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EFFECTS OF STENOSES ON CASSON FLOW OF BLOOD THROUGH ARTERIES

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ABSTRACT:

Blood flow through an axially symmetric but radially non-symmetric stenosed artery has been investigated in this paper. Blood has been represented by a non-Newtonian fluid obeying Casson fluid equation. Variation in wall shear stress with increasing stenosis height for different stenosis length and shape parameter has been shown. It is observed that the wall shear stress increases for the increasing stenoses height.The increase in stenosis height results the increase in wall shear stress. It is due to the narrowing of artery size. Wall shear stress decreases with increasing trend of shape parameter. The variation of flow rate with axial distance has been also studied for different shape parameter.

Keywords: Casson Fluid, Stenosis, Resistance to flow, wall Shear Stress (WSS), Flow rate

Introduction:

Casson fluid model is a non-Newtonian fluid with yield stress which is widely used for modeling blood flow in narrow arteries. Many researchers have used the Casson fluid model for mathematical modeling of blood flow in narrow arteries at low shear rates. Blair [1] demonstrated that the Casson fluid model is adequate for the representation of the simple shear behavior of blood in narrow arteries. Casson [2] studied the validity of Casson fluid model in his studies pertaining to the flow characteristics of blood and reported that at low shear rates the yield stress for blood is nonzero. Charm and Kurland [3] pointed out in their investigational findings that the Casson fluid model could be the best representative of blood when it flows through constricted arteries at low shear rates and that it could be useful to human blood at a wide range of hematocrit and shear rates. Aroesty and Gross [4] have developed a Casson fluid theory for pulsatile blood flow through narrow uniform arteries. Chaturani and Samy [5] analyzed the pulsatile flow of Casson fluid through stenosed arteries using the perturbation method.

Varghese and Frankel [6] obtained numerical predictions for computational pulsatile flow through different axisymmetric stenosis within the frame work of two equation turbulence models. Niu et al. [7] investigated the formation of the shear stress on the three-dimensional realistic health and dissected aortic dissection models under the laminar Newtonian flow assumption with Reynolds numbers 100 to 600. Bhardwaj and Kanodia [8] considered the pulsatile flow of blood through a stenosed porous medium under periodic body acceleration without considering magnetic effect. Balossino et al. [9] investigated spatial and temporal distribution of arterial wall shear stress (WSS) after the expansion of stents of different designs and different strut thicknesses. They also detected common oscillatory WSS behavior in all stent models. A mathematical model of blood flow through an irregular arterial mild stenosis is developed by Jain et al. [10] and they studied if the viscosity of fluid increases the velocity of fluid decreases in the presence of stenosis. Srivastava and Mishra [11] investigated the effects of an overlapping stenosis on blood flow characteristics in a narrow artery. They investigated that the impedance increases with the non-Newtonian behaviour of blood as well as with the stenosis size.

Mishra et al. [12] proposed a fluid mechanical study on the effects of the permeability of the wall through an artery with a composite stenosis. They derived expressions for the blood flow characteristics, flow resistance, the wall shear stress, shearing stress at the stenosis throat. Biswas and Laskar [13] investigated that axial velocity and flow rate increase with slip but decrease with yield stress and wall shear stress increases in Herschel-Bulkley fluid in comparison with corresponding Newtonian fluid. Nanda and Bose [14] developed a mathematical model for studying blood flow through a narrow artery with multiple stenoses. They observed that the stenosis height and axial velocity of flow effectively influence the shear stress in a stenosed artery. Various mathematical models have been studied by some researchers to explore the various aspects of blood flow in stenosed artery. Singh and Singh [15] discussed a mathematical model to study the effects of stenosis shape and height on the resistance to flow for different values of yield stress, blood viscosity and flux through an axially symmetric and radially non–symmetric artery under stenotic condition by considering blood as Casson fluid. Singh and Singh [16] studied the effect of blood yield stress, viscosity and flux on the resistance –to-flow ratio for Bingham plastic flow of blood through vessels contaning abnormal segments. They observed that as the yield stress increases, the resistance to flow ratio moves further from one. Resistance to flow shows no significant variation for variable blood viscosity and it decreases and moves closer to one as flux decreases.

Mathematical Formulation:

Let us consider an artery with stenosis symmetrical about the axis but non-symmetrical with respect to radial co-ordinates. The mathematical expression for geometry is given by

$$\frac{R}{R_0} = 1 - \varepsilon\left[l_0^{s-1}(z-d) - (z-d)^s \right]; d \le z \le d + l_0 \quad (1)$$

= 1; Otherwise

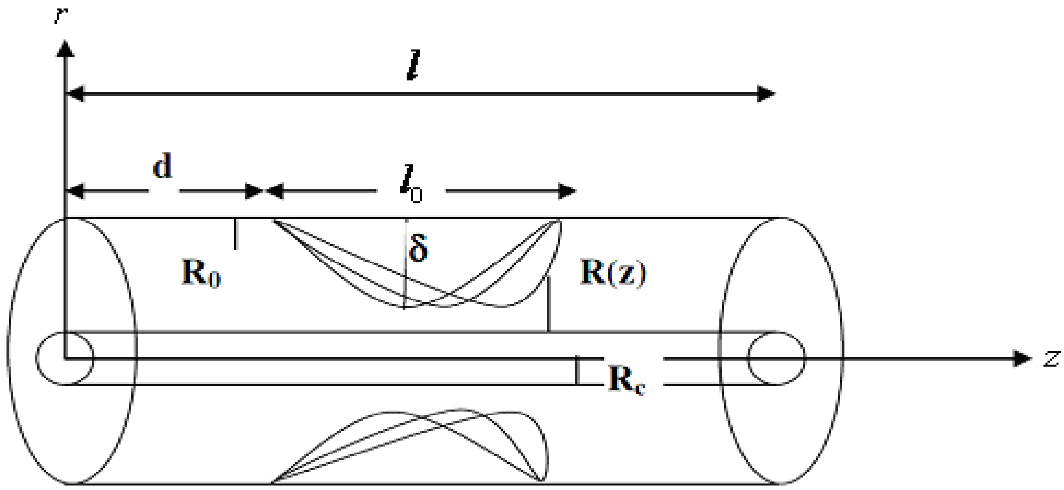

Fig 1: Geometry of an axially symmetric and radially non-symmetric stenosed artery

Where R, R_0 are tube radius (with, without stenosis), $s \geq 2$ is a shape parameter determining stenosis shape, l_0 is stenosis length, d indicates its location.

$$\varepsilon = \frac{\delta}{R_0} \frac{s^{s-1}}{l_0^{\,s}(s-1)}$$

δ be the maximum height of the stenosis located at

$$z = d + \frac{l_0}{s^{\frac{1}{s}-1}}$$

$$\dot{\beta} = f(\tau) = -\frac{\partial w}{\partial r} = \frac{(\sqrt{\tau} - \sqrt{\tau_0})^2}{\mu}; \tau \geq \tau_0 \tag{2}$$

Volumetric flow rate is given by

$$Q = \int_0^R 2\pi r w dr$$

$$= 2\pi \left[\left(w.\frac{r^2}{2} \right)_0^R - \int_0^R \left(\frac{dw}{dr} \right).\frac{r^2}{2} dr \right]$$ (Integrating by parts)

Using the condition $w = 0$ when $r = R$, we obtain

$$= 2\pi \int_0^R \left(-\frac{dw}{dr} \right).\frac{r^2}{2} dr = \pi \int_0^R f(\tau).r^2 dr \tag{3}$$

Since $\tau = -\frac{r}{2}\frac{dp}{dz}, \tau_w = -\frac{R}{2}\frac{dp}{dz}$ (4)

From (4)

$$\frac{\tau}{\tau_w} = \frac{r}{R} \Rightarrow r = \frac{R}{\tau_w}\tau \Rightarrow dr = \frac{R}{\tau_w}d\tau \quad (5)$$

Lower limit of (3) is $\tau = 0$ when $r = 0$

Upper limit of (3) is $\tau = \tau_w$ when $r = R$

Substituting the values of r and dr from (5) in (3).

$$Q = \pi \int_0^{\tau_w} f(\tau)\frac{R^2}{\tau_w^{\ 2}}\tau^2 . \frac{R}{\tau_w} d\tau = \frac{\pi R^3}{\tau_w^{\ 3}} \int_0^{\tau_w} \tau^2 f(\tau) d\tau$$

Putting the value of $f(\tau)$ from (2)

$$Q = \frac{\pi R^3}{\tau_w^{\ 3}} \int_0^{\tau_w} \tau^2 \frac{(\sqrt{\tau} - \sqrt{\tau_0})^2}{\mu} d\tau$$

$$= \frac{\pi R^3}{\mu \tau_w^{\ 3}} \int_0^{\tau_w} \tau^2 (\tau + \tau_0 - 2\sqrt{\tau\tau_0}) d\tau$$

$$= \frac{\pi R^3}{\mu \tau_w^{\ 3}} \int_0^{\tau_w} (\tau^3 + \tau_0 \tau^2 - 2\tau^{5/2}\sqrt{\tau_0}) d\tau$$

$$= \frac{\pi R^3}{\mu \tau_w^{\ 3}} \left[\frac{\tau^4}{4} + \tau_0 \frac{\tau^3}{3} - 2\sqrt{\tau_0} . \frac{2}{7}\tau^{7/2} \right]_0^{\tau_w}$$

$$= \frac{\pi R^3}{\mu \tau_w^{\ 3}} \left[\frac{\tau_w^{\ 4}}{4} + \tau_0 \frac{\tau_w^{\ 3}}{3} - 2\sqrt{\tau_0} . \frac{2}{7}\tau_w^{\ 7/2} \right]$$

$$= \frac{\pi R^3}{\mu \tau_w^{\ 3}} \tau_w^{\ 4} \left[\frac{1}{4} + \frac{1}{3}\frac{\tau_0}{\tau_w} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_w}} \right]$$

$$Q = \frac{\pi R^3 \tau_w}{\mu} \left[\frac{1}{4} + \frac{1}{3}\frac{\tau_0}{\tau_w} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_w}} \right] \quad (6)$$

When $\frac{\tau}{\tau_R} \ll 1$ replacing 1/3 by 16/49 in the second term.

$$Q = \frac{\pi R^3 \tau_w}{\mu}\left[\frac{1}{4} + \frac{16}{49}\frac{\tau_0}{\tau_w} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_w}}\right]$$

$$Q = \frac{\pi R^3 \tau_w}{\mu}\left[\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_w}}\right]^2$$

$$\left[\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_w}}\right]^2 = \frac{\mu Q}{\pi R^3 \tau_w}$$

$$\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_w}} = \sqrt{\frac{\mu Q}{\pi R^3}}\frac{1}{\sqrt{\tau_w}}$$

(7)

Multiplying by $2\sqrt{\tau_R}$ on both sides of (7), we get

$$\sqrt{\tau_w} - \frac{8}{7}\sqrt{\tau_0} = 2\sqrt{\frac{\mu Q}{\pi R^3}}$$

$$\sqrt{\tau_w} = \frac{8}{7}\sqrt{\tau_0} + 2\sqrt{\frac{\mu Q}{\pi R^3}}$$

Squaring on both sides to obtain

$$\tau_w = \left[\frac{8}{7}\sqrt{\tau_0} + 2\sqrt{\frac{\mu Q}{\pi R^3}}\right]^2$$

Using $\tau_w = -\frac{R}{2}\frac{dp}{dz}$

$$-\frac{R}{2}\frac{dp}{dz} = \left[\frac{8}{7}\sqrt{\tau_0} + 2\sqrt{\frac{\mu Q}{\pi R^3}}\right]^2$$

$$\frac{dp}{dz} = -\frac{2}{R}\left[\frac{8}{7}\sqrt{\tau_0} + 2\sqrt{\frac{\mu Q}{\pi R^3}}\right]^2$$

$p = p_1$ at $z = 0$ and $p = p_2$ at $z = l$

$$p_1 - p_2 = -\frac{128\tau_0}{49R_0}\int_0^l\left(\frac{R}{R_0}\right)^{-1}dz - \frac{64}{7R_0^{5/2}}\sqrt{\frac{\mu\tau_0 Q}{\pi}}\int_0^l\left(\frac{R}{R_0}\right)^{-5/2}dz$$

$$-\frac{8\mu Q}{\pi R_0^4}\int_0^l\left(\frac{R}{R_0}\right)^{-4}dz$$

$$\lambda = -\frac{128\tau_0}{49R_0Q}\int_0^l\left(\frac{R}{R_0}\right)^{-1}dz - \frac{64}{7R_0^{5/2}}\sqrt{\frac{\mu\tau_0}{Q\pi}}\int_0^l\left(\frac{R}{R_0}\right)^{-5/2}dz$$

$$-\frac{8\mu}{\pi R_0^4}\int_0^l\left(\frac{R}{R_0}\right)^{-4}dz$$

$$\frac{p_1 - p_2}{Q} = -\frac{128\tau_0}{49R_0Q}\int_0^l\left(\frac{R}{R_0}\right)^{-1}dz - \frac{64}{7R_0^{5/2}}\sqrt{\frac{\mu\tau_0}{Q\pi}}\int_0^l\left(\frac{R}{R_0}\right)^{-5/2}dz$$

$$-\frac{8\mu}{\pi R_0^4}\int_0^l\left(\frac{R}{R_0}\right)^{-4}dz$$

$$\lambda = -\frac{128\tau_0}{49R_0Q}\int_0^l\left(\frac{R}{R_0}\right)^{-1}dz - \frac{64}{7R_0^{5/2}}\sqrt{\frac{\mu\tau_0}{Q\pi}}\int_0^l\left(\frac{R}{R_0}\right)^{-5/2}dz$$

$$-\frac{8\mu}{\pi R_0^4}\int_0^l\left(\frac{R}{R_0}\right)^{-4}dz \qquad (8)$$

Suppose $f_1 = \frac{128\tau_0}{49R_0Q}$, $f_2 = \frac{64}{7R_0^{5/2}}\sqrt{\frac{\mu\tau_0}{Q\pi}}$, $f_3 = \frac{8\mu}{\pi R_0^4}$

$$\lambda = -f_1\left\{\int_0^d\left(\frac{R}{R_0}\right)^{-1}dz + \int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-1}dz + \int_{d+l_0}^{l}\left(\frac{R}{R_0}\right)^{-1}dz\right\}$$

$$-f_2\left\{\int_0^d\left(\frac{R}{R_0}\right)^{-5/2}dz+\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-5/2}dz+\int_{d+l_0}^{l}\left(\frac{R}{R_0}\right)^{-5/2}dz\right\}$$

$$-f_3\left\{\int_0^d\left(\frac{R}{R_0}\right)^{-4}dz+\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-4}dz+\int_{d+l_0}^{l}\left(\frac{R}{R_0}\right)^{-4}dz\right\}$$

(9)

$$\lambda=-(l-l_0)(f_1+f_2+f_3)-(f_1I_1+f_2I_2+f_3I_3) \quad (10)$$

Where $I_1=\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-1}dz$, $I_2=\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-5/2}dz$, $I_3=\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-4}dz$

If there is no stenosis i.e. in the normal condition, we have

$$\lambda_N=-(f_1+f_2+f_3)l \quad (11)$$

The resistance to flow is given by

$$\bar{\lambda}=1-\frac{l_0}{l}+\frac{f_1I_1+f_2I_2+f_3I_3}{(f_1+f_2+f_3)l} \quad (12)$$

From eq. (4) and (5), the wall shear stress is given by

$$\tau_w=\frac{64}{9}\tau_0+\frac{4\mu Q}{\pi R^3}+\frac{32}{7}\sqrt{\frac{\mu Q\tau_0}{\pi R^3}} \quad (13)$$

Wall shear stress in normal situation is written as

$$\tau_N=\frac{4\mu Q}{\pi R_0^{\,3}} \quad (14)$$

The wall shear stress ratio $\bar{\tau}_w$ can be obtained as

$$\bar{\tau}_w=\frac{\tau_w}{\tau_N}=\frac{16\pi\tau_0R_0^{\,3}}{49\mu Q}+\frac{1}{\left(\frac{R}{R_0}\right)^3}+\frac{8}{7}\left(\frac{\tau_0}{\mu Q}\right)^{1/2}\frac{R_0^{\,3/2}}{\left(\frac{R}{R_0}\right)^{3/2}} \quad (15)$$

The wall shear stress ratio at the mid point of the stenosis is

$$(\overline{\tau}_w)_{mid\,point} = \frac{16\pi\tau_0 R_0^{\ 3}}{49\mu Q} + \frac{1}{\left(1-\frac{\delta}{R_0}\right)^3} + \frac{8}{7}\left(\frac{\tau_0}{\mu Q}\right)^{1/2} \frac{R_0^{\ 3/2}}{\left(1-\frac{\delta}{R_0}\right)^{3/2}}$$

RESULTS AND DISCUSSION:

It is complicated to hold the problem with the extensive concept considered here, but calculations with MATLAB 7.0 makes it easier to illustrate the numerical results graphically for the present study. The flow analysis has been carried out by studying the consequence of entity factors like stenoses shape parameter S , stenoses length l_0, stenoses size and axial distance z which depends upon hematocrit (red blood cells). To attain the numerical results for flow rate and wall shear stress, some parameters have been taken constant with the values $R_0 = 1.5, l = 5, \mu = .000345 Pa.s, \tau_0 = .02 n/m^2$.

Figure (2) depicts the variation of wall shear stress for different values of stenoses length against stenoses height. The wall shear stress is increasing for the increasing stenoses length.The increase in stenoses height results the increase in wall shear stress. It is due to the narrowing of artery size. **Figures (3) & (4)** illustrate the wall shear stress against stenoses height for different values of stenoses shape parameter S. The wall shear stress decreases at (z=0.45) and increases at (z=0.55) for increasing values of stenoses shape parameter S . Because at (z=0.45) the height of stenoses decreases while at (z=0.55) it increases for increasing values of stenoses shape parameter S due to non-symmetric property of stenoses.

From figures (5) & (6), it is obvious that the flow rate decreases with an increase in the value of stenoses size. It is also clear from figures that at (z=0.45) the flow rate increases, while it is decreasing at (z=0.55) for increasing stenoses shape parameter S . **Figure (7)** elucidates the variation in flow rate for different values of stenoses shape parameter S against axial distance *z*. In this case the length of stenoses has been taken $l_0 = l/3$ to investigate the flow rate behavior at different stenoses length and height properly. As it is clear from figure that the flow rate is minimum at the centre of artery for $s = 2$. Because symmetric artery has the maximum height at centre of the artery, which causes minimum flow rate. It is also observe that first flow rate decreases upto centre of the artery afterthat it increases. Because stenoses height first increases upto centre of artery afterthat it dereases for symmetric artery.

Figure (8) describes the effects of yield stress on resistance to flow against stenoses size. The resistance to flow increases with an increase in the value of yield stress. It is also increasing for increasing values of stenoses size. It is due to narrowing of artery which results the increase in wall shear stress. That causes increase in resistance to flow.

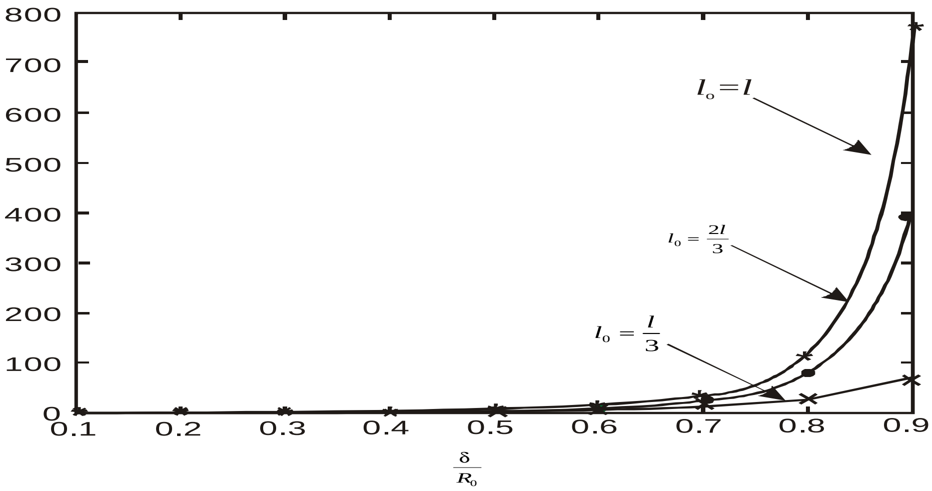

Fig: 2: Wall shear stress a against stenosis height for different stenosis length

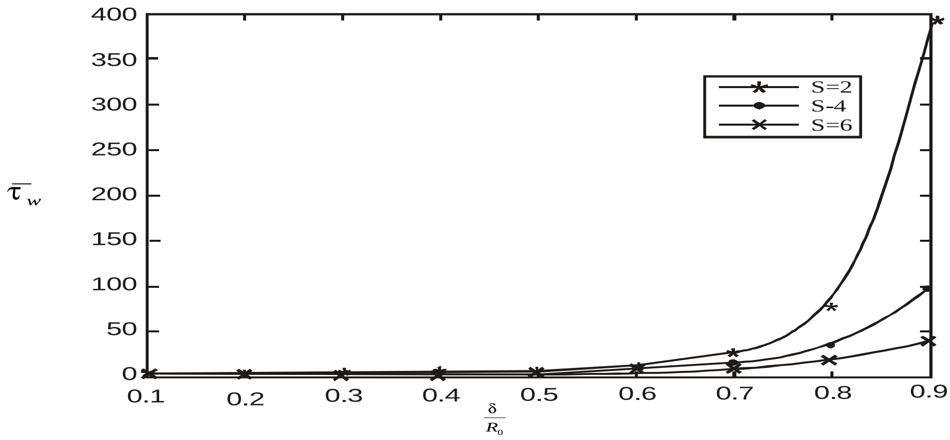

Fig: 3: Wall shear stress against stenosis height for different values of *s* at axial distance z = .10

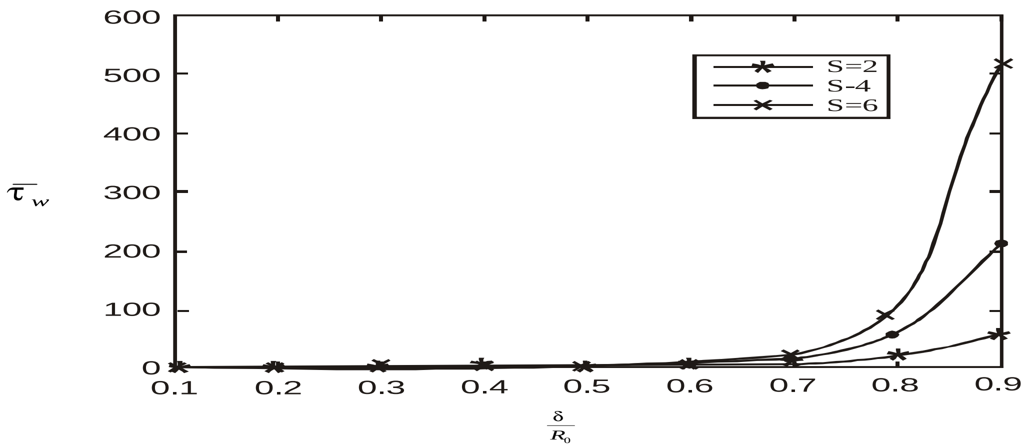

Fig: 4: Wall shear stress against stenosis height for different stenosis height for different values of *s* at axial distance *z* = .15

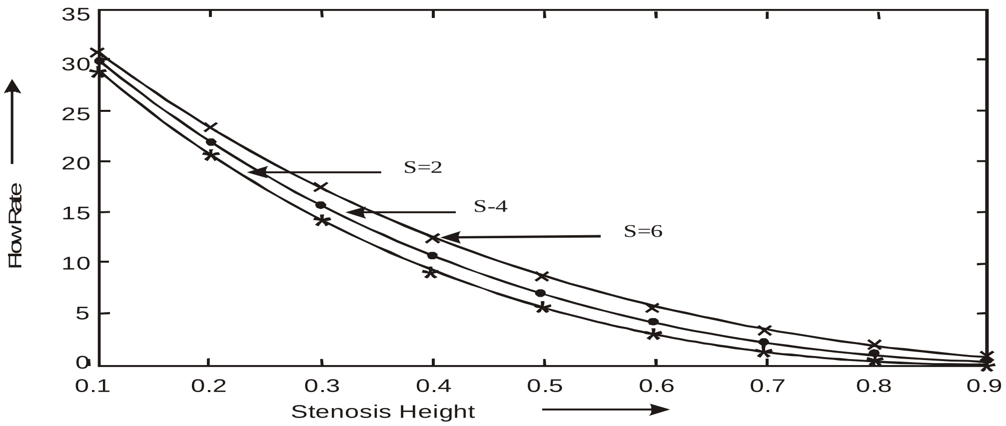

Fig: 5: Flow rate against different stenosis height for different values of s at z = .10

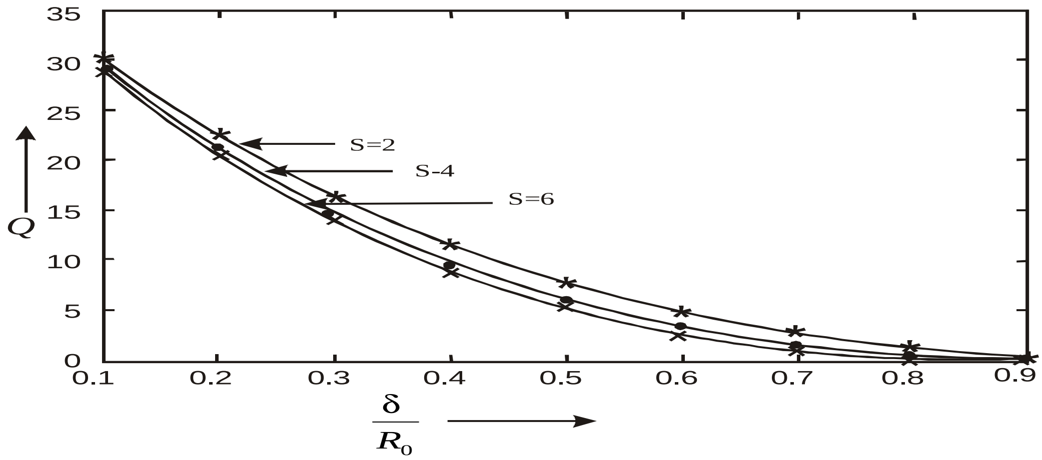

Fig: 6: Flow rate against different stenosis height for different values of s at z = .15

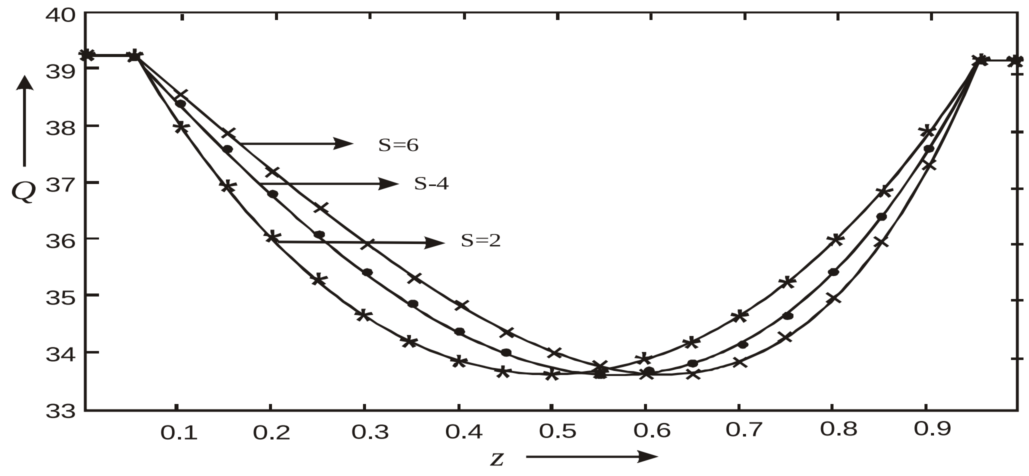

Fig: 7: Flow rate against axial distance z for different values of s

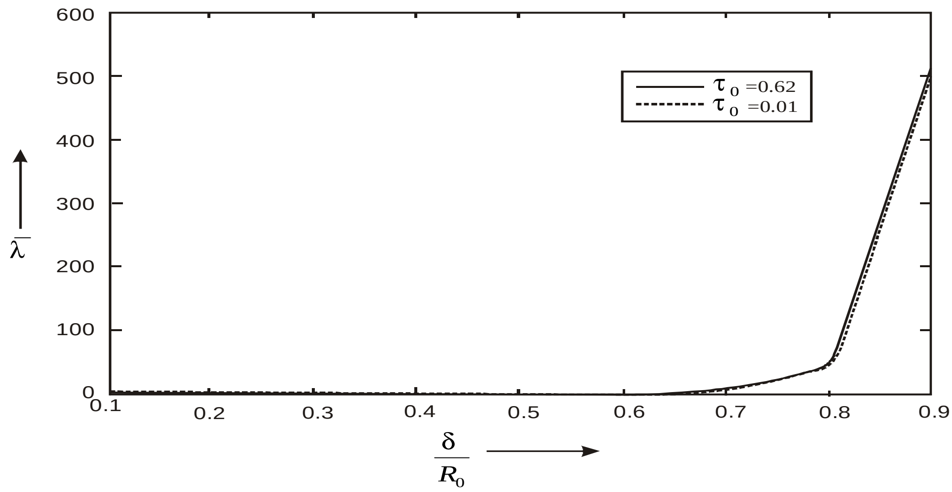

Fig: 8: Flow resistance against stenosis height for different values of yield stress τ_0

CONCLUSIONS:

The irregular growth of stenoses in blood vessels consequences the severe diseases like blood pressure, Atherosclerosis, heart attack and brain hemorrhage. This can not be ignored in modeling of problems occurs in blood flow through human arteries. A mathematical model can be developed for the deformation of stenoses with help of drug delivery models in human arteries. The nearby research will be very helpful in prediction of flow characteristics in blood vessels having mild and severe stenosis.

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