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BOUNDS FOR THE GROWTH RATE OF A PERTURBATION IN RIVLIN-ERICKSEN VISCOELASTIC FLUID IN THE PRESENCE OF MAGNETIC FIELD

Ajaib S. Banyal[1*], Daleep K. Sharma[2] and G. C. Rana[3]

[1*]Department of Mathematics, Govt. College Nadaun, Dist. Hamirpur, (HP) INDIA 177033
[2]Department of Mathematics, Rajiv Gandhi G. C. Kotshera, Shimla (HP), INDIA 171004
[3]Department of Mathematics, NSCBM, Govt. College Hamirpur, (HP) INDIA 177005
*** Corresponding Author** E mail: ajaibbanyal@rediffmail.com, Fax No. : 0197223268

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ABSTRACT

A layer of Rivlin-Ericksen viscoelastic fluid heated from below is considered in the presence of uniform vertical magnetic field. Following the linearized stability theory and normal mode analysis, the paper through mathematical analysis of the governing equations of Rivlin-Ericksen viscoelastic fluid convection with a uniform vertical magnetic field, for any combination of perfectly conducting free and rigid boundaries of infinite horizontal extension at the top and bottom of the fluid, established that the complex growth rate σ of oscillatory perturbations, neutral or unstable for all wave numbers, must lie inside a semi-circle

$$\sigma_r^2 + \sigma_i^2 = \left\{ \frac{Rp_2}{\pi^2 p_1 (1 + 2p_2 + \pi^2 F)} \right\}^2 ,$$

in the right half of a complex $\sigma_r \sigma_i$-plane, where R is the thermal Rayleigh number, p_1 is the thermal Prandtl number, p_2 is the magnetic Prandtl number and F is the viscoelastic parameter of the Rivlin-Ericksen fluid, which prescribes the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in the couple-stress fluid heated from below in the presence of uniform vertical magnetic field.

Key Words: Thermal convection; Rivlin-Ericksen Fluid; Magnetic field; PES; Rayleigh number; Chandrasekhar number.

MSC 2000 No.: 76A05, 76E06, 76E15; 76E07; 76U05.

I. INTRODUCTION

Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics, and has been investigated by several authors (e.g., Bénard[1], Rayleigh[2], Jeffreys[3]) under different conditions. A detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by

Chandrasekhar [4]. The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. There are many elastic-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's [5] constitutive relations. Two such classes of fluids are Rivlin-Ericksen's and Walter's (model B') fluids. Rivlin-Ericksen [6] have proposed a theoretical model for such one class of elastic-viscous fluids.

Bhatia and Steiner [7] have considered the effect of uniform rotation on the thermal instability of a viscoelastic (Maxwell) fluid and found that rotation has a destabilizing influence in contrast to the stabilizing effect on Newtonian fluid. The thermal instability of a Maxwell fluid in hydromagnetics has been studied by Bhatia and Steiner [8]. They have found that the magnetic field stabilizes a viscoelastic (Maxwell) fluid just as the Newtonian fluid. Sharma [9] has studied the thermal instability of a layer of viscoelastic (Oldroydian) fluid acted upon by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect. In another study Sharma [10] has studied the stability of a layer of an electrically conducting Oldroyd fluid [5] in the presence of magnetic field and has found that the magnetic field has a stabilizing influence.

Sharma and kumar [11] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Kumar et al. [12] considered effect of rotation and magnetic field on Rivlin-Ericksen elastico-viscous fluid and found that rotation has stabilizing effect where as magnetic field has both stabilizing and destabilizing effects. A layer of such fluid heated from below or under the action of magnetic field or rotation or both may find applications in geophysics, interior of the Earth, Oceanography, and the atmospheric physics.

Pellow and Southwell [13] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [14] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [15] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al. [16]. However no such result existed for non-Newtonian fluid configurations, in general and in particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal [17] have characterized the non-oscillatory motions in couple-stress fluid.

Keeping in mind the importance of non-Newtonian fluids and magnetic field, as stated above, the present paper is an attempt to prescribe the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude, in a layer of incompressible Rivlin-Ericksen fluid heated from below, in the presence of uniform vertical magnetic field, opposite to force field of gravity, when the bounding surfaces are of infinite horizontal extension, at the top and bottom of the fluid and are perfectly conducting with any combination of dynamically free and rigid boundaries.

The result is important since the exact solutions of the problem investigated in closed form, are not obtainable, for any arbitrary combination of perfectly conducting dynamically free and rigid boundaries

II. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Considered an infinite, horizontal, incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0 and ρ_0 and at the upper surface z = d are T_d and ρ_d respectively, and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The fluid is acted upon by a uniform vertical magnetic field $\vec{H}(0,0,H)$, parallel to the force field of gravity $\vec{g}(0,0,-g)$.

The equation of motion, continuity, heat conduction, and Maxwells equations governing the flow of Rivlin-Ericksen viscoelastic fluid in the presence of magnetic field (Rivlin and Ericksen[6]; Chandrasekhar[4] and Kumar et al[12]) are

$$\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=-\nabla\left(\frac{p}{\rho_o}\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}+\frac{\mu_e}{4\pi\rho_o}(\nabla\times\vec{H})\times\vec{H}, \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \quad (3)$$

$$\nabla.\vec{H}=0, \quad (4)$$

$$\frac{\partial \vec{H}}{\partial t}=(\vec{H}.\nabla)\vec{q}+\eta\nabla^2\vec{H}, \quad (5)$$

Where ρ, p, T, ν, ν' and $\vec{q}(u,v,w)$ denote respectively the density, pressure, temperature, kinematic viscosity, kinematic viscoelasticity and velocity of the fluid, respectively and $\vec{r}(x,y,z)$.

The equation of state for the fluid is

$$\rho=\rho_0[1-\alpha(T-T_0)], \quad (6)$$

Where the suffix zero refer to the values at the reference level z = 0. Here $\vec{g}(0,0,-g)$ is acceleration due to gravity and α is the coefficient of thermal expansion. In writing the equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equation of motion except the external force term. The magnetic permeability μ_e, thermal diffusivity κ, and electrical resistivity η, are all assumed to be constant.

The initial state is one in which the velocity, density, pressure, and temperature at any point in the fluid are, respectively, given by

$$\vec{q} = (0,0,0)\ , \rho = \rho\ (z),\ p=p(z),\ T= T(z), \tag{7}$$

Assume small perturbations around the basic solution and let $\delta\rho$, δp, θ, $\vec{q}(u,v,w)$ and $\vec{h} = (h_x, h_y, h_z)$ denote respectively the perturbations in density ρ, pressure p, temperature T, velocity $\vec{q}(0,0,0)$ and the magnetic field $\vec{H} = (0,0,H)$. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\rho + \delta\rho = \rho_0[1-\alpha(T+\theta-T_0)] = \rho - \alpha\rho_0\theta, \text{ i.e. } \quad \delta\rho = -\alpha\rho_0\theta\ . \tag{8}$$

Then the linearized perturbation equations are

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}\,\alpha\theta + \left(\nu + \nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}, \tag{9}$$

$$\nabla.\vec{q} = 0, \tag{10}$$

$$\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \tag{11}$$

$$\nabla.\vec{h} = 0, \tag{12}$$

$$\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}.\nabla\right)\vec{q} + \eta\nabla^2\vec{h}\ . \tag{13}$$

Within the framework of Boussinesq approximation, equations (9) – (13), become

$$\frac{\partial}{\partial t}\nabla^2 w = \left(\nu + \nu'\frac{\partial}{\partial t}\right)\nabla^4 w + \frac{\mu_e H}{4\pi\rho_0}\nabla^2\left(\frac{\partial h_z}{\partial z}\right) + g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right), \tag{14}$$

$$\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta \tag{15}$$

$$\frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \eta\nabla^2 h_z \tag{16}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

III. NORMAL MODE ANALYSIS

Analyzing the disturbances into normal modes, we assume that the Perturbation quantities are of the form

$$[w, \theta, h_{z,}] = [W(z), \Theta(z), K(z)]\,\mathrm{Exp}\left(ik_x x + ik_y y + nt\right), \tag{17}$$

Where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k = \left(k_x^{\,2} + k_y^{\,2}\right)^{\frac{1}{2}}$, is the resultant wave number, and n is the growth rate which is, in general, a complex constant.

Using (17), equations (14) - (16), in non-dimensional form transform to

$$\left(D^2 - a^2\right)\left[(1 + F\sigma)\left(D^2 - a^2\right) - \sigma\right]W = Ra^2\Theta - Q\left(D^2 - a^2\right)DK\,, \tag{18}$$

$$\left(D^2 - a^2 - p_1\sigma\right)\Theta = -W\,, \tag{19}$$

And

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW\,, \tag{20}$$

Where we have introduced new coordinates (x', y', z') = (x/d , y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number; $p_2 = \frac{\nu}{\eta}$, is the magnetic Prandtl number; $F = \frac{\nu'}{d^2}$, is the Rilvin-Ericksen kinematic viscoelasticity parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$, is the Chandrasekhar number. Also we have Substituted $W = W_\oplus$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$, $K = \frac{Hd}{\eta}K_\oplus$, and $D_\oplus = dD$, and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z=0 and z=1, as the case may be, and are perfectly conducting. The boundaries are maintained at constant temperature, thus the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (18) -- (20), must possess a solution are

W = 0= Θ, on both the horizontal boundaries, (21) DW=0, on a rigid boundary, (22)

$D^2W = 0$, on a dynamically free boundary, (23)

K = 0, on both the boundaries as the regions outside the fluid are perfectly conducting, (24)

Equations (18) -- (20), along with the appropriate boundary conditions, (21) – (24), pose an eigenvalue problem for σ and we wish to characterize σ_i when $\sigma_r \geq 0$.

We first note that since W, Θ and K satisfy $W(0) = 0 = W(1)$, $\Theta(0) = 0 = \Theta(1)$ and $K(0) = 0 = K(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [18]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz, \int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \text{ and} \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz, \quad (25)$$

Further, for $W(0) = 0 = W(1)$ and $K(0) = 0 = K(1)$, Banerjee et al. [19] have shown that

$$\int_0^1 \left|D^2W\right|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz \text{ and } \int_0^1 \left|D^2K\right|^2 dz \geq \pi^2 \int_0^1 |DK|^2 dz, \quad (26)$$

IV. MATHEMATICAL ANALYSIS

***Lemma 1*:** For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{|D\Theta|^2 + a^2|\Theta|^2\right\} dz \leq \frac{1}{p_1|\sigma|} \int_0^1 |W|^2 dz$$

Proof: Multiplying equation (19) and its complex conjugate, and integrating by parts each term on right hand side of the resulting equation for an appropriate number of times and making use of boundary conditions on Θ namely $\Theta(0) = 0 = (1)$, we get

$$\int_0^1 \left|\left(D^2 - a^2\right)\Theta\right|^2 dz + 2p_1\sigma_r \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz + p_1{}^2|\sigma|^2 \int_0^1 |\Theta|^2 dz = \int_0^1 |W|^2 dz,$$

since $\sigma_r \geq 0, \sigma_i \neq 0$ therefore this equation gives,

$$\int_0^1 \left|\left(D^2 - a^2\right)\Theta\right|^2 dz \langle \int_0^1 |W|^2 dz, \quad (27)$$

And

$$\int_0^1 |\Theta|^2 dz \langle \frac{1}{p_1{}^2|\sigma|^2} \int_0^1 |W|^2 dz, \quad (28)$$

It is easily seen upon using the boundary conditions (21) that

$$\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right) dz = \text{Real part of } \left\{-\int_0^1 \Theta^*\left(D^2 - a^2\right)\Theta dz\right\} \leq \left|\int_0^1 \Theta^*\left(D^2 - a^2\right)\Theta dz\right|,$$

$$\leq \int_0^1 \left|\Theta^*\left(D^2-a^2\right)\Theta\right|dz \leq \int_0^1 \left|\Theta^*\right|\left|\left(D^2-a^2\right)\Theta\right|dz = \int_0^1 \left|\Theta\right|\left|\left(D^2-a^2\right)\Theta\right|dz$$

$$\leq \left\{\int_0^1 |\Theta|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 \left|\left(D^2-a^2\right)\Theta\right|^2 dz\right\}^{\frac{1}{2}}, \quad \text{(Utilizing Cauchy-Schwartz-inequality)}$$

Upon utilizing the inequality (27) and (28), this inequality gives

$$\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz \leq \frac{1}{p_1|\sigma|}\int_0^1 |W|^2 dz, \tag{29}$$

This completes the proof of lemma**.**

We prove the following theorems:

Theorem 1**:** If $R \rangle 0$, $F \rangle 0$, $Q \rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, K) of equations (18) - (20) and the boundary conditions (21) and (24), and any combination of (22) and (23) is that

$$|\sigma| \langle \frac{Rp_2}{\pi^2 p_1(1+2p_2+\pi^2 F)} .$$

Proof: Multiplying equation (18) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$(1+F\sigma)\int_0^1 W^*\left(D^2-a^2\right)^2 W dz - \sigma\int_0^1 W^*(D^2-a^2)W dz = Ra^2\int_0^1 W^*\Theta dz - Q\int_0^1 W^* D\left(D^2-a^2\right)K dz, \tag{30}$$

Taking complex conjugate on both sides of equation (19), we get

$$\left(D^2-a^2-p_1\sigma^*\right)\Theta^* = -W^*, \tag{31}$$

Therefore, using (31), we get

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta\left(D^2-a^2-p_1\sigma^*\right)\Theta^* dz, \tag{32}$$

Also taking complex conjugate on both sides of equation (20), we get

$$\left[D^2-a^2-p_2\sigma^*\right]K^* = -DW^*, \tag{33}$$

Therefore, using (20) and using boundary condition (21), we get

$$\int_0^1 W^* D\left(D^2-a^2\right)K dz = -\int_0^1 DW^*\left(D^2-a^2\right)K dz = \int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz, \tag{34}$$

Substituting (32) and (34) in the right hand side of equation (30), we get

$$(1+F\sigma)\int_0^1 W^*\left(D^2-a^2\right)^2 W dz-\sigma\int_0^1 W^*(D^2-a^2)W dz$$

$$=-Ra^2\int_0^1 \Theta\left(D^2-a^2-p_1\sigma^*\right)\Theta^* dz-Q\int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^* dz, \qquad (35)$$

Integrating the terms on both sides of equation (35) for an appropriate number of times by making use of the appropriate boundary conditions (21) - (24), we get

$$(1+F\sigma)\int_0^1\left(\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right)dz+\sigma\int_0^1\left(|DW|^2+a^2|W|^2\right)dz=Ra^2\int_0^1\left\{|D\Theta|^2+a^2|\Theta|^2\right\}dz+Ra^2p_1\sigma^*\int_0^1|\Theta|^2dz$$

$$-Q\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz-Qp_2\sigma^*\int_0^1\left(|DK|^2+a^2|K|^2\right)dz\,. \qquad (36)$$

And equating the real and imaginary parts on both side of Eq. (36), and cancelling $\sigma_i(\neq 0)$ throughout, we get

$$(1+F\sigma_r)\int_0^1\left(\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right)dz+\sigma_r\int_0^1\left(|DW|^2+a^2|W|^2\right)dz=Ra^2\int_0^1\left\{|D\Theta|^2+a^2|\Theta|^2\right\}dz$$

$$-Q\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz+\sigma_r\left\{Ra^2p_1\int_0^1|\Theta|^2dz-Qp_2\int_0^1\left(|DK|^2+a^2|K|^2\right)dz\right\}, \qquad (37)$$

and

$$F\int_0^1\left(\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right)dz+\int_0^1\left(|DW|^2+a^2|W|^2\right)dz=-Ra^2p_1\int_0^1|\Theta|^2dz+Qp_2\int_0^1\left(|DK|^2+a^2|K|^2\right)dz, \qquad (38)$$

Equation (38) implies that,

$$Ra^2p_1\int_0^1|\Theta|^2dz-Qp_2\int_0^1\left(|DK|^2+a^2|K|^2\right)dz, \qquad (39)$$

is negative definite and, utilizing inequalities (25) and (26), equation (38) gives,

$$Q\int_0^1\left\{|DK|^2+a^2|K|^2\right\}dz\geq\frac{\pi^2(1+\pi^2F)}{p_2}\int_0^1\left|W^2\right|dz, \qquad (40)$$

Now R 〉 0, $Q\rangle$ and $\sigma_r\geq 0$, utilizing the inequalities (25), (26), (29), (39) and (40), the equation (37) gives,

$$a^2\left[\left(2\pi^2+\frac{\pi^2(1+\pi^2F)}{p_2}\right)-\frac{R}{p_1|\sigma|}\right]\int_0^1|W|^2dz+I\ \langle\ 0, \qquad (41)$$

Where

$$I = (1+F\sigma_r)\int_0^1\left(\left|D^2W\right|^2 + a^4|W|^2\right)dz + 2a^2F\sigma_r\int_0^1|DW|^2\,dz + \sigma_r\int_0^1\left(|DW|^2 + a^2|W|^2\right)dz + Q\int_0^1\left(\left|D^2K\right|^2 + a^2|DK|^2\right)dz,$$

is positive definite and therefore, we must have

$$|\sigma| \langle \frac{Rp_2}{\pi^2 p_1(1+2p_2+\pi^2F)}\ , \qquad (42)$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0, \text{ then} |\sigma| \langle \frac{Rp_2}{\pi^2 p_1(1+2p_2+\pi^2F)}.$$

And this completes the proof of the theorem.

And this completes the proof of the theorem.

V. CONCLUSIONS

The inequality (40) for $\sigma_r \geq 0$ and $\sigma_i \neq 0$, can be written as

$$\sigma_r^{\ 2} + \sigma_i^{\ 2} \langle \left\{\frac{Rp_2}{\pi^2 p_1(1+2p_2+\pi^2F)}\right\}^2,$$

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension heated form below, having top and bottom bounding surfaces of infinite horizontal extension, at the top and bottom of the fluid and are perfectly conducting with any arbitrary combination of dynamically free and rigid boundaries, in the presence of uniform vertical magnetic field parallel to the force field of gravity, the complex growth rate of an arbitrary oscillatory motions of growing amplitude, lies inside a semi-circle in the right half of the $\sigma_r\ \sigma_i$ - plane, with the center at the origin and radius is equal to $\left\{\frac{Rp_2}{\pi^2 p_1(1+2p_2+\pi^2F)}\right\}$, where R is the thermal Rayleigh number, p_1 is the thermal Prandtl number, p_2 is the magnetic Prandtl number and F is the viscoelasticity parameter of the Rivlin-Ericksen fluid. The result is important since the exact solutions of the problem investigated in closed form, are not obtainable, for any arbitrary combinations of perfectly conducting dynamically free and rigid boundaries.

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