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ELEMENTS OF MODERN ALGEBRA

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ABSTRACT

Modern algebra is the study of algebraic structures and also about their properties. Modern algebra is the set of several advanced topics of algebra which deals with the *algebraic* structures other than the number systems. The important structures of the modern algebraic structures are fields, groups and rings. This study discusses about all the basic elements of the modern algebra such as groups, abelian group, rings and lattices.

Index Terms—Groups, Rings, Lattices, Fields and Modern algebra

I. INTRODUCTION TO MODERN ALGEBRA

Modern algebra is the area of mathematics which studies the algebraic structures such as fields, rings, modules, groups, algebras and vector spaces [4]. The phrase modern algebra was coined in the 20th century from algebra which studies focus on manipulating algebraic expressions and formulae such as real and complex numbers but now which is called as elementary algebra [3]. Modern algebra studies the patterns and properties which disparate the mathematical concepts in common. Modern algebra is also known as Abstract algebra, Computer algebra and Boolean algebra [14].

Some of the examples of algebraic structures with single binary operations are:

- Magmas,
- Quasigroups,
- Semigroups,
- Monoids and
- Groups.

In addition to these, there are some complicated examples that included in algebraic structures are:

- Fields,
- Rings,
- Lattices and
- Modules

II. BASIC ELEMENTS OF MODERN ALGEBRA

This section discusses about the basic elements of modern algebra such as groups, fields, rings and lattices.

a) GROUPS:

Group is the set on which it can able to define the binary operation which is associative, has the inverses for each of its elements and also has the identity element. Group is the algebraic structure that consists of set together with the operation which combines any two elements in order to form third element [7]. To be a group, the operation and set must satisfy 4 conditions called the group axioms [12]. The group axioms are:

- Closure,
- Associativity,
- Identity and
- Inverses

A group $(G$ is the nonempty set together with the binary operation on so, the following conditions should be hold to be group.

Closure: For all $a,$ in $G(a,b \in$, element $a \cdot$ is the uniquely defined element of the .

Associativity: For all For all $a,$ and in $G(a,b,c \in$, we have

$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$

Identity: There exists an identity element in $G(e \in$ such that,

$e \cdot a = a$ and $a \cdot e = a$

for all $a \in$.

Inverses: For each in $G(a \in$ there exists the inverse element a in $G(a^{-1} \in$ such that

$a \cdot a^{-1} = e$ and $a^{-1} \cdot a = e.$

Examples:

Real numbers with addition:

- $x, y \in R \Rightarrow x + y \in$
- $(x + y) + z = x + (y +$
- The identity element is as $x + 0 = 0 + x =$
- The inverse of is - as $x + (-x) = (-x) + x =$
- $x + y = y +$

In modern algebra, field is a commutative ring that contains multiplicative inverse for the nonzero elements and then the nonzero elements may form the abelian group under multiplication [1]. Field is the algebraic structure with notation of multiplication, division, addition and subtraction that the appropriate distributive law and abelian group equations [11]. The commonly used fields are the field of rational numbers, the field of complex numbers, and the field of real numbers but there are also some finite fields, algebraic number fields and field of functions.

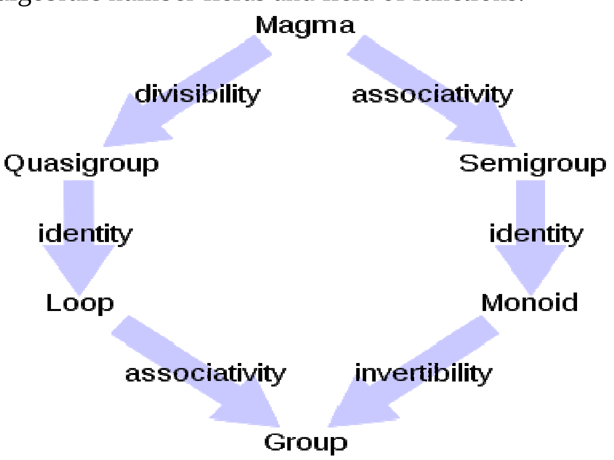

Figure: Elements of Group
Source: Allenby, R.B.J.T. (1991), *Rings, Fields and Groups*, Butterworth-Heinemann

The following figure indicates the various elements of the groups:

Magma: Magma is one of the basic kinds of the algebraic structure. Magma consists of the set M that equipped with the binary operation $M \times M \rightarrow M$. The binary operation is closed by the definitions and there will be no other axioms are involved in the operation.

Semigroup: Semigroup is the algebraic structure that consists of the set together with the associative binary operation. Semigroup generalizes the monoid and it does not need identity element. Semigroup is also a group and it does not have any inverse. The binary operation of the semigroup is denoted multiplicatively $x.y$ or xy and it results while applying in semigroup is just like the ordered pair (x, y).

Quasigroup: Quasigroup is the algebraic structure which resembles the group where, division is always possible in it. Quasigroup mainly differs from the groups and they are not need to be associative. The quasigroup with the identity element is known as loop.The quasigroup $(Q,*)$,where the set is Q with the binary operation$*$ (which is magma), such that: Foreach a and b in Q, there exist the unique elements x and y in Q such that:

- $a * x = b$;
- $y * a = b$.

b)FIELDS:

In modern algebra, field is a commutative ring that contains multiplicative inverse for the nonzero elements and then the nonzero elements may form the abelian group under multiplication [15]. Field is the algebraic structure with notation of multiplication, division, addition and subtraction that the appropriate distributive law and abelian group equations [9]. The commonly used fields are the field of rational numbers, the field of complex numbers, and the field of real numbers but there are also some finite fields, algebraic number fields and field of functions [5].

The field is a set which is equipped with 4 main operations such as: subtraction, addition, multiplication and division which have the usual properties [10]. The fields do not have other

operations like R has powers, roots, logs and other myriad functions like $sinex$, $cos\, x$.
Example:
Field of Rational numbers:
It consists of numbers that is written as fractions $a/b$where a and b are the integers and $b \neq 0$. The additive inverse of that fraction is $-a/b$, and multiplicative inverse is b/a.

$$\frac{b}{a} \cdot \frac{a}{b} = \frac{ba}{ab} = 1.$$

The required field axioms are reduced to the standard properties of the rational numbers (the distributivity law or associativity law or commutativity law).

$$\frac{a}{b} \cdot \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \cdot \left(\frac{c}{d} \cdot \frac{f}{f} + \frac{e}{f} \cdot \frac{d}{d}\right) = \frac{a}{b} \cdot \left(\frac{cf}{df} + \frac{ed}{fd}\right) = \frac{a}{b} \cdot \frac{cf+ed}{df} = \frac{a(cf+ed)}{bdf} = \frac{acf}{bdf} + \frac{aed}{bdf} = \frac{ac}{bd} + \frac{ae}{bf} = \frac{a}{b} \cdot \frac{c}{d} + \frac{a}{b} \cdot \frac{e}{f},$$

c) RINGs:

Ring is the algebraic concept which generalizes and abstracts the algebraic structure of the integers, most particularly two operations such as addition and multiplication. The concepts of rings mostly appear in the fields including geometry and mathematical analysis with the groups [8].
A ring is the triple $(R, +, \cdot)$ where R is the set, and $\cdot$ and $+$ are the binary operations on R (called multiplication and addition respectively) so that:

- $(R, +)$ is the abelian group (its identity is denoted by 0 and the inverse of $x \in R$ denoted as $-x$, as usual.)
- Multiplication is associative.
- The following distributive laws may hold $\forall x, y, z \in R$;
- Right distributive law:
$(x + y)z = xz + yz$
- Left distributive law:
$x(y + z) = xy + xz$

d)LATTICE:

Lattice is the partially ordered set, where any two elements have infimum (also known as meet or greatest lower bound) and supremum (also known as join or least upper bound). Lattices are also algebraic structures which satisfies particular axiomatic identities (Olsder et al, 1990). If two definitions are equivalent, then the lattice theory draws on both universal algebra and order theory (Grätzer, 1971).
The following figure illustrates the general concepts of partially ordered sets (lattice) of the modern algebra.

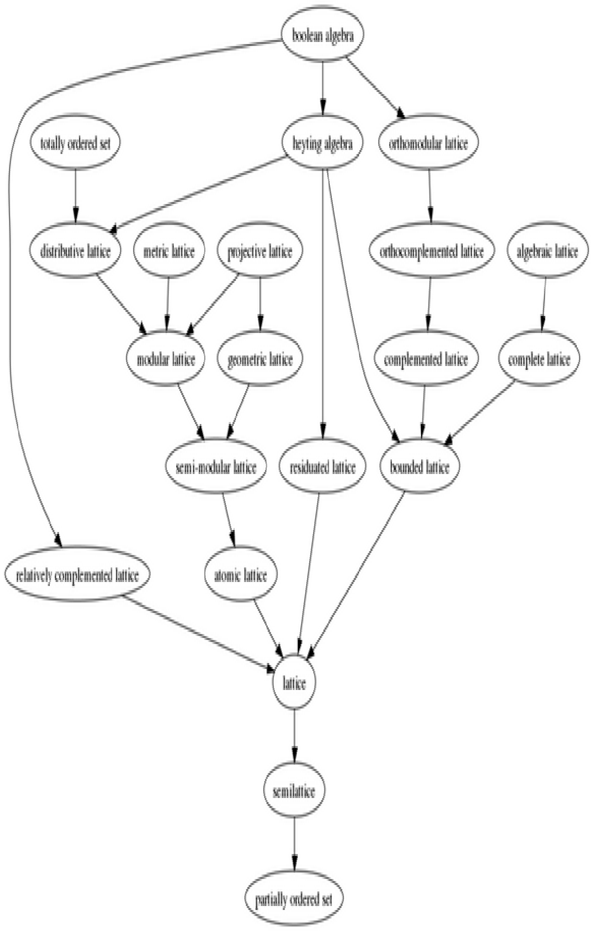

Figure: Partially Ordered Sets (Lattice)
Source:Burris, Stanley N., and H.P. Sankappanavar, H. P., (1981): A cource of universal algebra, Springer-Verlag. ISBN 3-540-90578-2

The algebraic structure $(L, \vee, \wedge)$ consists of the set L and two binary operations $\vee$, and $\wedge$, on the lattice L, if following axiomatic identities can hold the elements a, b, c of L.

Commutative laws

$a \vee b = b \vee a$,

$a \wedge b = b \wedge a$.

Associative laws

$a \vee (b \vee c) = (a \vee b) \vee c$,

$a \wedge (b \wedge c) = (a \wedge b) \wedge c$.

Absorption laws

$a \vee (a \wedge b) = a$,

$a \wedge (a \vee b) = a$.

The two identities are usually considered as the axioms, and they are followed from two absorption laws:

Idempotent laws

$$a \vee a = a,$$
$$a \wedge a = a.$$

Here, both $(L, \vee)$ and $(L, \wedge)$ are the semilattices.

III. CONCLUSION

It is concluded that modern algebra uses various basic structures and functions to perform the operations. The basic functions and operations are building blocks of the modern algebra.Modern algebraic expressions and formulae help to solve various complex programs. Modern algebra is used many applications and programming languages. In addition to these, modern algebra is used in various industries such as physics, chemistry, and so on.This research concludes that groups, fields, rings and lattice are most basic and general concepts that used in the modern algebra. These are the basic elements that come throughout the modern algebra.

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