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Elastic Wave Propagation Phenomenon at Solid-Solid Interface

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ABSTRACT

In this paper, the reflection and transmission phenomenon at a plane interface between viscoelastic solid half space and elastic solid half space is studied. P-wave or SV-wave is considered to be incident on the plane interface through viscoelastic solid half space. The amplitude ratios of various reflected and transmitted waves to that of incident wave are obtained. After obtaining the amplitude ratios, they have been computed numerically for a particular model and results thus obtained are depicted graphically with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties of the medium. A particular case of reflection at free surface of viscoelastic solid half space has been deduced and discussed.

1. INTRODUCTION

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structure material. To a laser extent linear elasticity describes the mechanical behaviour of other common solid materials i.e. concrete .coal and wood. However, the observed attenuation of the seismic waves in the earth, an important source of information regarding the composition and state of the deep interior of the earth cannot be explained by assuming the earth to be an elastic solid. Keeping this view, several problems of reflection and refraction in a linear viscoelastic solid have been discussed by many researchers like Cooper & Reiss (1965), Cooper (1967), Kumar & Gogna (1992), Tomar (1995), Kumar & Singh (2000), etc.

In the present problem, we consider the reflection and transmission of elastic waves (P-wave and SV-wave) which is obliquely incidents at an interface of linear viscoelastic solid half space and linear elastic solid half space. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically and discussed accordingly. A particular case of reflection at free surface of viscoelastic solid half space is also considered and discussed. The model considered is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model will be of great use in the fields related to earth sciences.

2. Basic equations

For M_1 (linear viscoelastic solid medium)

Following Kumar and Singh (2000), the equation governing the small motions in a linear viscoelastic solid may be written as

$$(K' + 4M'/3)\nabla(\nabla.\mathbf{u}') - M'\nabla \times (\nabla \times \mathbf{u}') = \rho_1\ddot{\mathbf{u}}',$$

where symbols K' is the complex bulk modulus, M' is the shear modulus, ρ_1 is the density of linear viscoelastic solid and $\mathbf{u}'$ is the displacement vector. Superposed dots on right hand side of equation (1) stand for second partial derivative with respect to time.

The stresses in the linear viscoelastic solid are given by

$$\sigma_{kl}' = (K' - 2M'/3)\theta\delta_{kl} + 2M'e_{kl},$$

where

$$e_{kl} = \frac{1}{2}\left(\frac{\partial u_k'}{\partial x_l} + \frac{\partial u_l'}{\partial x_k}\right), \quad \theta = \nabla.\mathbf{u}', \tag{3}$$

Using Helmholtz's theorem

$$\mathbf{u}' = \nabla\phi' + \nabla \times \boldsymbol{\psi}', \quad \nabla.\boldsymbol{\psi}' = 0, \tag{4}$$

We can show that ϕ' and ψ' satisfy

$$\alpha^2\nabla^2\phi' = \ddot{\phi}' \text{ and } \beta^2\nabla^2\psi' = \ddot{\psi}', \tag{5}$$

where

$$\alpha^2 = \left(K' + \frac{4M'}{3}\right)/\rho_1, \qquad \beta^2 = M'/\rho_1, \tag{6}$$

and

$$\psi' = -(\boldsymbol{\psi}')_y, \tag{7}$$

$$u' = \frac{\partial\phi'}{\partial x} + \frac{\partial\psi'}{\partial z}, \qquad w' = \frac{\partial\phi'}{\partial z} - \frac{\partial\psi'}{\partial x}. \tag{8}$$

For M_2 (Homogeneous isotropic elastic solid medium)

The equation governing the small motions in a homogeneous isotropic elastic are

$$\mu^*\nabla^2\mathbf{u}^* + (\lambda^* + \mu^*)\ \nabla(\nabla.\mathbf{u}^*) = \rho^*\ddot{\mathbf{u}}^*, \tag{9}$$

where symbols λ^*, μ^* are Lame's constants, ρ^* is the density and $\mathbf{u}^*$ is the displacement vector. Superposed dots on right hand side of eq. (9) stand for second partial derivative with respect to time.

The stress strain relation in the isotropic elastic medium is given by

$$\sigma_{ij}^* = \lambda^* e_{kk}^*\ \delta_{ij} + 2\mu^*\ e_{ij}^*, \tag{10}$$

where

$$e_{ij}^* = \frac{1}{2}\left(\frac{\partial u_i^*}{\partial x_j} + \frac{\partial u_j^*}{\partial x_i}\right), \tag{11}$$

are the components of the strain tensor, e_{kk}^* is the dilatation and σ_{ij}^* are the components of stress tensor in the isotropic elastic medium.

For the two dimensional problem, the displacement vector $\mathbf{u}'$ is taken as

$$\mathbf{u}^* = (u^*, 0, w^*), \tag{12}$$

The displacement components u^* and w^* are related to potential functions ϕ^* and ψ^* as

$$u^* = \frac{\partial \phi^*}{\partial x} + \frac{\partial \psi^*}{\partial z}, \quad w^* = \frac{\partial \phi^*}{\partial z} - \frac{\partial \psi^*}{\partial x}, \tag{13}$$

Using equations (12) and (13) in equation (9), we obtain as

$$\nabla^2 \phi^* = \frac{1}{v_1^{*2}} \frac{\partial^2 \phi^*}{\partial t^2}, \tag{14}$$

$$\nabla^2 \psi^* = \frac{1}{v_2^{*2}} \frac{\partial^2 \psi^*}{\partial t^2}, \tag{15}$$

where

$v_1^* = \sqrt{\frac{\lambda^* + 2\mu^*}{\rho^*}}$ and $v_2^* = \sqrt{\frac{\mu^*}{\rho^*}}$ are the velocities of longitudinal wave (P-wave) and transverse wave (SV-wave) in isotropic elastic medium respectively.

3. Formulation of the problem and its solution.

Considering a two dimensional problem by taking the z-axis pointing into lower half-space and the plane interface z=0 separating the linear viscoelastic solid half space M_1 $[z > 0]$ and elastic solid half space M_2 $[z < 0]$ (see figure1). A longitudinal wave (P-wave) or transverse wave (SV-wave) propagates through linear viscoelastic solid half space medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to each incident wave (P-wave or SV-wave), we get two reflected waves P-wave and SV-wave in the medium M_1 and two transmitted waves P-wave and SV-wave in medium M_2.

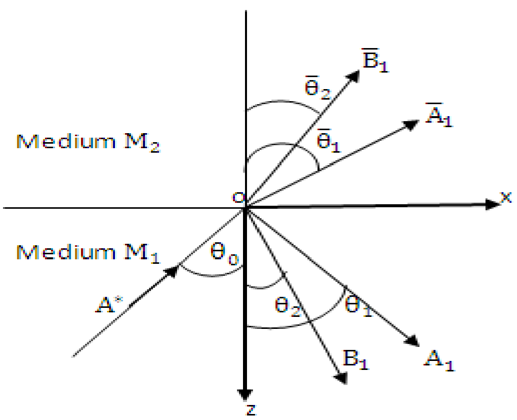


Fig.1 Geometry of the problem

In medium M_1

The potential function satisfying the equation (8) can be taken as

$$\phi' = A_0 \exp[ik_1(x\sin\theta_0 - z\cos\theta_0) + i\omega_1 t] + A_1 \exp[ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t] \tag{16}$$

$$\psi' = B_0 \exp[ik_2(x\sin\theta_0 - z\cos\theta_0) + i\omega_2 t] + B_1 \exp[ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t] \tag{17}$$

where A_0 and B_0 are amplitudes of the incident P-wave and SV-wave, respectively and A_1, B_1 are amplitudes of the reflected P-wave and SV-wave respectively and to be determined from boundary conditions.

In medium M_2

The potential function satisfying the equation (14)-(15) can be taken as

$$\phi^* = \bar{A}_1 \exp\left[i\bar{k}_1(x\sin\bar{\theta}_1 - z\cos\bar{\theta}_1) + i\bar{\omega}_1 t\right], \tag{18}$$

$$\psi^* = \bar{B}_1 \exp\{i\bar{k}_2(x\sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\bar{\omega}_2 t\}, \tag{19}$$

where $\bar{k}_1$ and $\bar{k}_2$ are wave numbers of transmitted P-wave and transmitted SV-wave, respectively. $\bar{A}_1$ and $\bar{B}_1$ are amplitudes of transmitted P-wave and transmitted SV-wave and are unknown to be determined from boundary conditions.

4. Boundary conditions. The appropriate boundary conditions at the interface z=0 are the continuity of displacement and stresses. Mathematically, these boundary conditions can be expressed as:

$$\sigma_{zz}' = \sigma_{zz}^*, \quad \sigma_{zx}' = \sigma_{zx}^*, \quad u' = u^*, \quad w' = w^*, \tag{20}$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{V}_2}, \tag{21}$$

Also

$$k_1 V_1 = k_2 V_2 = \bar{k}_1 \bar{V}_1 = \bar{k}_2 \bar{V}_2 = \omega, \quad \text{at } z = 0, \tag{22}$$

For P-wave,

$$V_0 = V_1, \quad \theta_0 = \theta_1, \tag{23}$$

For SV-wave,

$$V_0 = V_2, \quad \theta_0 = \theta_2, \tag{24}$$

For incident longitudinal wave at the interface z=0, putting $B_0 = 0$ in equation (17) and for incident transverse wave putting $A_0 = 0$ in equation (16). Substituting the expressions of potentials given by (16)-19) in equations (2),(8),(10) and (13) and using equations (20)-(24), we get a system of four non homogeneous which can be written as

$$\sum_{j=0}^{4} a_{ij} Z_j = Y_i, \quad (i = 1,2,3,4) \tag{25}$$

where

$$Z_1 = \frac{A_1}{A^*}, \quad Z_2 = \frac{A_2}{A^*}, \quad Z_3 = \frac{\bar{A}_1}{A^*}, \quad Z_4 = \frac{\bar{B}_1}{A^*} \tag{26}$$

Also a_{ij} in non dimensional form can be written as

$$a_{11} = -\frac{K'}{M'} - 2\sin^2\theta_0 + \frac{4}{3}, \quad a_{12} = 2\sin\theta_2\cos\theta_2\frac{k_2^{\,2}}{k_1^{\,2}}, \quad a_{13} = \frac{\lambda^*\bar{k}_1^{\,2}}{M' k_1^{\,2}},$$

$$a_{14} = \frac{\mu^*\bar{k}_2^{\,2}}{k_1^{\,2} M'}\sin 2\bar{\theta}_2, \quad a_{21} = -2\sin\theta_1\cos\theta_1, \quad a_{22} = -\frac{k_2^{\,2}}{k_1^{\,2}}(\cos^2\theta_2 - \sin^2\theta_2),$$

$$a_{23} = -\frac{\mu^*\bar{k}_1^{\,2}}{M' k_1^{\,2}}\sin 2\bar{\theta}_1, \quad a_{24} = -\frac{\mu^*\bar{k}_2^{\,2}}{M' k_1^{\,2}}\cos 2\bar{\theta}_2, \quad a_{31} = \sin\theta_1,$$

$$a_{32} = \frac{k_2}{k_1}\cos\theta_2, \quad a_{33} = -\frac{\bar{k}_1}{k_1}\sin\bar{\theta}_1, \quad a_{34} = \frac{\bar{k}_2}{k_1}\cos\bar{\theta}_2,$$

$$a_{41} = \cos\theta_1, \quad a_{42} = -\frac{k_2}{k_1}\sin\theta_2, \quad a_{43} = \frac{\bar{k}_1}{k_1}\cos\bar{\theta}_1 \quad a_{44} = \frac{\bar{k}_2}{k_1}\sin\bar{\theta}_2, \tag{27}$$

For incident longitudinal wave:

$$A^* = A_0, B_0 = 0, Y_1 = -a_{11}, Y_2 = a_{21}, Y_3 = -a_{31}, Y_4 = a_{41}, \tag{28}$$

For incident transverse wave:

$$A^* = B_0, A_0 = 0, Y_1 = a_{12}, Y_2 = -a_{22}, Y_3 = a_{32}, Y_4 = -a_{42}, \tag{29}$$

5. Particular case:

When upper half space is not present in the given formulation.

Considering a linear viscoelastic solid medium with free boundary surface, i.e. upper half space is not present in the given formulation. A plane wave (P-wave) or SV-wave propagating through the linear viscoelastic solid medium making an angle θ_0 with z-axis at the free surface z=0. Corresponding to each incident wave, we get two reflected waves. Boundary conditions for this case reduces to

$$\sigma_{zz}' = \sigma_{zz}^*, \quad \sigma_{zx}' = \sigma_{zx}^*, \tag{30}$$

And hence we obtain a system of two non-homogeneous equations which can be written as

$$\sum_{j=1}^{2} a_{ij} Z_j = Y_i, \quad (i = 1,2) \tag{31}$$

where a_{11}, a_{12}, a_{21}, a_{22}, are given by equation (27)

6. Numerical results and discussion

The theoretical results obtained above indicate that the amplitude ratios Z_i $(i = 1,2,3)$ depend on the angle of incidence of incident wave. In order to study in more detail the behaviour of various amplitude ratios on the angle of incidence, we have computed them numerically by taking the following values relevant elastic parameters.

In medium M_1, Following Silva (1976), the physical parameters representing the crust as a linear viscoelastic solid are as follows

$Q_P = 100$, $Q_S = 45$, $\rho_1 = 2.6\ gm/cm^3$, $V_P = 6.1\ km/s$, $V_S = 3.5\ km/s$

(32)

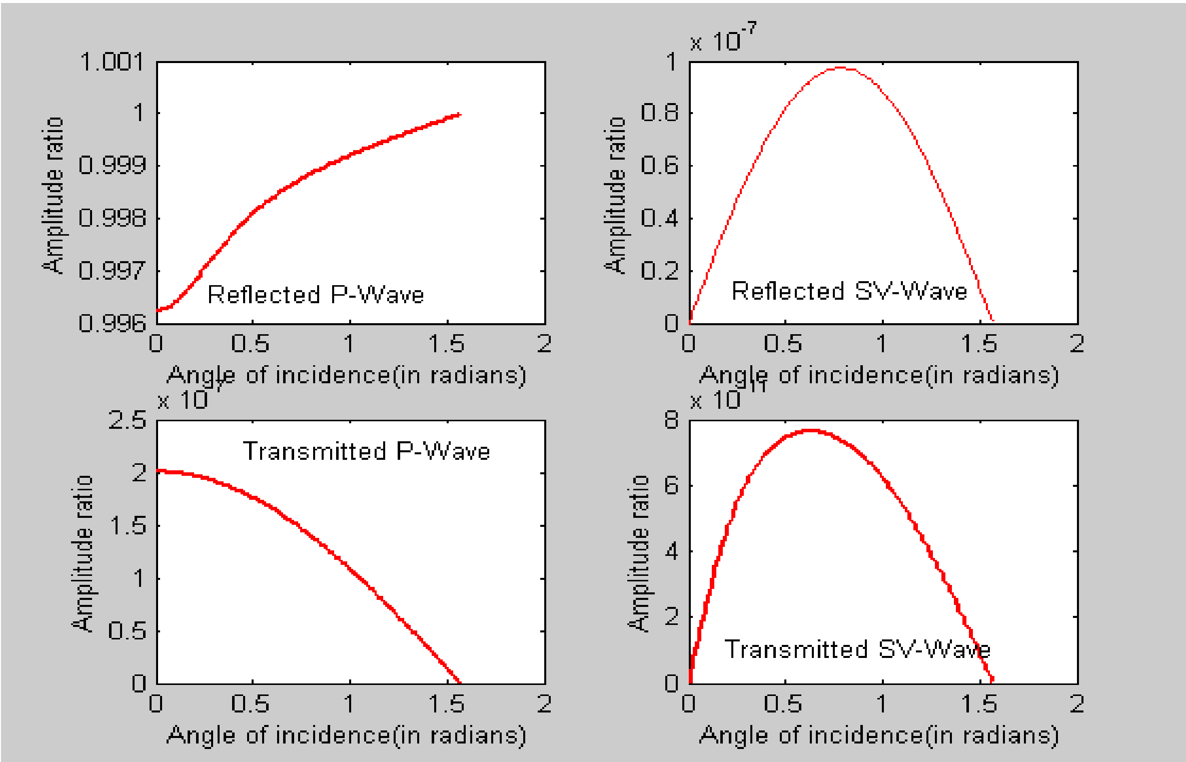
In medium M_2, the physical parameters for isotropic elastic solid are as follows

$$\rho' = 2.65\frac{Mg}{m^3}, \quad \mu' = 2.238\frac{MN}{m^2}, \quad \lambda' = 2.238\frac{MN}{m^2}, \tag{33}$$

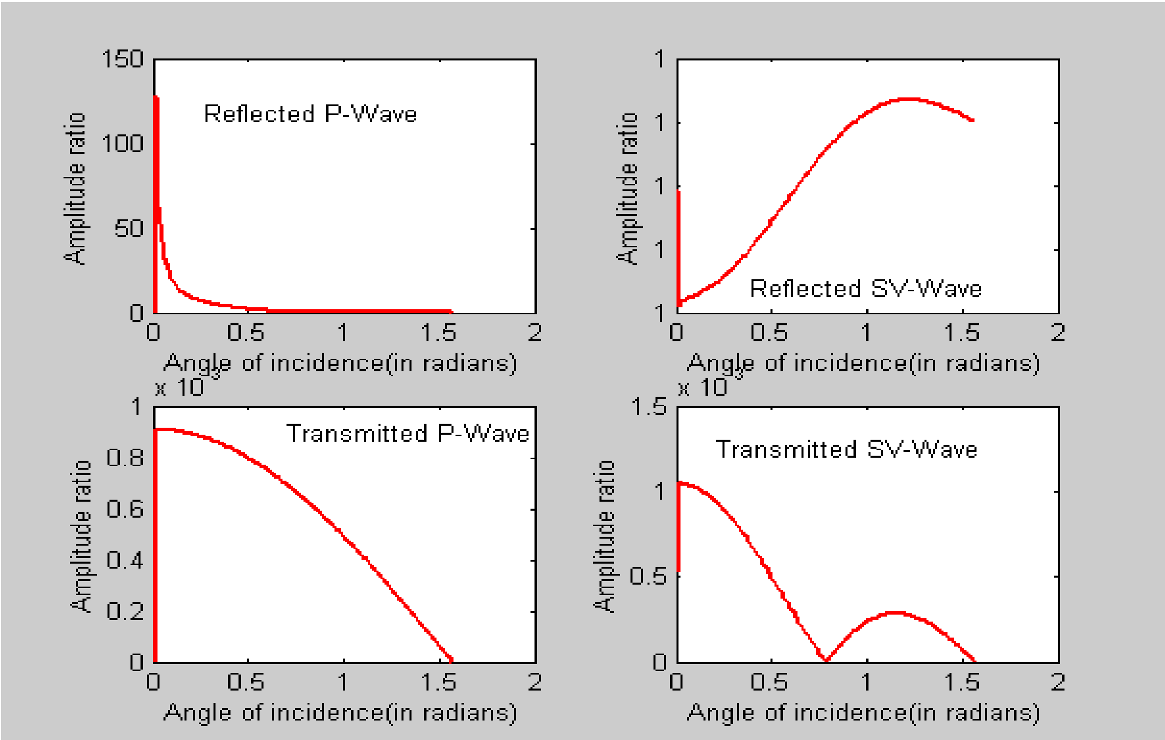
Using MATLAB, a computer programme has been developed and modulus of amplitude ratios $|Z_i|$, $(i = 1,2,3,4,)$ for various reflected and transmitted waves have been computed. $|Z_1|$ and $|Z_2|$ represent the modulus of amplitude ratios for reflected P and reflected SV-wave respectively. Also, $|Z_3|$ and $|Z_4|$ represent the modulus of amplitude ratios for transmitted P and transmitted SV-wave respectively. The variations in all the figures are shown for the range $0^0 \leq \theta \leq 90^0$.

Figures (2)-(5) represent the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave in the case (elastic slid half space lying over viscoelastic solid half space Figures (6)-(9) show the variations of the amplitude ratios for reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of the incident SV-wave. In figures (10)-(13), variation of the amplitude ratios of reflected P-wave reflected SV-wave with angle of incidence P and SV-wave showing effect of free surface.

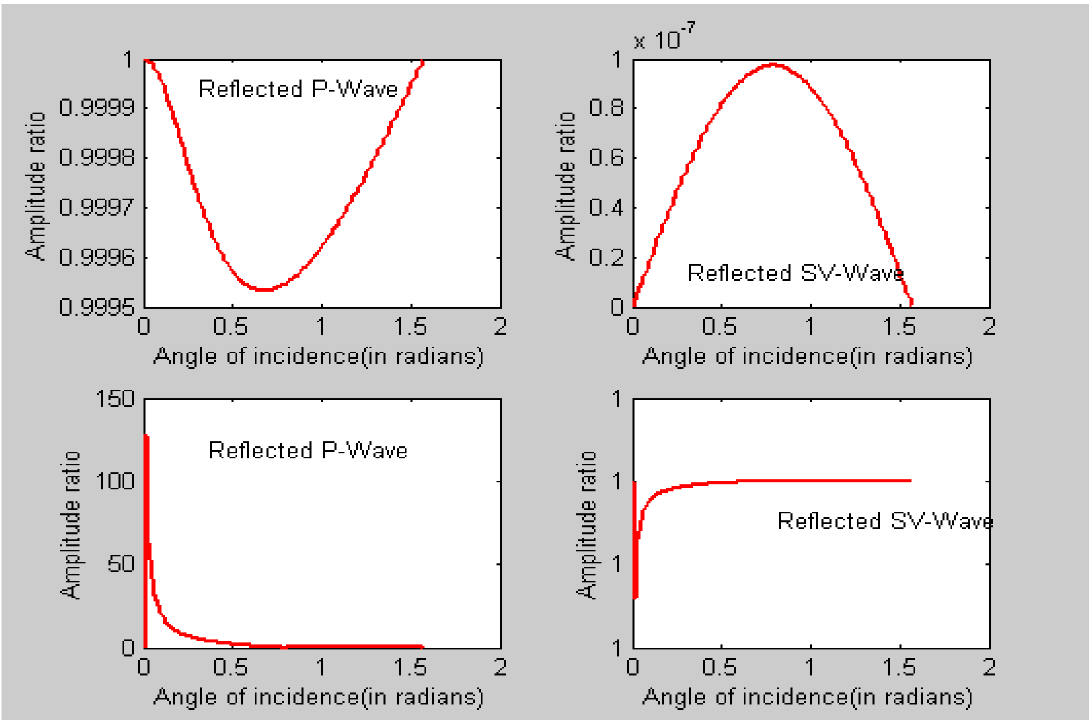
In figure (2), the curve increases in whole range and reach to their maximum value i.e. one. The behaviour of the curves in figures (3), (5) is same i.e. the curves first attain their maximum values and then start to decrease smoothly and reach to their minimum value i.e. zero, while in figure (4), the curve is smoothly decreasing and approaches to zero. In case of incident SV wave, the curves for amplitude ratios of reflected P wave (figure 6), transmitted P-wave (figure 8) and transmitted SV-wave (figure 9), first attain their maximum values very quickly and then start to decrease and reach to their minimum value i.e. zero. But in figure (7), the modulus of maximum amplitude ratio reaches to one. In figures (10) and (11), the behaviour of the curves is opposite. In figure (12), the curve takes its maximum value at $\theta = 0^0$ and minimum value at $\theta = 90^0$. But in figure (13), the trend of curve is opposite to as in figure (12).



Figs. 2-5. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave.



Figs. 6-9. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave.



Figs. 10-13. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave with angle of incidence of P AND SV-WAVE SHOWING EFFECT OF FREE SURFACE.

7. CONCLUSION

The phenomenon of reflection and transmission of incident elastic waves at the interface between a linear viscoelastic solid half space and elastic solid half space has been studied when P-wave or SV-wave is incident. It is observed that the amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties. A significant difference in the values of amplitudes ratios for reflected waves is noticed in the case of free surface boundary. The model presented in this paper is one of the more realistic forms of the earth models. It may be of some use in engineering, seismology and geophysics etc.

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