omega,\xi_{2n+1}(\omega))-ST(\omega,\xi_{2n+1}(\omega))\|^2.\|I(\omega,I(\omega,\xi_{2n}(\omega)))-AB(\omega,I(\omega,\xi_{2n}(\omega))\|^2}{\|I(\omega,I(\omega,\xi_{2n}(\omega)))-J(\omega,\xi_{2n+1}(\omega))\|^2}$$

$+ \beta(\omega)\ \|I(\omega, I(\omega,\xi_{2n}(\omega))) - J(\omega, \xi_{2n+1}(\omega))\|^2$.

Letting $n \to \infty$ and using above results, we get

$\|I(\omega, \xi(\omega)) - \xi(\omega)\|^2$

$$\leq \alpha(\omega)\frac{\|\xi(\omega)-\xi(\omega)\|^2 . \|I(\omega,\xi(\omega)) - I(\omega,\xi(\omega))\|^2}{\|I(\omega,\xi(\omega)) - \xi(\omega)\|^2}$$

$+ \beta(\omega)\ \|I(\omega, \xi(\omega)) - \xi(\omega)\|^2$

$\|I(\omega, \xi(\omega)) - \xi(\omega)\|^2 \leq \beta(\omega)\|I(\omega, \xi(\omega)) - \xi(\omega)\|^2$

yielding thereby

$I(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Step II. For any $\omega \in \Omega$,

$\|AB(\omega, \xi(\omega)) - ST(\omega, \xi_{2n+1}(\omega))\|^2$

$$\leq \alpha(\omega)\frac{\|\xi(\omega)-\xi(\omega)\|^2 . \|I(\omega,\xi(\omega)) - I(\omega,\xi(\omega))\|^2}{\|I(\omega,\xi(\omega)) - \xi(\omega)\|^2}$$

$+ \beta(\omega)\ \|I(\omega, \xi(\omega))) - J(\omega, \xi_{2n+1}(\omega))\|^2$.

Taking limit $n \to \infty$ and using the results of Step I of Case II, we have

$\|AB(\omega, \xi(\omega)) - \xi(\omega)\|^2$

$$\leq \alpha(\omega)\frac{\|\xi(\omega)-\xi(\omega)\|^2 . \|\xi(\omega) - AB(\omega,\xi(\omega)))\|^2}{\|\xi(\omega) - \xi(\omega)\|^2}$$

$+ \beta(\omega)\ \|\xi(\omega) - \xi(\omega)\|^2$

i.e. $\|AB(\omega, \xi(\omega)) - \xi(\omega)\|^2 \leq 0$

Hence, $AB(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$

Thus, $AB(\omega, \xi(\omega)) = I(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

As $AB(\omega,X) \subseteq J(\omega,X)$, there exists a measurable mapping $g' : \Omega \to X$ such that

$AB(\omega, \xi(\omega)) = J(\omega, g'(\omega))$.

Therefore, $\xi(\omega) = AB(\omega, \xi(\omega)) = I(\omega, \xi(\omega)) = J(\omega, g'(\omega))$.

Step III. For any $\omega \in \Omega$,

$\|AB(\omega, \xi_{2n}(\omega)) - ST(\omega, g'(\omega))\|^2$

$$\leq \alpha(\omega)\frac{\|J(\omega,g'(\omega)) - ST(\omega,g'(\omega))\|^2 . \|I(\omega,\xi_{2n}(\omega)) - AB(\omega,\xi_{2n}(\omega))\|^2}{\|I(\omega,\xi_{2n}(\omega)) - J(\omega,g'(\omega))\|^2}$$

$+ \beta(\omega)\ \|I(\omega, \xi_{2n}(\omega))) - J(\omega, g'(\omega))\|^2$

Taking limit $n \to \infty$ and using the results from above steps, we have

$\|\xi(\omega) - ST(\omega, g'(\omega))\|^2$

$$\leq \alpha(\omega)\frac{\|\xi(\omega) - ST(\omega,g'(\omega))\|^2 . \|\xi(\omega) - \xi(\omega)\|^2}{\|\xi(\omega) - \xi(\omega)\|^2}$$

$+ \beta(\omega)\ \|\xi(\omega) - \xi(\omega)\|^2$

$\|\xi(\omega) - ST(\omega, g'(\omega))\|^2 \leq 0$

so that $ST(\omega, g'(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.
Therefore,

$$ST(\omega, g'(\omega)) = J(\omega, g'(\omega)) = \xi(\omega) \text{ for each } \omega \in \Omega.$$

Now using the weak compatibility of (ST, J), we have

$$ST(\omega, J(\omega, g'(\omega))) = J(\omega, ST(\omega, g'(\omega))),$$
$$ST(\omega, \xi(\omega)) = J(\omega, \xi(\omega)) \text{ for each } \omega \in \Omega.$$

Step IV. For any $\omega \in \Omega$,

$$\|AB(\omega, \xi(\omega)) - ST(\omega, \xi(\omega))\|^2$$
$$\leq \alpha(\omega)\frac{\|J(\omega,\xi(\omega)) - ST(\omega,\xi(\omega))\|^2 . \|I(\omega,\xi(\omega)) - AB(\omega,\xi(\omega))\|^2}{\|I(\omega,\xi(\omega)) - J(\omega,\xi(\omega))\|^2}$$
$$+ \beta(\omega) \|I(\omega, \xi(\omega))) - J(\omega, \xi(\omega))\|^2$$

$$\|\xi(\omega) - J(\omega, \xi(\omega))\|^2$$
$$\leq \alpha(\omega)\frac{\|J(\omega,\xi(\omega)) - J(\omega,\xi(\omega))\|^2 . \|\xi(\omega) - \xi(\omega)\|^2}{\|\xi(\omega) - J(\omega,\xi(\omega))\|^2}$$
$$+ \beta(\omega) \|\xi(\omega) - J(\omega, \xi(\omega)\|^2$$

i.e. $\|\xi(\omega) - J(\omega, \xi(\omega))\|^2 \leq \beta(\omega)\|\xi(\omega) - J(\omega, \xi(\omega))\|^2$.
Hence, $\xi(\omega) = J(\omega, \xi(\omega))$.
Thus, $AB(\omega, \xi(\omega)) = ST(\omega, \xi(\omega)) = I(\omega, \xi(\omega)) = J(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Hence, $\xi(\omega)$ is a common random fixed point of the random operators AB, ST, I and J.

Similarly, we can also complete the proof when ST is continuous.

Uniqueness.

Let $h: \Omega \to X$ be another common random fixed point of the random operators AB, ST, I and J.

Then for each $\omega \in \Omega$,

$$\|\xi(\omega) - h(\omega)\|^2 \leq \|AB(\omega, \xi(\omega)) - ST(\omega, h(\omega))\|^2$$
$$\leq \alpha(\omega)\frac{\|J(\omega,h(\omega)) - ST(\omega,h(\omega))\|^2 . \|I(\omega,\xi(\omega)) - AB(\omega,\xi(\omega))\|^2}{\|I(\omega,\xi(\omega)) - J(\omega,h(\omega))\|^2}$$
$$+ \beta(\omega) \|I(\omega, \xi(\omega)) - J(\omega, h(\omega))\|^2$$
$$\leq \alpha(\omega)\frac{\|h(\omega) - h(\omega)\|^2 . \|\xi(\omega) - \xi(\omega)\|^2}{\|\xi(\omega) - h(\omega)\|^2}$$
$$+ \beta(\omega) \|\xi(\omega) - h(\omega)\|^2$$

$\|\xi(\omega) - h(\omega)\|^2 \leq \beta(\omega) \|\xi(\omega) - h(\omega)\|^2$ yielding thereby $\xi(\omega) = h(\omega)$.

Hence, $\xi(\omega)$ is a unique common random fixed point of AB, ST, I and J.

References

[1]. Badshah, V.H. and Gagrani, S.Common random fixed points of random multivalued operators on Polish space, Journal of Chungcheong Mathematical Society, Korea 18 (1), 2005, 33-40.

[2]. Badshah, V.H. and Sayyed, F.Common random fixed point of random multivalued operators on Polish spaces, Indian Jour. Pure Appl. Math. 33 (4), (2001), 573-582.

[3]. Badshah, V.H. and Shrivastava, N.Semi-compatibility and random fixed points on Polish spaces, Varahmihir Jour. Math. Sci., Vol. 6 No. 2, (2006), 561-568

[4]. Beg, I. and Shahzad, N.Random fixed points of random multivalued operators on Polish spaces, Nonlinear Anal. 20 (1993), No. 7, 835-847.

[5]. Beg, I. and Shahzad, N.Random approximations and random fixed point theorems, J. Appl. Math. Stochastic Anal. 7 (1994), No. 2, 145-150.

[6]. Choudhury, B.S. Convergence of a random iteration scheme to a random fixed point, J. Appl. Math. Stochastic Anal. 8 (1995), No. 2, 139-142.

[7]. Choudhury, B.S.A common unique fixed point theorem for two random operators in Hilbert spaces, IJMMS, 32: 3 (2002), 177-182.

[8]. Papageorgiou, N.S.Random fixed point theorems for measurable multifunctions in Banach space, Proc. Amer. Math. Soc. 97 (1986), No. 3, 507-514.

[9]. Sehgal, V.M. and Waters, C.Some random fixed point theorems for condensing operators, Proc. Amer. Math. Soc. 90 (1984), No. 3, 425-429.

[10]. Xu, H.K. Some random fixed point theorems for condensing and non expansive operators, Proc. Amer. Math. Soc. 110 (1990), No. 2, 395-400.