International Journal of Advanced Computer and Mathematical Sciences
ISSN 2230-9624. Vol 5, Issue1, 2014, pp1-8
http://bipublication.com

STOCHASTIC OPTIMIZATION PROGRAMMING PROBLEMS OF MULTIPLE SCLEROSIS

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[Received-21/11/2013, Accepted-01/01/2014]

ABSTRACT

Measuring the neurological disorders due to the cause of Multiple sclerosis is one of the researches which require the attention of operational research scientists. In this paper we have developed statistical measures and two optimization programming problems are (1) Minimizing the expected sizes of the scars and (2) Maximizing the expected size of defense MS cells, with suitable subject to the constraints. The decision variables on the growth and loss rates of MS, the growth and loss rates of defense MS cells are explored. The model behavior and the drug sensitivity were analyzed through numerical illustrations. It is observed that the size of the scar is influenced by the loss rate of myelin sheath and the growth & loss rates of infection causing T cells. Development of computer assisting decision tools is the scope of the study to convert it in to user friendly in healthcare industry.

Keywords: Optimal Control Strategy, Multiple Sclerosis, Decision tools, Healthcare Management.

1. INTRODUCTION:

Multiple sclerosis was introduced by Jean-Martin Chrcot in 1868, MS is a chronic inflammatory demyelinating disease that affects the central nervous system and called another names as "disseminated sclerosis" or "encephalomyelitis disseminata". The name multiple sclerosis refers to scars (scleroses-better known as plaques or lesions). It may be badly affected with the bacterial attacks and viral infections. There are numerous reasons for central nervous system to get exposure to infections. Inflammation and loss of immunity are the causing factors, which have significant influence of adverse results with defense MS cells and proactive results with multiple sclerosis. It is observed that improved immunity of the body system will act as catalyst to boost the protective mechanisms of central nervous system so as the myelin sheath by generating good number of defense MS cells.

Frederik Barkhof *et al.* have identified strategies to optimize the utilization of MRI in examine the MS disease activity in treatment trials [1]. Rachel Makay *et al.* [7] have developed hidden markov models to lesion count data of individual multiple sclerosis patients and also described the behavior of lesions over time and efficient method is used for original model validity check [8]. Ollivier Hyrien *et al.* have developed a stochastic model to find out the oligodendrocytes generation in cell nature and also observed model validation with time-lapse data [4]. Ruiz-Pena *et al.* (2008) have optimized multiple sclerosis treatment by avoiding subjective interpretation. A software based tool was developed to automatize the recommendations of Canadian multiple sclerosis patients [2]. Matthew W. Tanner *et al.* (2008) have formulated a stochastic programming problem for observe the optimal vaccination policy in two phases one is when vaccine supply is limited and another one is a cost- benefit scenario for controlling diseases under parameter uncertainty [3]. Vicente Pico–Ramirez *et al.* (2010) have developed a stochastic optimization programming problem for the treatment of human pathogenic disease using stochastic optimal control theory [9]. Rachel Miller Neilan *et al.* (2010) have introduced optimal control theory for applied on disease models and also provided examples for illustrate the optimal control [7]. P. Tirupathi Rao *et al.* (2012) have developed stochastic model and derived statistical measures of growth of multiple sclerosis [5]. Tirupathi Rao *et al.* (2013) have developed optimization programming problems based on stochastic models for MS [6].

However there is a little evidence on development of stochastic optimization programming problems to monitor the severity of MS disease. While developing optimization programming problems by Tirupathi Rao *et al.* they have developed objective functions directly i.e. minimization of average size of MS causing cells and maximization of average size of oligodendrocytes and also in each case they have observed two decision parameters only. They have ignored to observe the all parameters at a time of period. When we observe the behavior of MS, it is necessary for optimal monitoring of MS disease we need to observe all the decision parameters (α_1, α_2, $\beta_1$and β_2) at a time of period in all cases. The core objective of this work is to develop the programming problems that can minimize the severity of MS with several feasible constraints.

2. STOCHASTIC MODEL:

Notations

Δt= infinitesimal interval of time

α_1= Growth rate in MS causing cells per unit time

α_2= Growth rate in defense MS cells per unit time

β_1= Loss rate in MS causing cells per unit time

β_2= Loss rate in defense MS cells per unit time

I_0=Initial size of MS causing cells at a point of time t

J_0=Initial size of defense MS cells at a point of time t

t =Time of observation

A= Allowable maximum number of MS cells on average

B= Minimum required number of defense MS cells on average

C=Minimum variability in the size MS causing cells

D=Maximum allowable variability in defense MS cells

g=The coefficient in linear combination of growth of MS causing cells and defense MS cells

h=The coefficient in linear combination of loss of MS causing cells and defense MS cells

K_1 and K_2 are constants

A stochastic model is developed for multiple sclerosis with the following assumptions. Let us consider the event occurred in non-overlapping intervals of time are statistically independent. The probability of growth of MS causing cells during (t, t+ Δt) given that $\exists$ ‘r’ cells during (0,t) is

$r\alpha_1\Delta t + o(\Delta t)$; the probability of growth of

defense MS cells during (t, t+ Δt) given $\exists$exists ‘s’

cells during (0,t) is $s\alpha_2\Delta t + o(\Delta t)$; The probability of loss of MS causing cells during (t, t+ Δt) given that ∃ 'r' cells during (0,t) is $r\beta_1\Delta t + o(\Delta t)$; The probability of loss of defense MS cells during (t, t+ Δt) given that ∃ 's' cells during (0,t) is $s\beta_2\Delta t + o(\Delta t)$; The probability of no growth in MS causing cells during (t, t+ Δt) given that ∃ 'r' cells during (0,t) is $1-(r\alpha_1\Delta t + o(\Delta t))$; the probability of no growth in defense MS cells during (t, t+ Δt) given that ∃ 'r' cells during (0,t) is $1-(s\alpha_2\Delta t + o(\Delta t))$; The probability of no loss to MS causing cells during (t, t+ Δt) given that ∃ 'r' cells during (0,t) is $1-(r\beta_1\Delta t + o(\Delta t))$; The probability of no loss to MS cells during (t, t+ Δt) given that ∃ 's' cells during (0,t) is $1-(s\beta_2\Delta t + o(\Delta t))$; the probability of happing of more than one events during Δt time is $o(\Delta t)^2$.

Let $p_{r,s}(t)$ be the joint probability of 'r' MS causing cells and 's' defense MS cells in a myelin per unit time 't'. Then the difference differential equations of the model are:

$$p'_{rs}(t) = -p_{r,j}(t)[r(\alpha_1+\beta_1)+s(\alpha_2+\beta_2)] + p_{r-1,s}(t)(r-1)\alpha_1 + p_{r+1,s}(t)(r+1)\beta_1 + p_{r,s-1}(t)(s-1)\alpha_2 + p_{r,s+1}(t)(s+1)\beta_2;\ r,s \geq 1 \quad (2.1)$$

$$p'_{0,0}(t) = \beta_1 p_{1,0}(t) + \beta_2 p_{0,1}(t) \dots (2.2)$$

$$p'_{1,0}(t) = -(\alpha_1+\beta_1)p_{1,0}(t) + \beta_2 p_{1,1}(t) + 2\beta_1 p_{2,0}(t) \dots (2.3)$$

$$p'_{0,1}(t) = -(\alpha_2+\beta_2)p_{0,1}(t) + \beta_1 p_{1,1}(t) + 2\beta_2 p_{0,2}(t) \dots (2.4)$$

With initial conditions

$p_{r,s}(0) = 1\ \forall\ r,s > 0\ and$

$p_{r,s}(0) = 0\ \forall\ r = 0, s = 0$

Let p(x,y;t) be the joint probability generating function of $p_{r,s}(t)$, such that

$$p(x,y;t) = \sum_{r=0}^{\infty}\sum_{s=0}^{\infty} x^r y^s p_{r,s}(t)$$

Multiplying the equations 2.1 to 2.4 both sides with $x^r y^s$ and summing overall r & s, we obtain

$$\sum_{r=0}^{\infty}\sum_{s=0}^{\infty} x^r y^s p'_{r,s}(t)$$

$$= -(\alpha_1+\beta_1)\sum_{r=0}^{\infty}\sum_{s=0}^{\infty} r x^r y^s p_{r,s}(t) - (\alpha_2+\beta_2)\sum_{r=0}^{\infty}\sum_{s=0}^{\infty} s x^r y^s p_{r,s}(t)$$
$$+ \alpha_1\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}(r-1)x^r y^s p_{r-1,s}(t) + \beta_1\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}(r+1)x^r y^s p_{r+1,s}(t)$$
$$+ \alpha_2\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}(s-1)x^r y^s p_{r,s-1}(t) + \beta_2\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}(s+1)x^r y^s p_{r,s+1}(t)$$

… (2.5)

On simplifying the equation (2.5) we get,

$$\frac{\partial}{\partial t}p(x,y;t)$$

$$[-(\alpha_1+\beta_1)x+\alpha_1x^2+\beta_1]\frac{\partial}{\partial x}p(x,y;t)$$

$$+[-(\alpha_2+\beta_2)y+\alpha_2y^2+\beta_2]\frac{\partial}{\partial y}p(x,y;t) \quad \dots(2.6)$$

We can find the characteristics of the model by using the joint cumulant generating function of $p_{r,s}(t)$. Taking $x=e^m$ and $y=e^n$ and denoting k (m, n; t) as the joint cumulant generating function of $p_{r,s}(t)$, we can find the following:

$$\frac{\partial}{\partial t}k(m,n;t)=[-(\alpha_1+\beta_1)+\alpha_1e^m+\beta_1e^{-m}]\frac{\partial}{\partial m}k(m,n;t)+$$

$$[-(\alpha_2+\beta_2)+\alpha_2e^n+\beta_2e^{-n}]\frac{\partial}{\partial n}k(m,n;t)\dots(2.7)$$

3. DIFFERENTIAL EQUATIONS & STATISTICALs MEASURES:

Let $m_{r,s}(t)$ denote the moments of order (r, s) of MS causing cells and defense MS cells at time 't' . Then the differential equations are

$$\frac{d}{dt}m_{1,0}(t)=(\alpha_1-\beta_1)m_{1,0}(t) \quad \dots(3.1)$$

$$\frac{d}{dt}m_{0,1}(t)=(\alpha_2-\beta_2)m_{0,1}(t) \quad \dots(3.2)$$

$$\frac{d}{dt}m_{2,0}(t)=$$

$$(\alpha_1+\beta_1)m_{1,0}(t)+2(\alpha_1-\beta_1)m_{2,0}(t)$$

… (3.3)

$$\frac{d}{dt}m_{0,2}(t)=$$

$$(\alpha_2+\beta_2)m_{0,1}(t)+2(\alpha_2-\beta_2)m_{0,2}(t)$$

… (3.4)

Solving the above equations

The Expected number of MS causing cells at time 't' is

$$m_{1,0}(t)=e^{(\alpha_1-\beta_1)t}I_0$$

... (3.5)

Expected number of defense MS cells at time 't' is

$$m_{0,1}(t)=e^{(\alpha_2-\beta_2)t}I_0$$

... (3.6)

The variance of number of MS causing cells at time 't' is

$$m_{2,0}(t)=I_0\left[\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1}\right]e^{(\alpha_1-\beta_1)t}\left(e^{(\alpha_1-\beta_1)t}-1\right)$$

… (3.7)

The variance of number of defense MS cells at time 't' is

$$m_{0,2}(t)=I_0\left[\frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}\right]e^{(\alpha_2-\beta_2)t}\left(e^{(\alpha_2-\beta_2)t}-1\right)$$

… (3.8)

4. STOCHASTIC OPTIMIZATION PROGRAMMING PROBLEMS:

4.1 Optimization Programming Problem for Minimizing the Severity of MS

From the derived relations in section-3, the expected size of the MS causing cells at a point of time 't' is $e^{(\alpha_1-\beta_1)t}I_0$, and the expected size of the defense MS cells at a point of time 't' is $e^{(\alpha_2-\beta_2)t}I_0$. Here, the average number of MS causing cells is considered as unwanted component and average number of defense MS cells considered as wanted component. By using the concept of loss function, the relation between the difference of size in MS causing cells and size in defense MS cells is $Z_1=e^{(\alpha_1-\beta_1)t}I_0-e^{(\alpha_2-\beta_2)t}I_0$.

The objective is to minimize $Z_1=e^{(\alpha_1-\beta_1)t}I_0-e^{(\alpha_2-\beta_2)t}I_0$

The above mentioned objective function is in the influence of the following constraints. Let 'A' be the maximum threshold limit on the average number of MS causing cells. From section-3, the

average number of MS causing cells at a point of time is E (MS) = $e^{(\alpha_1-\beta_1)t}I_0$. Since E (MS) should not exceed the value of 'A'.

The constraint with 'A' and E (MS) is $e^{(\alpha_1-\beta_1)t}I_0 \leq A$

Let 'B' be the minimum threshold limit on the average number of defense MS cells. The expected number of defense MS cells at a point of time is equal to $e^{(\alpha_2-\beta_2)t}J_0$. Since the average size of defense MS cells should be more than a limit 'B'.

The constraint with 'B' and E(Defense MS cells) is $e^{(\alpha_2-\beta_2)t}J_0 \geq B$

Let 'C' be the minimum threshold limit on the variance of MS causing cells. The derived relation on variance of MS causing cells at a point of time is V (MS) $=I_0\left[\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1}\right]e^{(\alpha_1-\beta_1)t}\left(e^{(\alpha_1-\beta_1)t}-1\right)$.

Since, V (MS) should be greater than the value of C.

The constraint with 'C' and V (MS) is $I_0\left[\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1}\right]e^{(\alpha_1-\beta_1)t}\left(e^{(\alpha_1-\beta_1)t}-1\right)\geq C$

Let 'D' be the maximum threshold limit on the variance of defense MS cells. The derived relation on V(defense MS cells) at a point of time is $I_0\left[\frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}\right]e^{(\alpha_2-\beta_2)t}\left(e^{(\alpha_2-\beta_2)t}-1\right)$. Since V (Defense MS cells) should be less than the value of D.

The constraint with 'D' and V(Defense MS cells) is $I_0\left[\frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}\right]e^{(\alpha_2-\beta_2)t}\left(e^{(\alpha_2-\beta_2)t}-1\right)\leq D$

Let K_1 be the constant, which is equal to the linear combinations of both growth rates of MS causing cells and defense MS cells. Here, we are assuming that the number of MS causing cells and defense MS cells are having the growth such that one is influencing the other. The linear combination of both growth rates of MS causing cells and defense MS cells is $g\alpha_1+(1-g)\ \alpha_2$, where 'g' is the coefficient is convex combination and its limits are $0\leq g \leq 1$. Since, linear combinations of both growth rates should be equal to K_1.

The constraint with K_1 and linear combinations of both growth rates is $g\alpha_1+(1-g)\ \alpha_2=K_1$

Let K_2 be the constant equal to the linear combinations of both loss rates of MS causing cells and defense MS cells. Here, we are assuming that the number of MS causing cells and defense MS cells are having the loss such that one is influencing the other. The linear combination of both loss rates of MS causing cells and defense MS cells is $h\beta_1+(1-h)\beta_2$, where 'h' is the coefficient of convex combination and its limits are $0\leq h \leq 1$. Therefore linear combinations of both loss rates should be equal to K_2.

The constraint with K_2 and linear combinations of both loss rates is $h\beta_1+(1-h)\beta_2 =K_2$

Further it is assumed that the decision parameters α_1 (growth rate in MS causing cells per unit time); α_2 (growth rate in defense MS cells per unit time); β_1 (loss rate in MS causing cells per unit time) and β_2 (loss rate in defense MS cells per unit time) are non negative. In summary, the optimization programming problem is

To minimize $(Z_1)= e^{(\alpha_1-\beta_1)t}I_0 - e^{(\alpha_2-\beta_2)t}J_0$

Subject to the constraints:

$e^{(\alpha_1-\beta_1)t}I_0 \leq A$;

$e^{(\alpha_2-\beta_2)t}J_0 \geq B$;

$I_0\left[\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1}\right]e^{(\alpha_1-\beta_1)t}\left(e^{(\alpha_1-\beta_1)t}-1\right)\geq C$;

$I_0\left[\frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}\right]e^{(\alpha_2-\beta_2)t}\left(e^{(\alpha_2-\beta_2)t}-1\right)\leq D$;

$g\alpha_1+(1-g)\ \alpha_2=K_1$;

$h\beta_1+(1-h)\beta_2 =K_2$;

And α_1, α_2, $\beta_1$and $\beta_2\geq 0$. … (4.2.1)

4.2 Optimization Programming Problem for Maximizing the size of defense MS cells

From the derived relations in section-3, the expected size of the MS causing cells at a point of

time 't' is $e^{(\alpha_1-\beta_1)t}I_0$. The expected size of defense MS cells at a point of time 't' is $e^{(\alpha_2-\beta_2)t}J_0$. Here the average number of MS causing cells considered to be unwanted level of objective and average number of defense MS cells considered to be wanted level of objective. Notion of resulting positive benefit, the relation between the difference of defense MS cells and MS causing cells is

$$Z_2 = e^{(\alpha_2-\beta_2)t}J_0 - e^{(\alpha_1-\beta_1)t}I_0$$

Z_2 is the objective function to maximize the resulting size. The objective function is in the influence of the same constraints as in the previous problem along with decision parameters. In summary, the optimization programming problem is To maximize $(Z_2) = e^{(\alpha_2-\beta_2)t}J_0 - e^{(\alpha_1-\beta_1)t}I_0$

Subject to the constraints

$e^{(\alpha_1-\beta_1)t}I_0 \leq A$;

$e^{(\alpha_2-\beta_2)t}J_0 \geq B$;

$I_0\left[\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1}\right]e^{(\alpha_1-\beta_1)t}\left(e^{(\alpha_1-\beta_1)t}-1\right)\geq C$;

$J_0\left[\frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}\right]e^{(\alpha_2-\beta_2)t}\left(e^{(\alpha_2-\beta_2)t}-1\right)\leq D$;

$g\alpha_1+(1-g)\ \alpha_2=K_1$;

$h\beta_1+(1-h)\beta_2=K_2$;

And α_1, α_2, $\beta_1$and $\beta_2\geq 0$…. (4.2.2)

3.2.3 Numerical Illustrations and Sensitivity Analysis

The non linear programming problems 4.2.1 and 4.2.2 are solved with mathematical software LINGO 13 and the results were presented in table-1 and table-2. For computing the objective function and decision parameters, the hypothetical varying values of I_0, J_0, A, B, C, D, K_1 and K_2 are assumed.

Table-1

I_0	J_0	A	B	C	D	t	K_1	K_2	g	h	Z	α_1	β_1	α_2	β_2
2360	4620	5600	2700	1200	1000	2	220	220	0.3	0.3	-3870.78	733.1373	733.3333	0.08401	0
2370											-3864.02	733.1373	733.3333	0.08401	0
2375											-3860.64	733.1373	733.3333	0.08401	0
2380											-3857.26	733.1373	733.3333	0.08401	0
2385											-3853.89	733.1373	733.3333	0.08401	0
2370	4615	5600	2700	1200	1000	2	220	220	0.3	0.3	-3859.44	733.1371	733.3333	0.08409	0
	4620										-3864.02	733.1373	733.3333	0.08401	0
	4625										-3868.6	733.1375	733.3333	0.08394	0
	4630										-3873.18	733.1376	733.3333	0.08387	0
	4635										-3877.77	733.1378	733.3333	0.08380	0
2370	4620	5600	2700	1200	1000	1.9	220	220	0.3	0.3	-3864.021	733.127	733.3333	0.08844	0
						2					-3864.021	733.1373	733.3333	0.08401	0
						2.1					-3544.514	733.185	733.3333	0.06359	0
						2.12					-3544.506	733.1864	733.3333	0.06299	0
						2.13					-3544.501	733.1871	733.3333	0.06269	0
1200	1800	6600	1700	1000	1200	2	300	320	0.1	0.3	-1796.265	386.1037	388.9899	290.4329	290.4329
				1001							-1796.262	386.1043	388.99	290.4329	290.4329
				1002							-1796.258	386.1049	388.9902	290.4328	290.4328
				1003							-1796.255	386.1054	388.9903	290.4327	290.4327
				1005							-1796.249	386.1066	388.9906	290.4326	290.4326
800	1800	3600	1700	1200	1100	2	200	200	0.3	0.3	-2221.924	666.251	666.6667	0.17812	0
					1110						-2228.999	666.2486	666.6667	0.17917	0

					1120						**-2236.039**	**666.2462**	**666.6667**	**0.18021**	**0**
					1130						**-2243.046**	**666.2438**	**666.6667**	**0.18124**	**0**
					1140						**-2250.02**	**666.2414**	**666.6667**	**0.1823**	**0**

Table-2

I_0	J_0	A	B	C	D	t	K_1	K_2	g	h	Z	α_1	β_1	α_2	β_2
1200	**1800**	**6600**	**1700**	**1000**	**1200**	**2**	**300**	**320**	**0.1**	**0.3**	**1796.265**	**386.104**	**388.990**	**290.4329**	**290.4329**
1210											**1796.260**	**386.099**	**388.989**	**290.4335**	**290.4335**
1220											**1796.256**	**386.094**	**388.987**	**290.4340**	**290.4340**
1230											**1796.251**	**386.090**	**388.986**	**290.4345**	**290.4345**
1240											**1796.247**	**386.085**	**388.985**	**290.4350**	**290.4350**
1200	**1810**	**6600**	**1700**	**1000**	**1200**	**2**	**300**	**320**	**0.1**	**0.3**	**1806.27**	**386.104**	**388.990**	**290.433**	**290.4329**
	1820										**1816.27**	**386.104**	**388.990**	**290.433**	**290.4329**
	1830										**1826.27**	**386.104**	**388.990**	**290.433**	**290.4329**
	1840										**1836.27**	**386.104**	**388.990**	**290.433**	**290.4329**
	1850										**1846.27**	**386.104**	**388.990**	**290.433**	**290.4329**
1200	**1800**	**6600**	**1700**	**1000**	**1200**	**1.1**	**300**	**320**	**0.1**	**0.3**	**1793.77**	**383.543**	**388.326**	**290.717**	**290.717**
						1.12					**1793.87**	**383.640**	**388.351**	**290.707**	**290.707**
						1.14					**1793.96**	**383.734**	**383.734**	**290.696**	**290.696**
						1.16					**1794.05**	**383.825**	**388.399**	**290.686**	**290.686**
						1.17					**1794.09**	**383.869**	**388.411**	**290.681**	**290.681**
1200	**1800**	**6600**	**1700**	**1010**	**1200**	**2**	**300**	**320**	**0.1**	**0.3**	**1796.23**	**386.110**	**388.991**	**290.4323**	**290.4323**
				1020							**1796.20**	**386.115**	**388.993**	**290.4317**	**290.4317**
				1030							**1796.17**	**386.121**	**388.994**	**290.4310**	**290.4310**
				1040							**1796.14**	**386.126**	**388.996**	**290.4304**	**290.4304**
				1050							**1796.11**	**386.132**	**388.997**	**290.4298**	**290.4298**
1510	**1900**	**2300**	**1270**	**1200**	**1100**	**2**	**205**	**205**	**0.2**	**0.2**	**2298.36**	**1024.312**	**1025**	**0.17196**	**0**
					1101						**2299.22**	**1024.312**	**1025**	**0.17206**	**0**
					1102						**2300.08**	**1024.311**	**1025**	**0.17216**	**0**
					1103						**2300.94**	**1024.311**	**1025**	**0.17227**	**0**
					1105						**2302.66**	**1024.310**	**1025**	**0.17247**	**0**

From the table-1, it is observed that the objective function Z is increasing function of varying values of I_0 (Initial size of MS causing cells at a point of time t). The objective function Z, growth rate of defense MS cells are decreasing functions of J_0. The growth rate of MS causing cell is an increasing function of J_0 (Initial size of defense MS cells at a point of time t). It is observed that the objective function, growth and loss rates of MS causing cells are increasing functions of 'C'. Growth and loss rate of defense MS cells are decreasing functions of C (Minimum variability in the size MS causing cells). The objective function Z, growth rates of MS causing cells are decreasing functions and growth rates of defense MS cells is

increasing function of D (Maximum allowable variability in the size defense MS cells). The objective function, growth rate of MS causing cells are increasing functions and growth rate of defense MS cells are decreasing functions of 't' (time of observation) when all other parameters are constant.

From the table-2, it is observed that the objective function, growth and loss rate of MS causing cells are decreasing functions, growth and loss rate of defense MS cells are increasing functions of I_0 (Initial size of MS causing cells at a point of time). It is observed that the objective function Z is increasing function of J_0 (Initial size of defense MS cells at a point of time t). The objective function Z, growth and loss rate of defense MS cells are decreasing functions of 'C', growth and loss rate of MS causing cells are increasing functions of C (Minimum variability in the size MS causing cells). It is observed that the objective function, growth rates of defense MS cells are increasing functions and growth rates of MS causing cells are decreasing function of D (Maximum allowable variability in the size defense MS cells. And also it is observed that the objective function, growth rates of MS causing cells are increasing functions and loss rate of MS causing cells, growth and loss rates of defense MS cells are decreasing functions of t (time of observation) when other all parameters are constant.

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