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A COMMON RANDOM FIXED POINT THEOREM FOR SIX RANDOM MULTIVALUED OPERATORS SATISFYING A RATIONAL INEQUALITY

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ABSTRACT:

The purpose of this paper is to establish a common random fixed point theorem for six random multivalued operators satisfying a rational inequality using the concept of weak compatibility, semi compatibility and commutativity of random multivalued operators on Polish space.

Keywords: Commutativity, Polish space, semi compatible, weak compatible, random fixed point, random multivalued operators, measurable mapping.

Mathematical subject classification (2000): 47H10, 54H25.

1. INTRODUCTION

Probabilistic function analysis has come out as one of the important mathematical disciplines in view of its role in dealing with probabilistic model in applied sciences. The study of random fixed point forms a central topic in this area. Bharucha-Reid [8] have been given various ideas associated with random fixed point theory are used to form a particularly elegant approach for the solution of non linear random system. In recent years a vast amount of mathematical activity has been carried out to obtain many remarkable results showing the existence of random fixed point of single and multivalued random operators given by Spacek [13], Hans [9], Itoh [10], Beg [5], Beg and Shahzad [7], Badshah and Sayyed [2,3], Badshah and Gagrani [1], Beg and Abbas [6], Xu, H.K. [15], Tan and Yuan [14], O'Regan [11], Plubteing and Kuman [12] and other.
Recently Badshah and Shrivastava [4] introduced the concept of semi compatibility in Polish spaces and proved some random fixed point theorems for random multivalued operator on Polish spaces.

2. Prelminaries

We begin with establishing some preliminaries by $(\Omega, \sum)$ we denote a measurable space with $\sum$, a sigma algebra of subsets of Ω. Let (X,d) be a Polish space i.e. a separable complete metric space. Let 2^x be the family of all subsets of X and CB(X) denote the family of all non-empty bounded closed subset of X.

A mapping T: $\Omega \to 2^X$ is called measurable if for any open subset C of X,

$$T^{-1}(C) = \{\omega \in \Omega : T(\omega) \cap C \neq \varphi\} \in \Sigma$$

A mapping $\xi : \Omega \to X$ is called measurable selector of a measurable mapping T: $\Omega \to 2^X$, if ξ is measurable and for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$.

A mapping T: $\Omega \times X \to$ CB(X) is called a random multivalued operator if for every x$\in X$, T (., X) is measurable.

A measurable mapping $f : \Omega \times X \to X$ is called a random operator if for any x$\in X$, f (.,x) is measurable.

A measurable mapping $\xi : \Omega \to X$ is called random fixed point of random multivalued operator T: $\Omega \times X \to CB(X)\ (f : \Omega \times X \to X)$, if for every $\omega \in \Omega$, $\xi(\omega) \in T(\omega, \xi(\omega))$, $(f(\omega), \xi(\omega)) = \xi(\omega))$

Definition 2.1. [7] Let X be a Polish space i.e. a separable complete metric space. Mappings f, g: $X \to X$ are compatible if $\lim_{n\to\infty} d(fg(x_n), gf(x_n)) = 0$ provided that $\lim_{n\to\infty} f(x_n)$ and $\lim_{n\to\infty} g(x_n)$ exist in X and $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n)$. Random operators S, T: $\Omega \times X \to X$ are compatible if S(ω,.) and T(ω,.) are compatible for each $\omega \in \Omega$.

Definition 2.2. Let X be a Polish space. Random of S, T: $\Omega \times X \to X$ are weakly compatible if $T(\omega, \xi(\omega)) = S(\omega, \xi(\omega))$ for some measurable mapping $\xi : \Omega \to X$ and $\omega \in \Omega$, then T(ω, S(ω,ξ(ω)) = s(ω,T(ω,ξ(ω)) for every ω∈ Ω.

Definition 2.3. Let X be a Polish space. Random operators S, T : $\Omega \times X \to X$ are said to be commutative if S(ω,.) and T(ω,.) are commutative for each ω∈ Ω.

Definition 2.4. Let X be a Polish space. Random operators S,T : $\Omega \times X \to X$ are said to be semi compatible if

$$d(S(\omega,T(\omega,\xi_n(\omega))), T(\omega,\xi(\omega))) \to 0 \text{ as } n \to \infty$$

Whenever $\{\xi_n\}$ is a sequence of measurable mapping from $\Omega \times X \to X$ such that $d(S(\omega,\xi_n(\omega)), \xi(\omega)) \to 0$, $d(T(\omega,\xi_n(\omega)), \xi(\omega)) \to 0$ as $n \to \infty$ for each ω∈ Ω.

Definition 2.5. Let S, T: $\Omega\times X \to CB(X)$ be continuous random multivalued operators. S and T are said to weak compatible if they commute at their coincidence points i.e. $S(\omega, \xi(\omega)) = T(\omega,\xi(\omega))$ implies that $ST(\omega,\xi(\omega)) = TS(\omega, \xi(\omega))$.

S and T are said to be compatible if $d(ST(\omega,\xi_n(\omega)), TS(\omega,\xi_n(\omega))) \to 0$ as $n \to \infty$.

S and T are called semi compatible if $d(ST(\omega,\xi_n(\omega)), T(\omega,\xi(\omega))) \to 0$ as $n \to \infty$ whenever $\xi_n : \Omega \to X$, $n > 0$ be a measurable mapping such that

$d(T(\omega, \xi_n(\omega)), \xi(\omega)) \to 0$, $d(S(\omega, \xi_n(\omega)), \xi(\omega)) \to 0$ as $n \to \infty$

Clearly if the pair (S,T) is semi compatible then they are weak compatible.

3. Main Result:

Theorem 3.1. Let X be a Polish space and A, B, S, T, I and J : $\Omega\times X \to CB(X)$ be random multivalued operators satisfying

$AB(\omega,X) \subset J(\omega,X)$, $ST(\omega, X) \subset I(\omega,X)$ and for every $\omega \in \Omega$.

$$H(AB(\omega,x), ST(\omega,y)) \leq \frac{\alpha(\omega)[\{d(AB(\omega,x),J(\omega,y))\}^3 + \{d(ST(\omega,y),I(\omega,x))\}^3]}{\{d(AB(\omega,x),\ J(\omega,y))\}^2 + \{d(ST(\omega,y),\ I(\omega,x))\}^2}$$
$$+ \alpha_2(\omega)\,[d(AB(\omega,x),I(\omega,x))+d(ST(\omega,y), J(\omega,y))]$$
$$+ \alpha_3(\omega)\, d(I(\omega,x), J(\omega,y)) \qquad (3.1)$$

for every $\omega\in\Omega$ and $x,y\in X$ with $\alpha_I : \Omega \to X (i=1,2,3)$ are measurable mappings such that $2\alpha_1(\omega)+2\alpha_2(\omega)+\alpha_3(\omega)<1$. If either (AB,I) are semi compatible, I or AB is continuous and (ST,J) are weakly compatible or (ST,J) are semi compatible, J or ST is continuous and (AB, I) are weakly compatible.

Then AB, ST, I and J have a unique common random fixed point. Furthermore if the pairs (A,B), (A,I), (B,I), (S,T), (S,J) and (T,J) are commuting mappings then A,B,S,T,I and J have a unique common random fixed point.

Proof: Let $\xi_o, \xi_1, \xi_2: \Omega \to X$ be three measurable mappings such that -

$AB(\omega, \xi_0(\omega)) = J(\omega,\xi_1(\omega)$ and $ST(\omega,\xi_1(\omega)) = I(\omega, \xi_2(\omega))$.

In general we can choose sequences $\{\xi n\}$ and $\{\eta_n\}$ of measurable mappings such that.

$AB(\omega, \xi_{2n}(\omega)) = J(\omega, \xi_{2n+1}(\omega)) = \eta_{2n}(\omega)$

$ST(\omega,\xi_{2n}(\omega)) = I(\omega,\xi_{2n+2}(\omega)) = \eta_{2n+1}(\omega)$, $\forall$ n=0,1,2..... and $\omega\in\Omega$.

Then for each $\omega\in\Omega$.

$d(\eta_{2n}(\omega),\eta_{2n+1}(\omega))= d(AB(\omega,\xi_{2n}(\omega)), ST(\omega,\xi_{2n+1}(\omega))$

$$\le \alpha_1(\omega)\left[\frac{\{d(AB(\omega,\xi_{2n}(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^3+\{d(ST(\omega,\xi_{2n+1}(\omega)),I(\omega,\xi_{2n}(\omega)))\}^3}{\{d(AB(\omega,\xi_{2n}(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^2+\{d(ST(\omega,\xi_{2n+1}(\omega)),I(\omega,\xi_{2n}(\omega)))\}^2}\right]$$

$+\alpha_2(\omega)[d(AB(\omega,\xi_{2n}(\omega)), I(\omega,\xi_{2n}(\omega)))+d(ST(\omega, \xi_{2n+1}(\omega)), J(\omega, \xi_{2n+1}(\omega)))]$

$+ \alpha_3(\omega)\, d\, (I(\omega,\xi_{2n}(\omega)), J(\omega, \xi_{2n+1}(\omega)))$

$$\le \alpha_1(\omega)\frac{\left[\{d(\eta_{2n}(\omega),\eta_{2n}(\omega))\}^3+\{d(\eta_{2n+1}(\omega),\eta_{2n-1}(\omega))\}^3\right]}{\{d(\eta_{2n}(\omega),\eta_{2n}(\omega))\}^2+\{d(\eta_{2n+1}(\omega),\eta_{2n-1}(\omega))\}^2}$$

$+\alpha_2(\omega)\, [d(\eta_{2n}(\omega), \eta_{2n-1}(\omega)) +d(\eta_{2n+1}(\omega), \eta_{2n}(\omega))]$

$+\ \alpha_3(\omega)\, d(\eta_{2n-1}(\omega), \eta_{2n}(\omega))$

$\le \alpha_1(\omega)d(\eta_{2n+1}(\omega), \eta_{2n-1}(\omega)) +\alpha_2(\omega)\, d(\eta_{2n+1}(\omega), \eta_{2n}(\omega))+d(\eta_{2n}(\omega), \eta_{2n-1}(\omega))]$

$+\alpha_3(\omega)\, d(\eta_{2n-1}(\omega), \eta_{2n}(\omega))$

$\le \alpha_1(\omega)\, [d(\eta_{2n+1}(\omega), \eta_{2n}(\omega))+d(\eta_{2n}(\omega), \eta_{2n-1}(\omega))]+$

$\alpha_2(\omega)\, [d(\eta_{2n+1}(\omega), \eta_{2n}(\omega) + d(\eta_{2n}(\omega), \eta_{2n-1}(\omega))]$

$+ \alpha_3(\omega)d(\eta_{2n-1}(\omega), \eta_{2n}(\omega))$

$\le(\alpha_1(\omega)+\alpha_2(\omega))\, d(\eta_{2n+1}(\omega), \eta_{2n}(\omega))$

$+(\alpha_1(\omega)+\alpha_2(\omega)+\alpha_3(\omega))\, d(\eta_{2n}(\omega), \eta_{2n-1}(\omega))$

i.e. $d(\eta_{2n}(\omega), \eta_{2n+1}(\omega) \le \dfrac{\alpha_1(\omega)+\alpha_2(\omega)+\alpha_3(\omega)}{1-\alpha_1(\omega)-\alpha_2(\omega)}\ d(\eta_{2n}(\omega), \eta_{2n-1}(\omega))$

Or $\quad d(\eta_{2n}(\omega), \eta_{2n+1}(\omega) \le k\, d(\eta_{2n}(\omega), \eta_{2n-1}(\omega).$

where $k=\dfrac{\alpha_1(\omega)+\alpha_2(\omega)+\alpha_3(\omega)}{1-\alpha_1(\omega)-\alpha_2(\omega)}<1$

Similarly we can prove

$d(\eta_{2n+1}(\omega), \eta_{2n+2}(\omega)) \le k.d(\eta_{2n}(\omega), \eta_{2n+1}(\omega))$

$\le k.k.d(\eta_{2n-1}(\omega), \eta_{2n}(\omega))$

Similarly proceeding in the same way, by induction we get,

$d(\eta_{2n+1}(\omega), \eta_{2n+2}(\omega)) \le k^{2n+1}\, d(\eta_0(\omega), \eta_1(\omega))$

Furthermore for m > n , we have

$d(\eta_{2n}(\omega), \eta_{2m}(\omega)) \le d(\eta_{2n}(\omega), \eta_{2n+1}(\omega)) + d(\eta_{2n+1}(\omega), \eta_{2n+2}(\omega))$

$+ \dots + d(\eta_{2m-1}(\omega), \eta_{2m}(\omega))$

$\le k^{2n}\, d(\eta_0(\omega), \eta_1(\omega))+k^{2n+1}d(\eta_0(\omega),\eta_1(\omega)) + \dots + k^{2m-1}d\, (\eta_0(\omega), \eta_1(\omega))$

$\le [k^{2n}+k^{2n+1}+ \dots + k^{2m-1}]\, d(\eta_0(\omega), \eta_1(\omega))$

$\leq \dfrac{k^{2n}}{(1-k)}$ d($\eta_0(\omega)$, $\eta_1(\omega)$).

i.e. d($\eta_{2n}(\omega)$, $\eta_{2m}(\omega)$) $\leq \dfrac{k^{2n}}{(1-k)}$ d($\eta_0(\omega)$, $\eta_1(\omega)$) $\to 0$ as n, m $\to \infty$

Thus it follows that sequence $\{\eta_{2n}(\omega)\}$ is a Cauchy sequence. Since X is a separable complete metric space there exists a measurable mapping $\xi : \Omega \to X$ such that $\{\eta_{2n}(\omega)\}$ and its subsequences converges to $\xi(\omega)$.

So, AB $(\omega,\xi_{2n}(\omega)) \to \xi(\omega)$, J $(\omega, \xi_{2n+1}(\omega)) \to \xi(\omega)$ (3.2)

and ST $(\omega, \xi_{2n+1}(\omega)) \to \xi(\omega)$, $I(\omega,\xi_{2n+2}(\omega)) \to \xi(\omega)$ for each $\omega \in \Omega$. (3.3)

Case I. If I is continuous

In this case, we have

I (AB) $(\omega, \xi_{2n}(\omega)) \to I(\omega, \xi(\omega))$, $I^2(\omega, \xi_{2n}(\omega)) \to I(\omega, \xi(\omega)$ and semi compatibility of the pair (AB, I) gives $(AB)I(\omega, \xi_{2n}(\omega)) \to I(\omega,\xi(\omega))$ for each $\omega \in \Omega$.

Step 1. For each $\omega \in \Omega$,

$H((AB)I(\omega, \xi_{2n}(\omega)), ST(\omega, \xi_{2n+1}(\omega)))$

$$\leq \alpha_1(\omega)\frac{\{d(ABI(\omega,\xi_{2n}(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^3+\{d(ST(\omega,\xi_{2n+1}(\omega)),\ II(\omega,\xi_{2n}(\omega)))\}^3}{\{d(ABI(\omega,\xi_{2n}(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^2+\{d(ST(\omega,\xi_{2n+1}(\omega)),\ II(\omega,\xi_{2n}(\omega)))\}^2}$$

$+\alpha_2(\omega)$ $[d((AB)I(\omega,\xi_{2n}(\omega)),II(\omega,\xi_{2n}(\omega))) + d(ST(\omega, \xi_{2n+1}(\omega)), J(\omega,\xi_{2n+1}(\omega)))]$

$+\alpha_3(\omega)$ $[d(II(\omega,\xi_{2n}(\omega)), J(\omega,\xi_{2n+1}(\omega)))]$.

Taking limit $n \to \infty$ using (3.2), (3.3) we get

$$d(I(\omega,\xi(\omega)), \xi(\omega)) \leq \alpha_1(\omega)\frac{\{d(I(\omega,\xi(\omega)),\xi(\omega))\}^3+\{d(\xi(\omega),I(\omega,\xi(\omega)))\}^3}{\{d(I(\omega,\xi(\omega)),\xi(\omega))\}^2+\{d(\xi(\omega),I(\omega,\xi(\omega)))\}^2}$$

$+\alpha_2(\omega)$ $[d(I(\omega,\xi(\omega)), I(\omega,\xi(\omega))) + d(\xi(\omega), \xi(\omega))]$

$+\alpha_3(\omega)$ $d(I(\omega,\xi(\omega)), \xi(\omega))]$

$= \alpha_1(\omega)[d(I(\omega,\xi(\omega)),\xi(\omega))] + \alpha_3(\omega)d(I(\omega,\xi(\omega)),\xi(\omega))$

$= (\alpha_1(\omega)+\alpha_3(\omega))\ d(I(\omega,\xi(\omega)), \xi(\omega))$

i.e. $(1-\alpha_1(\omega)-\alpha_3(\omega))\ d(I(\omega, \xi(\omega)), \xi(\omega)) \leq 0$.

Hence, $\xi(\omega) = I(\omega,\xi(\omega))$ for each $\omega \in \Omega$.

Step 2. For any $\omega \in \Omega$.

$H(AB(\omega, \xi(\omega)), ST(\omega, \xi_{2n+1}(\omega)))$

$$\leq \alpha_1(\omega)\frac{\{d(AB(\omega,\xi(\omega)),\ J(\omega,\xi_{2n+1}(\omega)))\}^3+\{d(ST(\omega,\xi_{2n+1}(\omega)),\ I(\omega,\xi(\omega)))\}^3}{\{d(AB(\omega,\xi(\omega)),\ J(\omega,\xi_{2n+1}(\omega)))\}^2+\{d(ST(\omega,\xi_{2n+1}(\omega)),\ I(\omega,\xi(\omega)))\}^2}$$

$+\alpha_2(\omega)[d(AB(\omega,\xi(\omega)), I(\omega,\xi(\omega))) + d(ST(\omega,\xi_{2n+1}(\omega)), J(\omega, \xi_{2n+1}(\omega)))]$

$+\alpha_3(\omega)$ $[d(I(\omega,\xi(\omega)), J(\omega,\xi_{2n+1}(\omega)))]$.

Taking limit $n \to \infty$ and using results of step 1 and (3.3), we get d(AB(ω,ξ(ω)), ξ(ω)) ≤ $\alpha_1(\omega)\dfrac{\{d(AB(\omega,\xi(\omega)),\xi(\omega))\}^3+\{d(\xi(\omega),\xi(\omega))\}^3}{\{d(AB(\omega,\xi(\omega)),\xi(\omega))\}^2+\{d(\xi(\omega),\xi(\omega))\}^2}$

$+\alpha_2(\omega)$ [d(AB(ω,ξ(ω)), ξ(ω)) + d(ξ(ω), ξ(ω))]

$+\alpha_3(\omega)$ d(ξ(ω), ξ(ω))

= $\alpha_1(\omega)$d(AB(ω,ξ(ω)),ξ(ω)) +$\alpha_2(\omega)$d(AB(ω,ξ(ω)), ξ(ω))

d(AB(ω,ξ(ω)),ξ(ω)) ≤ ($\alpha_1(\omega)$+$\alpha_2(\omega)$) d(AB(ω,ξ(ω)),ξ(ω))

implying thereby

AB(ω,ξ(ω)) = ξ(ω) for each ω ∈ Ω.

Hence AB(ω,ξ(ω)) = ξ(ω) = I(ω,ξ(ω)) for each ω ∈ Ω.

As AB(ω,X) ⊆ J(ω,X) then there exists a measurable mapping g $:\Omega \rightarrow X$ such that AB(ω, ξ(ω)) = J(ω,g(ω)).

Therefore ξ(ω) = AB(ω,ξ(ω)) = I(ω,ξ(ω)) = J(ω, g(ω)).

Step 3. For any ω∈ Ω

H(AB(ω,ξ_{2n}(ω)), ST(ω, g(ω))

$$\le \alpha_1(\omega)\frac{\{d(AB(\omega,\xi_{2n}(\omega)),\ J(\omega,g(\omega)))\}^3+\{d(ST(\omega,g(\omega)),I(\omega,\xi_{2n}(\omega)))\}^3}{\{d(AB(\omega,\xi_{2n}(\omega)),\ J(\omega,g(\omega)))\}^2+\{d(ST(\omega,g(\omega)),I(\omega,\xi_{2n}(\omega)))\}^2}$$

$+\alpha_2(\omega)$ [d(AB(ω,ξ_{2n}(ω)), I(ω,ξ_{2n}(ω))) + d(ST(ω,g(ω)), J(ω,g(ω)))]

$+\alpha_3(\omega)$ d(I(ω,ξ_{2n}(ω)), J(ω, g(ω))).

Taking limit as n $\rightarrow \infty$ and using the results from above steps, we obtain that

d(ξ(ω,ST(ω,g(ω))) ≤ $\alpha_1(\omega)\dfrac{\{d(\xi(\omega),\xi(\omega))\}^3+\{d(ST(\omega,g(\omega)),\ \xi(\omega))\}^3}{\{d(\xi(\omega),\xi(\omega))\}^2+\{d(ST(\omega,g(\omega)),\ \xi(\omega))\}^2}$

$+\alpha_2(\omega)$[d(ξ(ω), ξ(ω)) + d(ST(ω,g(ω)), ξ(ω))]

$+\alpha_3(\omega)$ d(ξ(ω), ξ(ω))

d(ξ(ω),ST(ω,g(ω)))≤$\alpha_1(\omega)$d(ST(ω,g(ω)),ξ(ω))+$\alpha_2(\omega)$ d(ST(ω,g(ω), ξ(ω))

i.e. d(ξ(ω),ST(ω,g(ω))) ≤ [$\alpha_1(\omega)$+$\alpha_2(\omega)$] d(ST(ω,g(ω)),ξ(ω))

implying thereby

ST(ω,g(ω)) = ξ(ω) for each ω ∈ Ω

Therefore,

ST(ω,g(ω)) = J(ω,g(ω)) = ξ(ω).

Now using the weak compatibility of (ST,J) we have

J(ST) (ω,g(ω)) = (ST) J(ω,g(ω))

i.e. ST(ω, ξ(ω)) = J(ω,ξ(ω)) for each ω∈ Ω

Step 4. For each ω∈ Ω, we have

H(AB(ω,ξ(ω)), ST(ω,ξ(ω)))

$$\le \alpha_1(\omega)\frac{\{d(AB(\omega,\xi(\omega)),\ J(\omega,\xi(\omega)))\}^3+\{d(ST(\omega,\xi(\omega)),\ I(\omega,\xi(\omega)))\}^3}{\{d(AB(\omega,\xi(\omega)),\ J(\omega,\xi(\omega)))\}^2+\{d(ST(\omega,\xi(\omega)),\ I(\omega,\xi(\omega)))\}^2}$$

$+\alpha_2(\omega)$[d(AB(ω,ξ(ω)),I(ω,ξ(ω)))+d(ST(ω,ξ(ω)), J(ω,ξ(ω)))]

$+\alpha_3(\omega)d(I(\omega,\xi(\omega)), J(\omega,\xi(\omega)))$

$$d(\xi(\omega), J(\omega,\xi(\omega))) \le \alpha_1(\omega)\frac{\{d(\xi(\omega),\ J(\omega,\xi(\omega)))\}^3 + \{d(\xi(\omega),\ J(\omega,\xi(\omega)))\}^3}{\{d(\xi(\omega),\ J(\omega,\xi(\omega)))\}^2 + \{d(\xi(\omega),\ J(\omega,\xi(\omega)))\}^2}$$

$$+\alpha_3(\omega)\, d(\xi(\omega), J(\omega, \xi(\omega)))$$

$d(\xi(\omega),J(\omega,\xi(\omega))) \le (\alpha_1(\omega)+\alpha_3(\omega))d(\xi(\omega), J(\omega,\xi(\omega)))$.

Hence $\xi(\omega) = J(\omega,\xi(\omega))$.

Thus $AB(\omega,\xi(\omega)) = ST(\omega,\xi(\omega)) = I(\omega, \xi(\omega)) = J(\omega, \xi(\omega)$ for each $\omega\in \Omega$.

Hence, $\xi(\omega)$ is a common random fixed point of the random multivalued operators AB, ST, I and J.

Case II. If AB is continuous.

In this case, we have,

$(AB)I(\omega, \xi_{2n}(\omega)) \to AB(\omega,\xi(\omega))$

and semi compatibility of the pair (AB,I) gives $(AB)I\ (\omega,\xi_{2n}(\omega)) \to I(\omega,\xi(\omega))$ for each $\omega\in \Omega$.

Step 1. For each $\omega\in \Omega$, we have

$H((AB)I(\omega, \xi_{2n}(\omega)), ST(\omega, \xi_{2n+1}(\omega)))$

$$\le \alpha_1(\omega)\frac{\{d((AB)I(\omega,\xi_{2n}(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^3 + \{d(ST(\omega,\xi_{2n+1}(\omega)),\ II(\omega,\xi_{2n}(\omega)))\}^3}{\{d((AB)I(\omega,\xi_{2n}(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^2 + \{d(ST(\omega,\xi_{2n+1}(\omega)),\ II(\omega,\xi_{2n}(\omega)))\}^2}$$

$+\alpha_2(\omega)\ [d((AB)I(\omega,\xi_{2n}(\omega)), II(\omega,\xi_{2n}(\omega))) + d(ST(\omega,\xi_{2n+1}(\omega)), J(\omega,\xi_{2n+1}(\omega)))]$

$+\alpha_3(\omega)\ d(II(\omega, \xi_{2n}(\omega)), J(\omega, \xi_{2n+1}(\omega)))$.

Taking limit $n \to \infty$ and using above result we get

$$d(I(\omega,\xi(\omega)), \xi(\omega)) \le \alpha_1(\omega)\frac{\{d(I(\omega,\xi(\omega)),\xi(\omega))\}^3 + \{d(\xi(\omega),\ I(\omega,\xi(\omega)))\}^3}{\{d(I(\omega,\xi(\omega)),\xi(\omega))\}^2 + \{d(\xi(\omega),\ I(\omega,\xi(\omega)))\}^2}$$

$$+\alpha_2(\omega)[d(I(\omega,\xi(\omega)), I(\omega,\xi(\omega)))+d(\xi(\omega),\xi(\omega))]$$

$$+\alpha_3(\omega)d(I(\omega,\xi(\omega)),\xi(\omega))$$

$d(I(\omega,\xi(\omega)),\xi(\omega))\le (\alpha_1(\omega)+\alpha_3(\omega))d(I(\omega,\xi(\omega)),\xi(\omega))$

yielding thereby

$I(\omega,\xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Step 2. For any $\omega \in \Omega$

$H(AB(\omega,\xi(\omega)), ST(\omega,\xi_{2n+1}(\omega)))$

$$\le \alpha_1(\omega)\frac{\{d(AB(\omega,\xi(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^3 + \{d(ST(\omega,\xi_{2n+1}(\omega)),I(\omega,\xi(\omega)))\}^3}{\{d(AB(\omega,\xi(\omega)),J(\omega,\xi_{2n+1}(\omega)))\}^2 + \{d(ST(\omega,\xi_{2n+1}(\omega)),I(\omega,\xi(\omega)))\}^2}$$

$+\ \alpha_2(\omega)\ [d(AB(\omega,\xi(\omega)), I(\omega,\xi(\omega)))+d(ST(\omega,\xi_{2n+1}(\omega)), J(\omega, \xi_{2n+1}(\omega)))]$

$+\ \alpha_3(\omega)d(I(\omega,\xi(\omega)), J(\omega,\xi_{2n+1}(\omega)))$.

Taking limit $n \to \infty$ and using the result of step 1 of case II, we get

$$d(AB(\omega,\xi(\omega)),\xi(\omega)) \le \alpha_1(\omega)\frac{\{d(AB(\omega,\xi(\omega)),\xi(\omega))\}^3 + \{d(\xi(\omega),\xi(\omega))\}^3}{\{d(AB(\omega,\xi(\omega)),\xi(\omega))\}^2 + \{d(\xi(\omega),\xi(\omega))\}^2}$$

$$+\alpha_2(\omega)[d(AB(\omega,\xi(\omega)), \xi(\omega))+d(\xi(\omega), \xi(\omega))]$$

$+\alpha_3(\omega)\, d(\xi(\omega), \xi(\omega))$

$\leq \alpha_1(\omega) d(AB(\omega,\xi(\omega)),\xi(\omega))+\alpha_2(\omega) d(AB(\omega,\xi(\omega)),\xi(\omega))$

i.e. $d(AB(\omega,\xi(\omega)), \xi(\omega)) \leq (\alpha_1(\omega)+\alpha_2(\omega))\, d(AB(\omega,\xi(\omega)),\xi(\omega))$.

Hence $AB(\omega,\xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$

Thus $AB(\omega,\xi(\omega)) = I(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

As $AB(\omega,X) \subseteq J(\omega,X)$ there exists a measurable mapping $g':\Omega \to X$ such that $AB(\omega,\xi(\omega)) = J(\omega,g'(\omega))$.

Therefore, $\xi(\omega) = AB(\omega, \xi(\omega)) = I(\omega,\xi(\omega)) = J(\omega,g'(\omega))$.

Step 3. For any $\omega \in \Omega$,

$H(AB(\omega, \xi_{2n}(\omega)), ST(\omega, g'(\omega)))$

$$\leq \alpha_1(\omega)\frac{\{d(AB(\omega,\xi_{2n}(\omega)),J(\omega,g'(\omega)))\}^3+\{d(ST(\omega,g'(\omega)),\ I(\omega,\xi_{2n}(\omega)))\}^3}{\{d(AB(\omega,\xi_{2n}(\omega)),J(\omega,g'(\omega)))\}^2+\{d(ST(\omega,g'(\omega)),\ I(\omega,\xi_{2n}(\omega)))\}^2}$$

$+ \alpha_2(\omega)\, [d(AB(\omega,\xi_{2n}(\omega)), I(\omega,\xi_{2n}(\omega)))+d(ST(\omega,g'(\omega)), J(\omega,g'(\omega)))]$

$+ \alpha_3(\omega)\, d(I(\omega,\xi_{2n}(\omega)), J(\omega,g'(\omega)))$.

Taking limit $n \to \infty$ and using the result from above steps, we obtain that

$d(\xi(\omega), ST(\omega,g'(\omega))) \leq (\alpha_1(\omega)+\alpha_2(\omega))\, d(\xi(\omega), ST(\omega, g'(\omega)))$

implying thereby

$ST(\omega,g'(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Therefore $ST(\omega,g'(\omega)) = J(\omega,g'(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$.

Now using the weak compatibility of (ST, J) we have

$J(ST)(\omega,g'(\omega)) = (ST)J(\omega,g'(\omega))$

i.e. $ST(\omega,\xi(\omega)) = J(\omega,\xi(\omega))$ for each $\omega \in \Omega$.

Step 4. For any $\omega \in \Omega$

$H(AB(\omega,\xi(\omega)), ST(\omega,\xi(\omega)))$

$$\leq \alpha_1(\omega)\frac{\{d(AB(\omega,\xi(\omega)),J(\omega,\xi(\omega)))\}^3+\{d(ST(\omega,\xi(\omega)),\ I(\omega,\xi(\omega)))\}^3}{\{d(AB(\omega,\xi(\omega)),J(\omega,\xi(\omega)))\}^2+\{d(ST(\omega,\xi(\omega)),\ I(\omega,\xi(\omega)))\}^2}$$

$+\alpha_2(\omega)[d(AB(\omega,\xi(\omega)),I(\omega,\xi(\omega))) + d(ST(\omega,\xi(\omega)), J(\omega,\xi(\omega)))]$

$+\alpha_3(\omega) d(I(\omega,\xi(\omega)),J(\omega,\xi(\omega)))$

$$d(\xi(\omega),J(\omega,\xi(\omega)))\leq \alpha_1(\omega)\frac{\{d(\xi(\omega),J(\omega,\xi(\omega)))\}^3+\{d(J(\omega,\xi(\omega)),\xi(\omega))\}^3}{\{d(\xi(\omega),J(\omega,\xi(\omega)))\}^2+\{d(J(\omega,\xi(\omega)),\xi(\omega))\}^2}$$

$+ \alpha_3(\omega)d(\xi(\omega), J(\omega, \xi(\omega)))$

$d(\xi(\omega), J(\omega,\xi(\omega))) \leq (\alpha_1(\omega)+\alpha_3(\omega))\, d(\xi(\omega), J(\omega,\xi(\omega)))$

Hence $\xi(\omega) = J(\omega,\xi(\omega))$.

Thus $AB(\omega,\xi(\omega)) = ST(\omega,\xi(\omega)) = I(\omega,\xi(\omega)) = J(\omega, \xi(\omega)) = \xi(\omega)$

for each $\omega \in \Omega$.

Hence $\xi(\omega)$ is a common random fixed point of the random multivalued operators AB, ST, I and J.

If the mapping ST of J is continuous instead of AB or I there the proof that $\xi(\omega)$ is a common random fixed point of AB, ST, I and J is similar.

For uniqueness : Let h : $\Omega \rightarrow X$ be another common random fixed point of random multivalued operators AB, ST, I and J. Then for each $\omega \in \Omega$.

H(AB(ω,ξ(ω)), ST(ω,h(ω)))

$$\leq \alpha_1(\omega)\frac{\{d(AB(\omega,\xi(\omega)),J(\omega,h(\omega)))\}^3+\{d(ST(\omega,h(\omega)),I(\omega,\xi(\omega)))\}^3}{\{d(AB(\omega,\xi(\omega)),J(\omega,h(\omega)))\}^2+\{d(ST(\omega,h(\omega)),I(\omega,\xi(\omega)))\}^2}$$

$$+\ \alpha_2(\omega)[d(AB(\omega,\xi(\omega)), I(\omega,\xi(\omega)))+d(ST(\omega,h(\omega)), J(\omega,h(\omega)))]$$

$$+\ \alpha_3(\omega)\ d(I(\omega,\xi(\omega)), J(\omega,h(\omega))).$$

Taking limit $n \rightarrow \infty$ and using the result we obtain that

$$d(\xi(\omega),h(\omega)) \leq \alpha_1(\omega)\ \frac{\{d(\xi(\omega),h(\omega))\}^3+\{d(h(\omega),\xi(\omega))\}^3}{\{d(\xi(\omega),h(\omega))\}^2+\{d(h(\omega),\xi(\omega))\}^2}$$

$$+\ \alpha_3(\omega)d(\xi\omega), h(\omega))$$

i.e. $d(\xi(\omega),h(\omega)) \leq (\alpha_1(\omega)+\alpha_3(\omega))\ d(\xi(\omega),h(\omega))$

yielding thereby

$$\xi(\omega) = h(\omega)$$

Hence ξ(ω) is a unique common random fixed point of AB, ST, I and J.

Finally we need to show that ξ(ω) is a common random fixed point of random multivalued operators A,B,S,T,I and J. For this ξ(ω) be the unique common random fixed point of both the pairs (AB,I) and (ST,J).

Then

A(ω,ξ(ω))=A(ω,AB(ω,ξ(ω))) = A(ω,BA(ω,ξ(ω)))=AB (ω, A(ω,ξ(ω))),

A(ω,ξ(ω))=A(ω,I(ω,ξ(ω)))=I(ω,A(ω,ξ(ω)))

B(ω,ξ(ω)) = B(ω,AB(ω,ξ(ω)))=BA (ω,B(ω,ξ(ω)))=AB(ω,B(ω,ξ(ω))),

B(ω,ξ(ω))=B(ω,I(ω,ξ(ω))) = I(ω,B(ω,ξ(ω)))

which shows that A(ω,ξ(ω)) and B(ω,ξ(ω)) is a common random fixed point of (AB,I) yielding thereby

A(ω,ξ(ω))= ξ(ω) = B(ω,ξ(ω)) = I(ω,ξ(ω)) = AB(ω,ξ(ω)) in the view of uniqueness of the common random fixed point of the pair (AB,I).

Similarly using the commutativity of (S,T) (S,J) and (T,J) it can be shown that

S(ω,ξ(ω)) = ξ(ω)=T(ω,ξ(ω)) = J(ω,ξ(ω)) = ST(ω,ξ(ω)).

Now we need to show that

A(ω,ξ(ω)) = S(ω,ξ(ω)) and B(ω,ξ(ω)) =T(ω,ξ(ω)) also in the view of uniqueness of the common random fixed point of the pair (AB,I).

Similarly using the commutativity if (S,T), (S,J) and (T,J) it can be shown that

S(ω,ξ(ω))=ξ(ω) = T(ω,ξ(ω)) = J(ω,ξ(ω)) = ST (ω,ξ(ω))

Now we need to show that

A(ω,ξ(ω)) = S(ω,ξ(ω)) and B(ω,ξ(ω)) = T(ω,ξ(ω)) also remains a common random fixed point of both the pairs (AB,I) and (ST,J).

For this

H(AB(ω,A(ω,ξ(ω))),ST(ω,S(ω,ξ(ω))))

$$\leq \alpha_1(\omega)\frac{\{d(AB(\omega,A(\omega,\xi(\omega))),J(\omega,S(\omega,\xi(\omega))))\}^3+\{d(ST(\omega,S(\omega,\xi(\omega))),I(\omega,A(\omega,\xi(\omega)))\}^3}{\{d(AB(\omega,A(\omega,\xi(\omega))),J(\omega,S(\omega,\xi(\omega))))\}^2+\{d(ST(\omega,S(\omega,\xi(\omega))),I(\omega,A(\omega,\xi(\omega)))\}^2}$$

$+ \alpha_2(\omega)[d(AB(\omega,A(\omega,\xi(\omega))),I(\omega,A(\omega,\xi(\omega))))]$

$+ \alpha_3(\omega)\, d(I(\omega,A(\omega,\xi(\omega))), J(\omega,S(\omega,\xi(\omega))))$.

Taking limit $n \to \infty$ we have

$d(A(\omega,\xi(\omega)), S(\omega,\xi(\omega)) = 0$

yielding thereby

$A(\omega,\xi(\omega)) = S(\omega,\xi(\omega))$, for each $\omega \in \Omega$.

Similarly it can be shown that

$B(\omega, \xi(\omega)) = T(\omega,\xi(\omega))$, for each $\omega \in \Omega$.

Thus $\xi(\omega)$ is a unique common random fixed point of random multivalued operators A,B,S,T,I and J.

Corollary: Theorem remains true if contraction condition 3.1 replaced by any one of the following condition.

(i) $$H(AB(\omega,x),ST(\omega,y)) \leq \alpha_1(\omega)\frac{[\{d(AB(\omega,x),J(\omega,y))\}^3+\{d(ST(\omega,y),I(\omega,x))\}^3]}{\{d(AB(\omega,x),J(\omega,y))]^2+\{d(ST(\omega,y),I(\omega,x))\}^2}$$

$$+\alpha_2(\omega)[d(AB(\omega,x),I(\omega,x))+d(ST(\omega,y),J(\omega,y))]$$

for every $\omega \in \Omega$ and $x,y \in X$ with $\alpha_1,\alpha_2:\Omega \to X$ are measurable mappings such that $2\alpha_1(\omega)+2\alpha_2(\omega) < 1$.

(ii) $$H(AB(\omega,x),ST(\omega,y)) \leq \alpha_1(\omega)\frac{[\{d(AB(\omega,x),J(\omega,y))\}^3+\{d(ST(\omega,y),I(\omega,x))\}^3}{\{d(AB(\omega,x),J(\omega,y))\}^2+\{d(ST(\omega,y),I(\omega,x))\}^2}$$

$$+\alpha_3(\omega)\, d(I(\omega,x), J(\omega,y))$$

for every $\omega \in \Omega$ and $x, y \in X$ with $\alpha_1,\alpha_3 : \Omega \to X$ are measurable mappings such that $2\alpha_1(\omega) + \alpha_3(\omega) < 1$

(iii) $$H(AB(\omega,x),ST(\omega,y)) \leq \alpha_2(\omega)\, [d(AB(\omega,x), I(\omega,x)) + d(ST(\omega,y),J(\omega,y))]$$
$$+\alpha_3(\omega)\, d(I(\omega,x), J(\omega,y))$$

for every $\omega \in \Omega$ and $x,y \in X$ with $\alpha_2,\alpha_3: \Omega \to X$ are measurable mappings such that $2\alpha_2(\omega)+\alpha_3(\omega) < 1$.

Proof. Corollaries corresponding to the contraction conditions (i),(ii) and (iii) can be deducted directly from theorem-by choosing $\alpha_3(\omega)=0$, $\alpha_2(\omega)=0$ and $\alpha_1(\omega) = 0$ respectively.

REFERENCE

1. Badshah, V.H. And Gagrani, S., Common random fixed points of random multi valued operators on Polish space, Journal of Chungcheong Mathematical society, Korea 18(1), 2005, 33-40.
2. Badshah, V.H. and Sayyed, F., Random fixed point of random multivalued operators on Polish space, Kuwait J. of science and Engineering, 27 (2000), 203-208.
3. Badshah V.H. Sayyed, F., Common random fixed point of random multivalued operators on Polish space, Indian Jour. Pure Appl. Math. 33(4), (2001), 573-582.
4. Badshah, V.H. and Shrivastava, N., Semi compatibility and random fixed points on Polish spaces, Varahmihir Jour. Math. Sci. Vol. 6 No. 2(2006), 561-568.

5. Beg, I., Random fixed points of random multivalued operators satisfying semi-contractivity conditions, Math-Japan., 46(1), (1997), 151-155.
6. Beg, I. and Abbas M., Common random fixed point of compatible random operators, Int. Jour. Math. Math. Sci, Vol. 2006, Article ID 23486, 1-15.
7. Beg, I. and Shahzad, N., Random fixed points of random multi-valued operators on Polish Spaces, Non-Linear Analysis, 20 (1993) 335-347.
8. Bharucha-Raid, A.T., Random Integral equations, Academic Press, New York, 1972.
9. Hans, O., Reduzierende Zufallige transformation, Czechoslovak Math. Jour. 7(1957), 154-158.
10. Itoh, S., A random fixed point theorem for a multi valued contraction mapping, Pacific, J. Math. 68(1977), 85-90.
11. O'Regan, D.Shahzad, N., Agrawal, R.P., Random fixed point theory in spaces with two metrices, Jour Appl.Math. Sto. Anal. 16(2), (2003), 171-176.
12. Plubtieng, S. and Kumar, P., Random fixed point theorems for multivalued non-expansive non-self random operators, Jour, Appl Math. Sto.Anal. Vol. 2006, Article ID 43796,1-9
13. Spacek, A., Zufallige Gleichungen, Czechoslovak Math. Jour. 5 (1955), 462-466
14. Tan, K.K. and Yuan, X.Z., on deterministic and random fixed point, Proc. Amer. Math. Soc. 119 (1993), 849-856.
15. Xu. H.K., Multivalued non expansive mappings in Banach spaces, Nonlinear Analysis 43 (2001), No. 6, 693-706.