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GAME THEORY IN ECONOMICS

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ABSTRACT

The theory of games provides a set of mathematical techniques for analyzing situations in which each agent's utility depends not only on his own actions but also on the actions of others; and all of the agents take these interdependencies into account when deciding their actions. Von Neumann and Morgenstern explained the difference between a game-theoretic problem and an optimization problem of the sort more traditionally solved in economic theory, such as a single consumer's maximization problem. This paper exhibits the rich versatility of these theories and lively interaction between the mathematical theory of games and significant economic problems.

Index Terms: Game theory, static games, cooperative games, non cooperative games, two person games, linear complementary problem and game theory and economic applications

I. INTRODUCTION

With the arrival of the New Industrial Organization Theory, a new instrument for analyzing and predicting oligopolistic, interdependent, behavior has been developed, i.e. 'game theory'. Although Neumann and Morgenstern developed the first seminal work in the field of Game Theory in 1944, it did not receive full attention until the 1970s. Since then, game theory has rapidly developed into a useful instrument, which can be used to gain insight into the dynamic of oligopolistic interdependence.

Everything revolves around the search for a possible equilibrium or equilibria, i.e. a combination of the strategies that represent the best strategy for every competitor, or player, who is presumed to make 'rational' decision in order to maximize profits.

Straffin [2] has described that a game theoretic perspective differs from earlier economic understandings in part because it assume that each manager adopts his or her best strategy, based on assessment of competitors best strategies, not simply that managers in oligopolistic markets attempt to take competitors likely reactions into account.

Two main branches of game theory exist: *cooperative* and *non cooperative game theory*. In cooperative game theory models competitors are allowed to make binding agreements that restrict their feasible strategies. As the particular problem of oligopoly is that competitors behave independently on the market, cooperative game theory is not generally used to analyze oligopoly games.

Myerson [3] has described that Non cooperative game theory is the currently accepted economic mode of analyzing oligopoly interactions. This branch of game theory begins with a non-cooperative viewpoint, i.e. it assumes that each firm's independent choice of its best strategy will result in equilibria (of strategies), that are non-cooperatively optimal given the others similarly calculated optimal strategies. It is presumed that firms cannot communicate, or they cannot, at least, rely on the contents of the communication.

1. STATIC GAMES:

According to Shoham and Leyton-Brown [4] the one-shot game model constitutes the simplest form of non-cooperative game theory. In such a static game, every player can move only once, without having knowledge of the move of the other player. It is presumed that both players move contemporarily, so that there is no possibility of knowing beforehand the move the other player makes. An example:

Prisoner's Dilemma*: Suppose two villains are caught in a stolen car after having robbed a bank. Both are being put into separate cells in prison; there is no possibility that they can communicate with each other. The District Attorney lacks the required proof in order to charge the villains with bank-robbery, and would need a confession of one of the crooks. She does however have sufficient proof to charge them with car stealing. The villains, being separately interrogated, are being given the following choice: confess or deny. Do both confess, they will be charged with six years of imprisonment each. If only one of them confesses, he will walk a free man, while the other, who denied, will have to face ten years in jail. Do both deny, they can only be charged with car stealing, the punishment for which is one year of imprisonment each. The choices can be reproduced in the following 'pay-off matrix'., indicating the years in prison:*

A/B	*Confess*	*Deny*
Confess	*6.6*	*0.10*
Deny	*10.0*	*1.1*

Choosing purely out of self-interest, A is better of confessing whatever choice B makes. The same holds true for B, the outcome then will be that both confess and both will be imprisoned for six years.

2. TWO-PERSON GAMES:

Helmes and Srinivasan [5] consider the problem of estimating an unknown parameter vector θ through a vector y which can be observed. More precisely, the question they address is to find a linear combination of the data y which minimizes the minimum risk of all such procedure. A solution to this problem is offered through fractional programming. They also present an efficient method to solve the fractional programming problem in some special cases.

Bapat and ijis [6] consider a matrix game in which the pay-off matrix is the vertex-edge incidence matrix of either a directed or undirected graph. For the directed incidence matrix game, they derive results on the value and the structure of optimal strategies when the graph has no directed cycle. The problem of determining strategies for the undirected incidence matrix games is shown to be related to the theory of 2-matchings.

Parthasarathy, Ravindran and Sabatini [7] study injectivity of cubic linear mappings which is related to the (real) Jacobian Conjecture. They derive results using results from completely mixed games due to Kalplansky.

Mishra and Krishna Kumar [8] consider the problem of obtaining a pure strategy equilibrium in matrix games with random pay-offs. In that context they generalize the notion of separation of diagonals due to von Neumann and Morgestern and give a set of necessary and sufficient conditions for the game to have mixed strategy equilibrium.

3. COOPERATIVE GAMES:

Sudholter and Beleg [9] gives a survey on modified nucleolus of a game, its definition, interpretation and a list of elementary properties. In the latter half of his paper, he discusses the notion of modified kernel as well as modified bargaining set of a game.

Athey [10] discuss the relationship between the prenucleolus and a new value, called ENPAC-value. The authors give several alternative sufficient conditions for the equality of the ENPAC-value to the prenucleolus.

Yanovskaya [11] considers three solutions, namely Core, (pre)nucleolus, and (pre)kernel for cooperative games for nontransferable utilities. She studies these solution concepts with the help of excess function. It is shown that both the prenucleolus and the prekernel do not possess the reduced game property for all excess functions satisfying Kalai's condition.

Axiomatic characterization of a core and the collection of cores are given for some excess functions.

Ewehart [12] focus their attention on a uniform treatment of a special type of one point solution for cooperative games, called egalitarian non-individual contribution (ENIC) value. The main goal of the authors is to provide an axiomatic characterization of the ENIC-Value in general to construct four particular ENIC-values.

4. NONCOOPERATIVE GAMES:

Harsanyi [13] attaches for each normalized NTU-game (N, V), a relevant strategy game Tu. Then they show that there is a nice correspondence between core allocations of (N, V) and Nash equilibria Tu. Further a relation is described between the pay-off function of the strategic game and remainder map considered by Driessen and Tijs.

Ali Khan, Kali Rath and Yeneng Sun [14] present an example of a nonatomic game without pure Nash Equilibria. They also present a theorem on the existence of pure strategy Nash Equilibria in nonatomic games in which the set of players is modeled on nonatomic Loeb measure space.

Abraham Neyman and Sylvain Sorin [15] in their paper on equilibria in repeated games, show that every two person game incomplete information in which the information to both players is identical and deterministic has equilibrium.

Vemeulen, Potters and Jansen [16] introduce a new kind of perturbations for normal form games and they investigate the stability of these perturbations. The CQ sets obtained in this manner satisfy the Kohlberg-Mertens program except invariance. In order to overcome this problem, the authors modify their solution concept in such a way that all properties formulated by Kohlberg Mertens are satisfied.

5. LINEAR COMPLEMENTARITY PROBLEM AND GAME THEORY:

Amit and Murthy [17] introduce a 'chain rule' and show that no Q0-matrix can satisfy this rule. They use this rule to answer a certain conjecture due to Stone for 5*5 matrices. Thus chain rule is quite handy in many situations to decide whether a given matrix is Q0 or not.

Mohan, Neogy and Parthasarathy [18] consider n-person stochastic games, with finite state and action spaces, in which player n controls the law of motion and where each plater wants to minimize his limiting average expected costs. For such games the authors show that stationary equilibrium strategies can be computed by applying Lemke's algorithm to solve a related linear complementarity problem. This result is quite useful from the point of view of algorithms.

Romans Snajder and Seetharama Gowda [19] investigate the Lipschitz continuity of the solution map in the settings of horizontal, vertical and mixed linear complementarity problems. In each of the settings, they show that the solution map is (globally) Lipschitzian if and only if the solution map is single valued. These generalize a similar result of Murthy, Parthasarathy and Sabatini proved in the LCP setting.

5. ECONOMIC APPLICATIONS

Guillermo Owen [20] considers the following: a bookie (pari-mutuel system), faced by several betters with different subjective probabilities, has the problem of choosing pay-off odds so as to avoid the risk of loss. It is shown under some conditions equilibrium set of pay-off odds exists. Some examples are worked out in detail.

Hubert Chin describes a heuristic approach for finding the nucleolus of assignment games using genetic algorithms. It is not clear how the algorithm proposed here compares with the other known algorithms.

Raghavan [21] gives a nice survey on algorithms to compute nucleolus for structured cooperative games. This is based on the current algorithms available to calculate nucleolus effectively for (i)

General games (Studied by Potters Reijnerse and Ansing) (ii) Assignment games (Raghavan and Solymosi) (iii) Tree games (Maschler, Owen, Granot and Zhao) (iv) Interval games (studied by Driessen, Solymosi and Aarts).

Bettina Klaus [22] considers the problem of reallocating the total endowment of an infinitely divisible good among agents with single-peaked preferences and study several properties of reallocation rules such as individual rationality, endowment monotonicity, envy freeness and bilateral consistency. Main result is the proof that individual rationality and endowment monotonicity imply Pareto Optimality. The result is then used to give two characterizations of the uniform reallocation rule.

Ahmet Alkan considers a model of sealed bid actions with resale. The policy question whether the seller would fare better under the multiprice rule (where winners pay their actual bids) or the uniprice rule (where winners pay the highest losing bid) has been discussed since the 60's and seen a recent revival. While theory has mostly recommended the uniprice rule, the results of the present author recommend the multiprice rule.

Alan Richards and Nirvikar Sing [23] analyses the impact of a two-level game for water allocations. Nash bargaining theory is used to derive several propositions on the consequences of different bargaining rules for water allocations. The effect on international negotiations of the ability to commit to having domestic negotiations is also examined. The authors cite several live examples of two-level games over water.

Meenakshi Rajeev [24] considers the role of money as a medium of exchange in a competitive set up. Her set-up is derived from the frame-work of Kiyotaki and Wright. She examines how monetized trading posts set-up manifest itself through (the agent's) behavior.

6. CONCLUSION

Dimand and Dore [25] analyses of oligopolies can be seen as a one-shot game, depending on the variable the competitors have chosen to determine their strategies. The choices oligopolists face in the market-game can be illustrated by the so-called 'prisoner's dilemma'.

Such a 'dilemma' can be transported to oligopolistic market situations, where firms have to choose between setting a high or a low price (or output, depending on the variable chosen).The outcome of the dilemma is that both firms will prefer not to run the risk of losing demand by being the only one charging the high price, and both firms will set a low price, earning lower profits than would have been possible by both setting the high price.

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