THEORETICAL APPROACH TO STUDY THE THERMAL STRESS ON HUMAN BRAIN TISSUE IN HYPOTHERMIC CONDITIONS

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ABSTRACT

The thermal equilibrium in human head is more stable as compared to other body organs. It is therefore desirable to develop a numerical model to monitor thermal changes in the head at hypothermic conditions. A theoretical model has been developed for the estimation of temperature in the human head surface, human brain surface and overlying layers at cold environmental temperatures. The study involves the essential factors responsible for the distribution in brain tissue including, the central blood flow, the heat transfer coefficient due to the exchange of the temperature between the atmospheric temperature and head surface. The effect of atmospheric temperature has been analyzed at micro level for rapid change in outer layers of the head. The estimation is based on bio-heat equation associated with biophysical and biochemical reactions taking place. The transient cooling of an anatomically correct realistic three-dimensional head with realistically varying local tissue properties was numerically simulated using the finite element method (FEM). The simulations performed in this study consider exposure of head at severe cold and contact with cold materials.

Keywords: heat transfer; mathematical model; FEM; Cold exposure.

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1. INTRODUCTION

Cold exposure to the body core is called hypothermia-(decrease in core temperature) and cold injury to body's shell is called frostbite. Hypothermia and frostbite commonly occur together, but they can occur separately as well. Deleterious effects of extreme cold are commonly seen in the high altitude areas and also cold zone regions.

Hypothermia and local cold injuries are the two main problems encountered due to severe cold conditions. Many cases of frostbite reported during various mountaineering expeditions and hence the problem of cold injuries has gained considerable importance for the civil population too.

It is known that the peripheral tissues cool more rapidly than the central tissues on exposure to cold, and hence they are more liable to cold injuries. As the extremities contain very little muscle mass, the capacity for heat production during cold exposure is less. In the cold areas of high altitude the problem becomes more intense since hypoxia is superimposed over cold stress which results in marked reduction of extremity blood flow. Besides frostbite, trench foot and chilblains are other forms of cold injuries generally seen in our mountainous areas. The brain is considered as the control unit for the whole body, therefore the thermostat of brain is not only important for the stability of the brain tissue but also to the rest of the body for proper functioning. Any adverse change in brain temperature can develop secondary to toxin exposure, metabolic derangements, infections, and dysfunction of the central nervous and endocrine system. The decrease in the brain temperature to a level at which normal muscular and cerebral function is impaired is the hypothermic condition. Thus, it is imperative to study the temperature changes in human brain tissue in hypothermic conditions. A mathematical model is presented to estimate the effect of cold ambient temperature to the surface of brain tissue, surface of head together with the response on the head layers including CSF, Skull and Scalp. The main objective of this study is to investigate the temperature profiles in human brain and peripherals at hypothermic conditions. The estimation of various types of thermal injuries taking place in cold ambient conditions has been studied by using finite element method. Due to the fact that the head has an irregular geometry it is worthwhile to say that the finite element method plays a key role to have realistic values of nodal temperatures at various regions of human head layers.

2. MATERIALS AND METHODS

The bioheat equation initially given by Penne's [7] in terms of diffusion, perfusion and rate of metabolic heat generation is the basic tool to estimation of temperature profiles in biological tissues.

Several computer-simulated models of temperature distribution in human body have been developed. While these models provide important insights into the problem, their results substantially depend on input parameters that are not always known and may vary in broad ranges. The goal of this study is to develop a model describing temperature distribution in various layers of head including human brain. The model provides analytical expressions that allow evaluation of changes in temperatures under the influence of measurable input parameters. Perl [1962] suggested a method for the study of temperature in biological tissues by combining Fick's perfusion principle with heat diffusion. Patterson [6] has experimentally determined temperature profiles of the outer skin using radio-camera. Chao and Yang [1] considered two simple models and obtained temperature distribution curve in skin and subcutaneous tissue for certain fixed values of parameters. Later on Saxena [8] and Saxena and Arya [9] and Saxena and Gupta [10] used variational finite element approach with linear shape functions to find one dimensional unsteady state temperature distribution in SST assuming rate of blood flow and rate of metabolic heat generation as variable in the dermis part. Saxena and his coworkers used variational finite element method to estimate temperature distribution in human limbs. Khanday and Saxena [3] used an advanced model to estimation of cold effect on human dermal parts, in which one dimensional steady state

case has been described over five layered skin and subdermal tissues. Also, in Khanday and Saxena [4,5] worked on the study of thermoregulation and fluid regulation in human head and human dermal regions respectively at cold environmental conditions using variational finite element method. The researchers in last few years have tried to formulate mathematical models in human head in cold conditions by using various techniques. Our group has formulated various models on human dermal regions and human head using variational finite element method. The models were developed in one and two dimensional cases, the present model is a three dimensional model in which the finite element method together with initial, boundary and interface conditions were suitably incorporated.

The research presented will be used to monitor the thermal stress on human head due to the use of a realistic 3d geometry of human head and using physiological properties of large number of layers in human head. The main layers are scalp, skull, cerebral spinal fluid (CSF) and brain tissue. The numerical approximation seems to be more accurate and efficient.

3. Formulation of the Model

The estimation of thermoregulation throughout the perfused tissue in human body is based on the solution of the bioheat equation.

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x} \right) + \rho_b c_b F\left(T_{art} - T\right) + S_m$$
$$\dots (1)$$

where, ρ, c, K, ρ_b, c_b, F, T_{art} represent tissue density, specific heat of the tissue, thermal conductivity, density of the blood, specific heat of the blood, blood flow of tissue, the arterial perfusion temperature and

S_m is the heat regulation due to metabolism. This rate can be treated as blood flow through the various capillaries in the tissue. Moreover, we presume temperature-independent material properties and heat source terms. Under these conditions analytic solutions can be found only for very simple geometries. For realistic solution for temperature profiles, we must resort to numerical estimations of the bio-heat transfer equation. A finite element method, which is described, was used earlier by Khanday and Saxena [4]. In the finite element method, the domain where the solution is sought is divided into a finite number of parts called elements. In this work, tetrahedral shaped elements were used exclusively since almost any complex 3d geometry, including a human head, can be decomposed into a finite number of well-shaped tetrahedral elements.

The boundary condition at the skin exposed to atmosphere is

$$-K \frac{\partial T}{\partial n} = h\left(T - T_a\right) + LE \quad \dots(2)$$

at the outer surface. $\partial/\partial n$ is the derivative along outward normal to the boundary surface. The first term on the right hand side of equation is the heat loss due to convection and radiation (h = h_r + h_c) and second term is the latent heat and sweat evaporation. Also T_a is the atmospheric temperature.

At the inner boundary surface, the core temperature being constant; we have

$$T = T_b \qquad \dots (3)$$

where T_b is the body core temperature.

Also at the interfaces between adjoining layers, we have

$$T_1 = T_2 \qquad \text{and}$$

$$\left(K \frac{\partial T}{\partial n} \right)_1 = \left(K \frac{\partial T}{\partial n} \right)_2 \qquad \dots (4)$$

The equation (4) represents respectively the continuity of temperature and heat flux between different subdomains of the region under study. Thus, the problem of heat migration in layers of human head ultimately reduces to finding a solution of partial differential equation (1) under the boundary and interface conditions given by equations (2) and (3).

4. Solution of the Model

Applying the method of weighted residuals to Eq. (1) over a single tetrahedral element Ω^e with weight function v results in

$$\int_\Omega v \left\{ \rho c \frac{\partial T}{\partial t} - k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - \rho_b c_b F(T_{art} - T) - S_m \right\} d\Omega = 0$$

…(5)

Integrating Eq. (5) by parts once creates the weak statement for the element

$$\int_{\Omega^e} v \rho c \frac{\partial T}{\partial t} d\Omega^e + \int_{\Omega^e} k \left(\frac{\partial v}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} + \frac{\partial v}{\partial z} \frac{\partial T}{\partial z} \right) + v \rho_b c_b F T d\Omega^e =$$
$$\int_{\Omega^e} v (\rho_b c_b F T_{art} + S_m) d\Omega^e - \int_{\Gamma^e} v (\vec{S} \cdot \vec{n}) d\Gamma^e$$

… (6)

Here, Γ^e is the element of the surface. Variation of the temperature across the element can be expressed by

$$T(x, y, z) = \sum_{i=1}^{m} N_i(x, y, z) T_i \quad \text{… (7)}$$

Here, i is an element local node number, m is the total number of element nodes, and N is the shape function associated with node i. Using Galerkin's method, the weight function v and the interpolation function for T are

chosen to be the same. By defining the matrix $[B]$ as

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_m}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_m}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \cdots & \frac{\partial N_m}{\partial z} \end{bmatrix}$$

… (8)

the weak statement in Eq. (6) can be written in the matrix form as

$$[C^e] \left\{ \frac{\partial T^e}{\partial t} \right\} + [K_C^e]\{T^e\} = \{R^e\}$$

… (9)

The local stiffness matrix, $[K^c_e]$, the local capacitance matrix, $[Ce]$, and the right hand side vector, $\{Re\}$, are evaluated as

$$[K_C^e] = \int_{\Omega_e} k[B]^T[B] + \rho_b c_b F\{N\}\lfloor N \rfloor d\Omega_e$$

… (10)

$$[C^e] = \int_{\Omega_e} \rho C_p \{N\}\lfloor N \rfloor d\Omega_e \quad \text{… (11)}$$

$$[R^e] = \int_{\Omega_e} (S_m + \rho_b c_b F T_{art})\{N\} d\Omega_e - \int_{\Gamma^e} S_s \{N\} d\Gamma^e$$

… (12)

for each element in the domain and then assembled into the global system of linear ordinary differential equations.

$$[C]\{\dot{T}\} + [K_C]\{T\} = \{R\} \quad \text{… (13)}$$

This global system of equations can be solved to obtain the approximate solution to the bioheat transfer equation of the entire domain. Here, the time derivative of temperature, $\dot{T}$, is discretized using a finite difference approximation in time. The Crank-Nicolson

algorithm is used and the final linear system of algebraic equations can be written as

$$\left(\frac{1}{2}[K_C]+\frac{1}{\Delta t}[C]\right)\{T_{n+1}\}=\left(-\frac{1}{2}[K_C]+\frac{1}{\Delta t}[C]\right)\{T_n\}+\frac{1}{2}(\{R\}_{n+1}+\{R\}_n)$$

… (14)

where the subscript $n+1$ denotes the current time step and n denotes previous time step. The steady state finite element formulation of the Pennes bio-heat equation is obtained similarly to Eq. (14) and takes the form

$$[K_C]\{T\}=\{R\}$$

…(15)

It is obvious from equations above that in general the finite element formulation of the Penne's bio-heat equation resembles that of the finite element formulation of conduction heat transfer in inanimate systems. The main difference is that in the stiffness matrix there is a term related to the volumetric blood flow in tissue and in the residual, forcing vector $\{R\}$, there are terms related to the arterial blood temperature and the metabolic heat generation. Stability and accuracy of numerical integration as a function of the equation matrices remains the same as in conventional inanimate heat transfer.

The approximate solution approaches the exact solution as the number of elements is increased and as the time step size, Δt, is reduced. However, increasing the number of elements and increasing the number of time steps also increases the computational cost.

5. Numerical Computation

It is important to note that due to hypothermic conditions, the vasoconstriction and other conditions results the decrease in central blood flow in the whole body and in particular to the brain tissue. The arterial blood temperature and the metabolic temperature shift in the brain and the scalp are $Ta_0 = Ta_3 = 37°C$, $S_{m0} = S_{m3} = 0.3°C$; the heat exchange between the scalp and the cooling intensity $T_e = 10°C$,

$\rho_0 = \rho_3 = 1.03 g/cm^3$, $\rho_b = 1.05 g/cm^3$, $c_b = 3.8 J/(g\ ^0C)$, $k_{0,1,2,3}$ = (5.03, 5.82, 6.5, 3.4) x 10^{-1} W/(m °C), F_3 = 10 ml/ (100 g min). The thicknesses of the layers (CSF, scalp. skull)): d_1 = 0.1 cm, d_2 = 0.3 cm, d_3 = 0.1 cm [Khanday, 2009]. In the figure-1, the temperature T, in the head for low central blood flow central blood flow F_0 (low) = 6 ml/100 g/min and normal central blood flow F_0 (norm) = 24 ml/100g/min. Lines 1, 2 correspond to h = h(norm) = 8 W/(m² °C). Lines 3, 4 correspond to the ideal cooling device ($h \rightarrow \infty$). A vertical line at x = 0 denotes the brain surface (interface brain/CSF) and other, vertical lines correspond to the interfaces CSF/skull, skull/scalp and head surface.
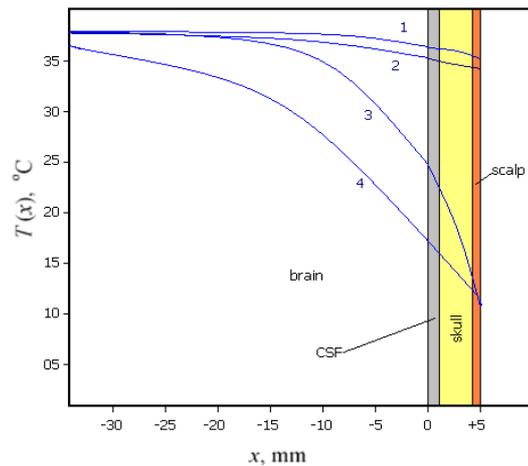

Figure-1: Temperature variations in Human Head for Normal and Hypothermic Conditions

In figure-3 and figure-4, the temperatures at the head surface T_{hs}, and the temperature at brain surface T_{bs} as functions of the heat transfer coefficient h for two values of central blood flow, F_0 (low) = 6 ml/100 g/min (lines 2, 4) and $F_0^{(norm)}$ = 24 ml/100 g/min (lines 1, 3) and the heat exchange between the scalp and the cooling intensity T_e, 10°C (lines 3, 4) and 20°C (lines 1, 2). For both the normal and low

central blood flow, the temperature at the head surface and at the brain surface decreases rather substantially as the heat transfer coefficient increases. However, for the case of low central blood flow, this temperature drops about 2–3°C (depending on T_e) when the heat transfer coefficient becomes 10–15 times higher than its normal value h (norm) = 8 W/(m^2°C) and then remains practically unchanged.
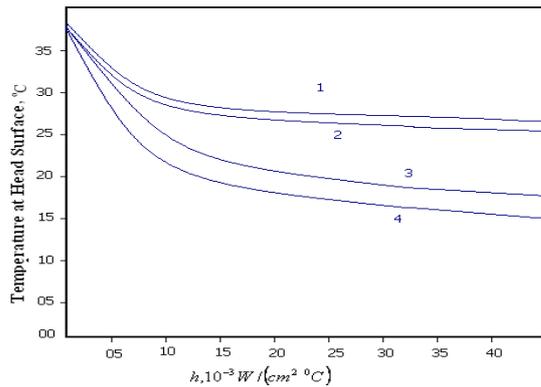
Figure-2: Temperature changes at Head surface vs heat transfer coefficient, h
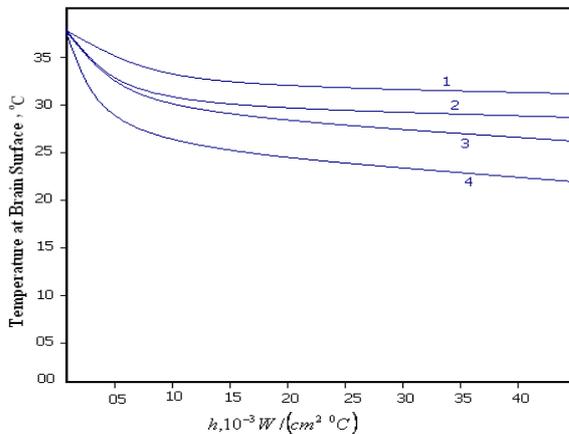
Figure-3: Temperature changes at Brain Surface vs heat transfer coefficient, h

6. DISCUSSION AND CONCLUSION

In the present study, the theoretical analysis using finite element approach has been used to estimate the temperature fluctuations in the human brain in hypothermic conditions. The essential factors responsible for the temperature distribution in the brain tissue are the central blood flow F_0, the temperature of the arterial blood flow T_a, the effective ambient temperature Te, and the heat transfer coefficient h correspond the exchange of temperature between cold ambient temperature and head surface. Practically, the core temperature T_b is homogeneous and slightly higher than T_a due to metabolic heat generation. The cerebral blood flow CSF acts as cooler for the brain tissue and removes heat produced by the brain metabolism. A different situation takes place in the superficial brain, where the temperature is usually lower than Ta due to the heat exchange with the cold ambient conditions (or with cold wind, coolers or any other cooling device). In this region, blood flow serves as a heater shielding the internal brain region from external cold assault. This effect plays the leading role in protecting the brain from external cooling, whereas the thermal resistance of the intermediate layers between the brain and the device (CSF, skull, scalp) plays a secondary role. The analysis shows that a targeted mild hypothermia (2–3°C temperature decrease) in deep brain (2–3 cm from the brain surface) can be achieved only if central blood flow is less than 10 ml/100 g/min. Thus, a prognosis for successful achievement of mild hypothermia in deep brain regions should be associated with sufficiently low central blood flow. This result makes it possible to introduce a necessary eligibility criterion for using brain cooling by means of decrease in surrounding temperature for achieving mild hypothermia therapy: Central blood flow should be lower than 10 ml/100 g/min. This leads to a long characteristic shielding length and allows ''cold assault'' to penetrate deeper in the brain. Note also that central blood flow in neonates can substantially fluctuate depending

on their activity, especially sleep/awake cycles.

These fluctuations will affect the characteristic temperature screening length, hence efficiency of brain cooling. Also, thermal receptors in the skin may react to head cooling by increasing metabolism of brown adipose tissue and consequently increasing temperature of arterial blood perfusing brain. This mechanism may affect successful achievement of mild hypothermia in deep brain regions.

The framework of the formulated model of temperature distribution in the brain tissue, it was established that a mild hypothermia can be successfully induced by means of external head cooling due to severe cold if two necessary conditions are fulfilled: sufficiently low cerebral blood flow and sufficiently high value of the heat transfer coefficient describing the heat exchange between the head surface and the intensity of the coldness.

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