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ANALYZING THE DIGITAL NOTE PROGRESSION OF RAGAS WITHIN A *THAAT* USING FRACTAL GEOMETRY

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ABSTRACT*:*

Music is one of the oldest forms of art, something to experience and enjoy. In recent times we have seen a great interest of modern science interacting with this highly emotional and experiential phenomenon of music. Music is organized sound that is capable of conveying emotion; hence melody has to be ordered successions of musical notes and it is of interest to investigate if the successions depict a fractal nature. Successions are *fractal* if the incidence frequency F and the interval between successive notes i in a musical piece bear the relation: $F = c/i^{D}$ where D is the fractional dimension and c is a constant of proportionality. The present work compares four ragas within a *thaat* (a raga-group based on scale) using fractals. Two of the ragas are of restful nature and the remaining two restless. Our findings are very interesting. Fractal nature is found to be far more prominent in both the restless ragas! We propose to extend the work to other *thaats* also as well as compare ragas between *thaats*.

Key Words— Fractals, melody, musical notes, *thaat*, raga

1. INTRODUCTION

Music is a way of expressing nature's harmony, and fractals are the tools that describe this harmony. Naturally it is of interest to explore the application of fractals in music. One direction of research could be to investigate whether a musical succession of digital notes depicts a fractal nature or not. Another interest can be if we can mathematically characterize the difference between the musical component of ragas (with different moods) within a *thaat* or those with similar moods between the *thaats* or finally can we make fractal music (music generated algorithmically using fractals) interesting? In this paper we are interested in comparing ragas within a *thaat* using fractals while comparing ragas between *thaats* using fractals is reserved as a future work.

1.1 Fractal and Music

Fractals are geometric shapes with interesting properties that set them apart from normal Euclidean shapes. The first interesting property is that of self similar nature. Another property of fractal is a non integer dimension which is related to the concept of self similarity. The term fractal was coined by Benoit Mandelbrot in 1975 to describe shapes that are "self-similar" – that is, shapes that look the same at different magnifications and we refer to his classic treatise [1] for an insight.

Mandelbrot's fractal geometry has provided a new qualitative and quantitative approach for the understanding of the complex shapes of nature. The calculation of fractal dimension is an important way to classify objects that exhibit fractal characteristics.

The relative abundance or the incidence frequency F, of notes of different acoustic frequency f in a musical composition is not fractal [2]. Unplanned striking of the keys in a piano or a harmonium will not create music. Music is *organized sound that conveys emotion*; hence melody has to be ordered successions of musical notes. These successions are fractal if the incidence frequency F of the interval between successive notes i in a musical piece bear the relation

$F=c/i^D$ where D is the fractional dimension and C is a constant of proportionality [3].

or, ln(F)=ln(c)-Dln(i)=C-Dln(i) where C=ln(c), another constant. ……. (1).

Voss and Clark [4] determined that music exhibits $1/f$ - power spectra at low frequencies. This fact allows us to consider music as a time series and analyze the fractal dimension of a particular piece of music. Bigerelle and Lost [5] found the global *D* to be an invariant for different types of music. In another work [6], *D* in the music of Mozart and Bach was calculated. Hsu and Hsu [3] discussed the application of *D* to music in detail and for a work of Bach, found *D* to be 2.418.

1.2 Ragas and *Thaats*

A raga, in Indian classical music (both Hindustani and Cartatic), may be defined as a melodic structure with fixed notes and a set of rules that characterize a particular mood conveyed by performance. According to Vishnu Narayan Bhatkhande (1860-1930) one of the most influential musicologist in the field of Hindustani Classical Music in the twentieth century, each one of the traditional ragas is based on, or is a variation of ten basic *thaats*, or musical scales or frameworks. The ten *thaats* are Bilawal, Kalyan, Khamaj, Bhairav, Poorvi, Marwa, Kafi, Asavari, Bhairavi and Todi. If one were to pick a raga at random, it should be possible to find that it is based on one or the other of these *thaats* [7]. For instance, the four ragas Bhimpalashi (or Bhimpalashree), Pilu, Bageshree and Kafi studied here all belong to the Kafi *thaat*. Table 1 gives the distinguishing musical features of these ragas.

Table 1: Musical Feautures of four ragas of the Kafi *thaat*

Musical feature:-	Raga 1: Bageshree	Raga 2: Pilu	Raga 3: Kafi	Raga 4: Bhimpalashi
Thaat	Kafi	Kafi	Kafi	Kafi
Arohan (ascent)	S g M D n **S**	*N* S g R g, M P, d P, n D P, **S**	S R g M P D n **S**	*n* S g M P n **S**
Awarohan (Descent)	**S** n D, M P D g, M g R S	**S** n D P M g, *N* S	**S** n D P M g R S	**S** n D P M g R S
Pakad (note assembly giving a catch of the raga)	S *n D*, S M D n D, M g R S	N S g N S, *P d N* S	S S R R g g M M P	*n* S M, M g, P M, g, M g R S
Vadi-Samvadi Swars (Most important and second most important notes)	M, S	g, N	P, S	M, S
Jati (raga group according to number of distinct notes allowed in ascent and descent)	*Aurabh-Sampoorna (5 distinct notes allowed in ascent; 7 in descent)*	*Sampoorna-Sampoorna 7 distinct notes allowed in ascent; 7 in descent)*	*Sampoorna-Sampoorna (7 distinct notes allowed in ascent; 7 in descent)*	*Aurabh-Sampoorna (5 distinct notes allowed in ascent; 7 in descent)*
Nyas Swars (stay notes in the raga)	g M D	G P N	R g M P	g M P n
Prakriti (nature)	Restful	Restless	Restless	Restful
Time of rendition	9PM-12PM	12.00-3PM	Midnight	1PM-3PM

Notes 1: The raga Pilu discussed here is actually *Mishra Pilu* and uses two notes N and d which are not used in the other three ragas of the Kafi thaat. In fact, these two notes are not among the parent notes of the Kafi *thaat*, namely, S R g M P D and n. However, these additional notes do make this raga more coulourful and suitable for thumris, a lighter form of Hindustani classical music and it is the lighter form of Pilu that is more popular and hence being analyzed here.

2: Kafi is both a *thaat* and a raga within this *thaat*. Raga Kafi should not be confused with Kafi as a *thaat* which is a raga-group and not an individual raga.

Abbreviations: The letters S, R, G, M, P, D and N stand for Sa, *Sudh* Re, *Sudh* Ga, *Sudh* Ma, Pa, *Sudh* Dha and *Sudh* Ni respectively. The letters r, g, m, d, n represent *Komal* Re, *Komal* Ga, *Tibra* Ma, *Komal* Dha and *Komal* Ni respectively. Normal type indicates the

note belongs to middle octave; italics implies that the note belongs to the octave just lower than the middle octave while a bold type indicates it belongs to the octave just higher than the middle octave. Sa, the tonic in Indian music, is taken at C. Corresponding Western notation is also provided. (see table 2) The terms "*Sudh*", "*Komal*" and "*Tibra*" imply, respectively, natural, flat and sharp.

2. METHODOLOGY

We take four different ragas from Kafi *thaat* and calculate intervals *i* as the absolute values of differences in pitch of two successive notes. For each sequence of notes, a frequency distribution is found of the intervals. Accordingly four tables of ln *F* versus ln *i* are formed, one for each raga. Calculations are made only for those values of *F* and *i* for which both ln *F* and ln *i* are defined. The note sequences are taken from a standard text [7] and not from any audio recording. There are some obvious advantages and disadvantages for doing so. If we go for audio recordings, it is not always necessary that the same raga performed by different artists (or even the same artist on different occasions) will exhibit the same fractal nature. Even if we analyze a single recording of an artist, it is not easy to say which part of the fractal nature is attributable to the raga itself and which part to the style. In a structure analysis, the style of the artist does not interfere with our analysis whereby the fractal nature can be studied for its presence (with dimension) in the raga structure itself in a general sense. The technique is to assign the number 0 to C (where the tonic Sa or S is taken), 1 to the next note Db (Komal Re or r) and so on (table 2). On the disadvantage side, we miss information on note duration and pitch movements between the notes which we could get in audio recordings.

Table 2: Numbers representing pitch of notes [8]

C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B
S	*r*	*R*	*g*	*G*	*M*	*m*	*P*	*d*	*D*	*n*	*N*
(lower octave)											
-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
S	r	R	g	G	M	m	P	d	D	n	N
(middle octave)											
0	1	2	3	4	5	6	7	8	9	10	11
S	**r**	**R**	**g**	**G**	**M**	**m**	**P**	**d**	**D**	**n**	**N**
(higher octave)											
12	13	14	15	16	17	18	19	20	21	22	23

3. EXPERIMENTAL RESULTS

Our experimental results are summarized in tables 3-6 and corresponding fig. 1-4 for the ragas Bageshree, Pilu, Kafi and Bhimpalashi respectively.

***Thaat*: Kafi** Raga: Bageshree

i	F	lni	lnF
0	10		2.30
1	41	0.00	3.71
2	67	0.69	4.20
3	11	1.10	2.40
4	32	1.39	3.47
5	15	1.61	2.71
6	3	1.79	1.10
7	1	1.95	0.00

Table 3: Fractal Analysis for Bageshree

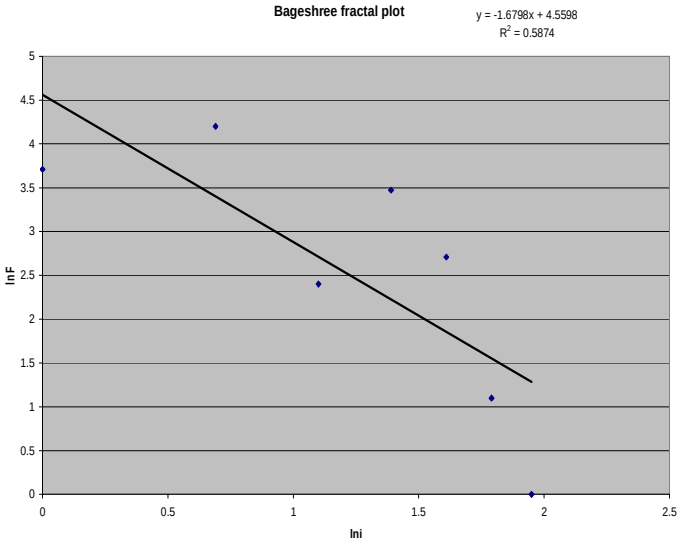

Fig. 1 Fractal Plot of Bageshree

i	F	lni	lnF
0	6		1.79
1	48	0.00	3.87
2	74	0.69	4.30
3	25	1.10	3.22
4	21	1.39	3.04
5	4	1.61	1.39
6	1	1.79	0.00
11	1	2.40	0.00

Table 4: Fractal Analysis for Pilu

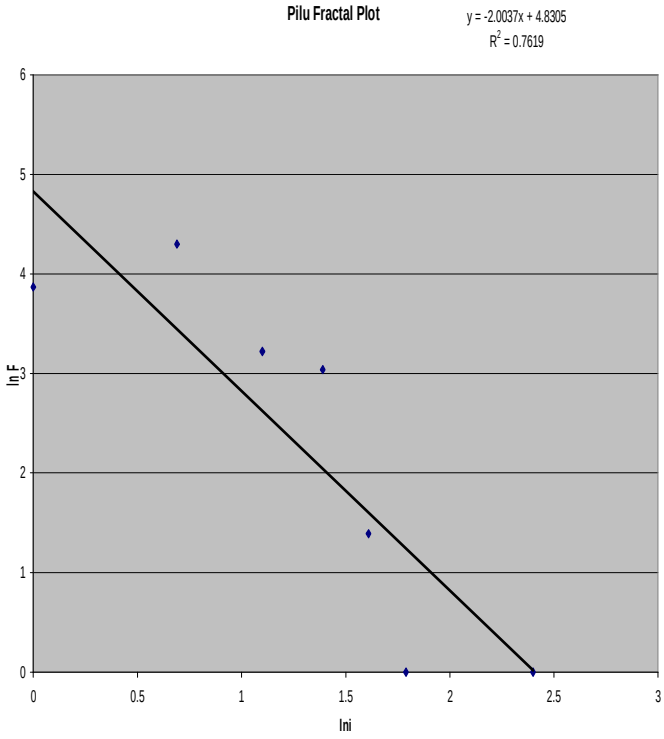

Fig. 2 Fractal Plot for Pilu

i	F	lni	lnF
0	12		2.48
1	43	0.00	3.76
2	108	0.69	4.68
3	7	1.10	1.95
4	4	1.39	1.39
5	3	1.61	1.10
6			
7	3	1.95	1.10

Table 5: Fractal Analysis for Kafi

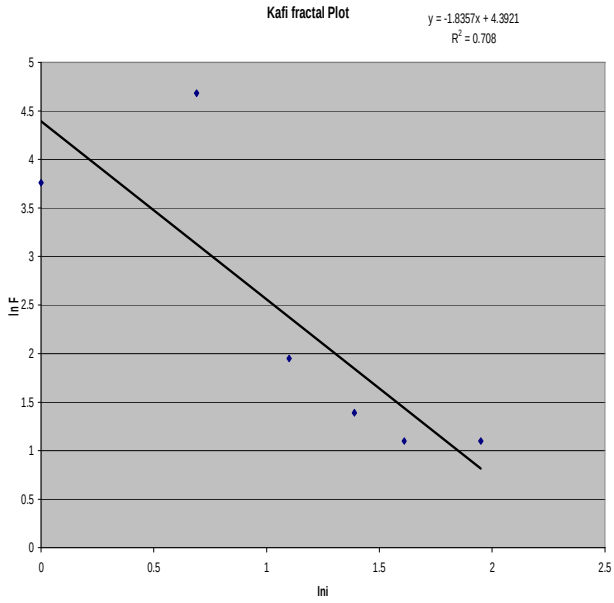

Fig. 3 Fractal Plot for Kafi.

i	F	lni	lnF
0	8		2.08
1	16	0.00	2.77
2	126	0.69	4.84
3	7	1.10	1.95
4	11	1.39	2.40
5	11	1.61	2.40
6	0	1.79	
7	1	1.95	0.00

Table 6: Fractal Analysis for Bhimpalashi

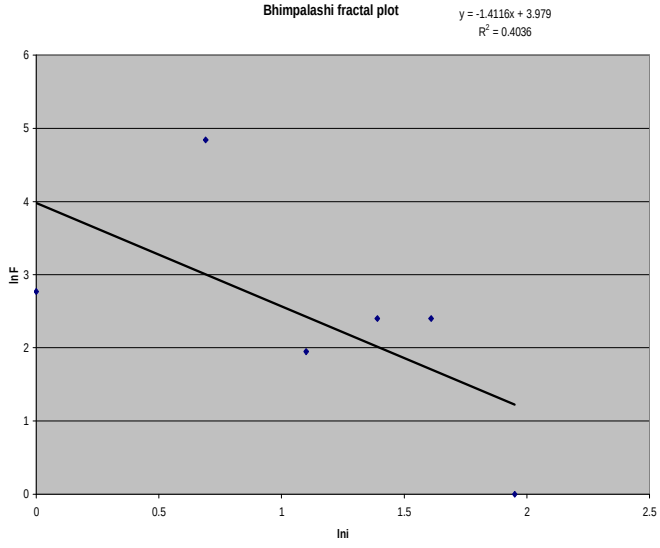

Fig. 4 Fractal plot for Bhimpalashi

4. DISCUSSION

The results on the four ragas are indeed very interesting. The ragas Kafi and Pilu are depicting fractal nature with comparatively higher dimension than the other two ragas (2.0037 for Pilu and 1.8357, i.e., nearly 2). The fractal nature in raga Bhimpalashree is not so prominent as R^2 is not very high with fractal dimension 1.4116 and same is true about the other raga Bageshree which has fractal dimension 1.6798. $100R^2$, also called % coefficient of determination, gives the percentage of variation in the response (here lnF) explained by the predictor (here lni) through the model (here a straight line). In a recent paper [9] it has been argued that fractal dimension is related with the *chalan* (melodic movement) of the raga. Thus our present finding suggests that Kafi and Pilu can be placed in one sub-group within the same *thaat* while Bageshree and Bhimpalashree can be placed in another sub-group within this *thaat*. Musically, we already know that Kafi and Pilu areboth of restless nature and therefore can be placed in one group (*Chanchal prakriti* or restlessness nature). Bageshree and Bhimpalashree are both restful ragas and can be placed in another group (*shant prakriti* or restful nature). Hence the present study confirms once again that fractals do provide interesting mathematical properties that may be related to the melodic movement (in this case, whether restful or restless) of a raga. We caution the reader, however, that the nature (restful or restless) of the raga is not exactly the same thing as the mood or emotional content of the raga. For that matter, Pilu evokes sadness (*karuna rasa*) like Bageshree and Bhimpalashi while Kafi evokes joy in a romantic sense (*shringar rasa*). The paper [10] compares Kafi and Bageshree probabilistically using entropy.

Remarks 1: This is a paper on structure analysis of ragas. Readers interested in performance analysis of a raga are referred to [11]. This paper also gives a brief outline of what a raga is and how Indian classical music differs from Western classical music.

2. It is not correct to conclude in general that the restless ragas are more likely to exhibit a fractal nature. Our study is confined to only one *thaat* and there too, we can study only a limited number of ragas even within a *thaat*. The present work is only illustrative and by no means exhaustive. We caution the reader that Malkauns, a restful raga of Bhairavi *thaat* (some experts say Asavari *thaat* is correct for this grouping this raga [7]), has been found to exhibit good fractal nature! [12]
3. The reader is also referred to the works of H. V. Sahasrabuddhe [13].

5. CONCLUSION

The present work compares four ragas within a *thaat* using fractals. Fractal nature is found to be far more prominent in two of the ragas which happen to be of restless nature as compared to the other two which happen to be restful ones. Although the finding is interesting, yet in the light of the second remark made in the previous section, we have extend the work to other *thaats* as well as compare ragas between *thaats* and investigate a good number of ragas in each of the ten *thaats* in order to concretize things.

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