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Impact of $\pi g\alpha$-Continuity and Semi- $\pi g\alpha$-Continuity on $\pi g\alpha o$- Compact Topological Spaces.

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ABSTRACT:

The aim of this paper is to project the concept of $\pi g\alpha$-open/closed sets in a topological space, which motivates to characterize $\pi g\alpha$- continuous, $\pi g\alpha$-irresolute and semi- $\pi g\alpha$-continuous maps via $\pi g\alpha$-closed sets and to relate the concept to the class of $\pi g\alpha o$-compact spaces. Further the notions of $\pi g\alpha$- $T_{1/2}$ space & $T_{\pi g\alpha}$ -spaces along with α- $T_{1/2}$ & α -maximal spaces are introduced to relate compactness as well as α -compactness to $\pi g\alpha o$ -compactness.

Keywords: α-$T_{1/2}$, $\pi g\alpha$-$T_{1/2}$, $\pi g\alpha o$-compact, $T_{\pi g\alpha}$ - space, notions of $\alpha\tau^*, \pi g\tau^*, \pi g\alpha\tau^*$.

§1. INTRODUCTION :

The concept of g-closed [6] ,s-open[7] and α -open [11] sets has a significant role in the generalization of continuity in topological spaces. The modified form of these sets and generalized continuity were further developed by many mathematicians [2,3].

Dontchev & Noiri studied new types of continuity along with quasi normal spaces & πg-closed sets[4] which are based upon the closure operator and π-open sets being as the union of regular open sets. An extensive study on generalized closedness appeared in recent years in the form of research work as the notions of generalized closed, generalized semi-closed, α-generalized closed & generalized α-closed sets were investigated[6,3,8].

This paper highlights the characteristic properties of $\pi g\alpha$- continuity & semi $\pi g\alpha$- continuity by using the notion of the $\pi g\alpha$-closed sets[5].

We, here, investigate the interrelationship between $\pi g\alpha$-continuous functions, $\pi g\alpha$-irresolute functions, semi $\pi g\alpha$-continuous functions and other related generalized forms of continuity. Further the concept of $\pi g\alpha o$-compactness and its behavior under $\pi g\alpha$-continuity etc. are introduced & observed. Naturally the concept of $\pi g\alpha o$-compactness is slightly weaker form of πg-compactness & g-compactness.

§2. PRELIMINARIES:

Spaces (X,τ),(Y,σ)&(Z,γ)(or simply X,Y,Z)always mean topological spaces on which no separation axioms are assumed unless explicitly stated.

We recall some definitions and properties essential in this paper.

Definition (2.1):

[I]A subset A of a topological space (X,τ) is called

(a) pre-open set[9] if A $\subset$ int(cl(A)).
(b) semi –open set[7] if A$\subset$ cl(int (A)) .
(c) α-open[11] if A$\subset$ int(cl(int (A))).
(d) β-open [13] if A$\subset$ cl(int(cl (A))).

Here, the notions cl(A) and int(A) stand as the closure of A and the interior of A respectively .The complements of these sets are obviously the same type of closed sets.

(II) Let (X,τ) be a topological space &A$\subseteq$X, then

(a) The finite union of regular open sets is said to be π–open where A is called regular open if A = int(cl (A)).
The complement of a π -open sets is said to be π -closed.

The family of all pre-open (resp. semi-open, α -open, β-open, regular open,π-open) subsets of a space (X,τ) is denoted by τ_p (resp. $\tau_s, \tau_\alpha, \tau_\beta, \tau_r, \tau_\pi$).

The family of all pre-closed(resp. semi-closed, α-closed,β-closed, regular-closed,π-closed) subsets of a space (X,τ) is denoted by $\tau_p{}^C$ (resp. $\tau_s^C, \tau_\alpha^C, \tau_\beta^C, \tau_r^C, \tau_\pi^C$).

(b) αint(A) stands for the α-interior of A, which is the union of all α-open subsets contained in A.
clearly, A = α int(A) means A is α-open.
(c) αcl(A) stands for the α -closure of A, which is the intersection of all α-closed subsets containing A.
clearly, A = αcl(A) means A is α-closed.

Definition(2.2): A subset A of a space (X, τ) is called

(a) g- closed[6] if cl(A) $\subseteq$ U whenever A $\subseteq$ U & U is open .

(b) πg-closed [4]if cl(A) $\subseteq$U whenever A $\subseteq$ U & U is π-open.

(c) αg-closed[8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ & U is open .

(d) $g\alpha$ -closed[8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ & U is α-open.

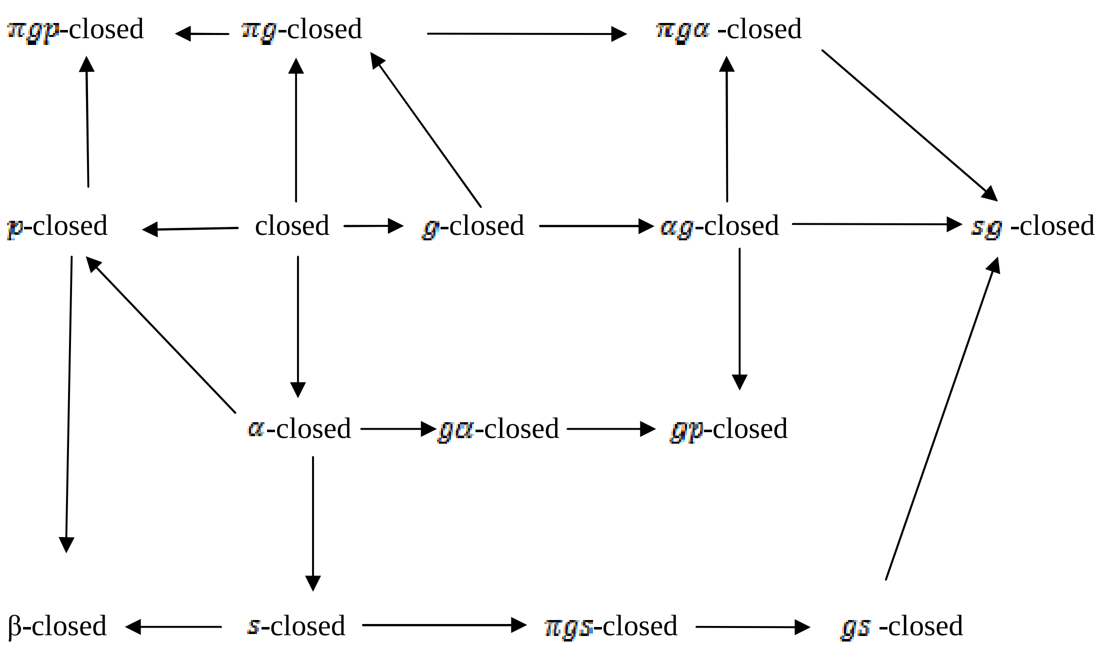
(e) $\pi g\alpha$-closed[5] set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ & U is π -open.

(f) πgp-closed[13] set if $pcl(A) \subseteq U$ whenever $A \subseteq U$ & U is π-open.

(g)πgs–closed[1] set if $scl(A) \subset U$ whenever $A \subset U$ & U is π–open.

The family of all g-closed (resp. πg-closed, αg-closed, $g\alpha$-closed, $\pi g\alpha$-closed, πgp-closed & πgs-closed) is denote by $\tau_g^{\ C}$ (res $\tau_{\pi g}^{C}, \tau_{\alpha g}^{C}, \tau_{g\alpha}^{C}, \tau_{\pi g\alpha}^{C}, \tau_{\pi gp}^{C}$ & $\tau_{\pi gs}^{C}$) in a space (X,τ).

The diagram given below gives relations among the above noted concepts.



Definition (2.3): A function $f : X \to Y$ is said to be

(a) g-continuous [12] if $f^{-1}(V)$ is g–closed in X for every closed set V of Y.

(b) πg–continuous [12] if $f^{-1}(V)$ is πg–closed in X for every closed set V of Y.

(c) almost πg-continuous [12] if $f^{-1}(V)$ is πg–closed in X for every regular closed set V of Y.

(d) contra-continuous [12] if $f^{-1}(V)$ is closed in X for every open set V of Y.

(e) perfectly–continuous[12] if $f^{-1}(V)$ is clopen in X for every open set V of Y.

(f) α-continuous[10] if $f^{-1}(V)$ is α -open in X for every open set V of Y.

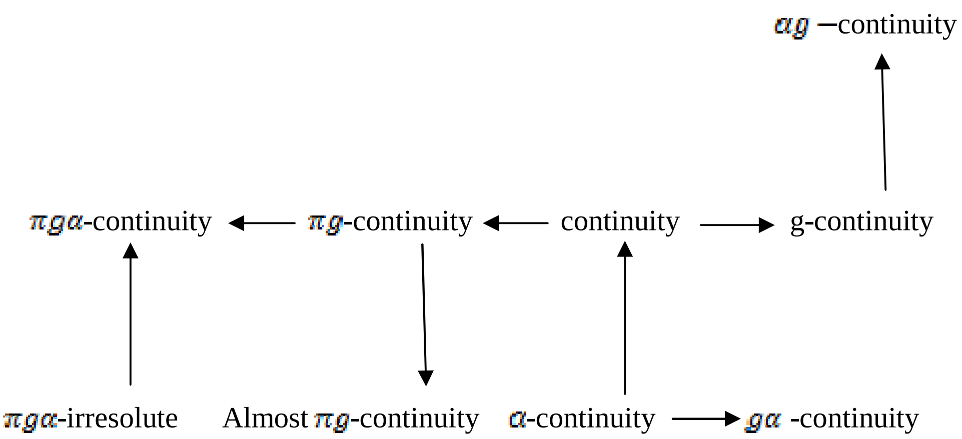
(g) αg-continuous[10] if $f^{-1}(V)$ is αg-closed in X for every closed set V of Y.

(h) $g\alpha$ -continuous[10] if $f^{-1}(V)$ is $g\alpha$ -closed in X for every closed set V of Y.

(i) $\pi g\alpha$-continuous [5] if $f^{-1}(V)$ is $\pi g\alpha$–closed in X for every closed set V of Y.

(j) $\pi g\alpha$ –irresolute [5] if f^{-1}(V) is $\pi g\alpha$ –closed in X for every $\pi g\alpha$ -closed set V of Y.

The above concepts provide the following applications:



* The following definitions appear as the introduction of the notion of $\pi g\alpha$-$T_{1/2}$ space, $T_{\pi g\alpha}$-space, α-$T_{1/2}$ space, & α-maximal space.

Definition (2.4): A space (X, τ) is called a $\pi g\alpha$- $T_{1/2}$ space if every $\pi g\alpha$- closed set is α-closed.

Definition (2.5): A space(X, τ) is $\pi g\alpha$–space(i.e. $T_{\pi g\alpha}$ -space) if every $\pi g\alpha$- closed set is closed.

Definition (2.6): A space (X, τ) is said to be α-$T_{1/2}$ space if every α- closed set is closed.

Definition (2.7): A space (X, τ) is said to be α-maximal if and only if every α-open set is open.

Example(2.1): (i) Let X = {a,b,c}, τ= {φ,X,{a},{b},{a,b}}. Then the simple computation provides that τ^C = {φ,X,{c},{b,c},{c,a}}.

$\tau_\alpha = \tau$, $\tau_\alpha^C = \tau^C$, $\tau_{\pi g\alpha}^C = \tau^C$ & $\tau_{\pi g\alpha}^C = \tau_\alpha^C$

Thus, (X,τ) is $\pi g\alpha$-$T_{1/2}$ space. Also it is $T_{\pi g\alpha}$ -space as well as α-$T_{1/2}$ space. Moreover , (X,τ) is a α -maximal.

(ii) Let X = {a,b,c}, τ = {φ,X,{a},{b},{a,b},{a,c}}. Then the simple computation provides that τ^C = {φ,X,{b},{c},{b,c},{c,a}}.

$\tau_\alpha = \tau$ & $\tau_\alpha^C = \tau^C$ but $\tau_{\pi g\alpha}^C$ = $\wp$(X)

Thus, (X,τ) is neither $\pi g\alpha$-$T_{1/2}$ space nor $T_{\pi g\alpha}$ -space . But (X,τ) is an α-$T_{1/2}$ space as well as α-maximal.

* The following definitions help in analyzing the structure framed in this paper.

Definition (2.8): A mapping $f:(X,\tau)\to(Y,\sigma)$ is called regular open if f sends π-open sets into π-open sets.

Definition (2.9): A mapping $f:(X,\tau)\to(Y,\sigma)$ is called pre α-open if f sends α-open sets into α-open sets or f sends α-closed sets into α-closed sets.

Definition (2.10): A mapping $f:(X,\tau)\to(Y,\sigma)$ is called strongly $\pi g\alpha$ -continuous if $f^{-1}(V)$ is open in X for every $\pi g\alpha$-open set V of Y.

§3. $\pi g\alpha$ –CLOSURES WITH BASIC PROPERTIES:

This section carries the notion of $\pi g\alpha$–closure of a set in a space and its fundamental properties.

Definition(3.1): We define the $\pi g\alpha$-closure of a subset A of topological space (X,τ) as follows:

$\pi g\alpha - cl(A) = \cap$ { F: F is $\pi g\alpha$–closed in X, A$\subset$ F}.

Since, finite intersection of $\pi g\alpha$-closed sets need not be $\pi g\alpha$-closed, hence, $\pi g\alpha - cl(A)$ is not necessarily a $\pi g\alpha$-closed set as illustrated by the following example.

Example (3.1): Let X = { a,b,c,d}, τ = { φ, {a},{b},{a,b},{a,b,c},{a,b,d},X}.

Then $\tau^C =$ the class of closed sets = {φ,{c},{d},{c,d},{a,c,d},{b,c,d},X}.

Now, $\tau_\pi =$ the class of π-open sets = {φ,X,{a},{b},{a,b}}.

$\tau_\alpha^C =$ the class of α-closed sets = {φ,X,{c},{d},{c,d},{a,c,d},{b,c,d}}.

& $\tau_{\pi g\alpha}^C =$ the class of $\pi g\alpha$-closed sets.

= {φ,X,{c},{d},{c,d},{a,c},{a,d},{b,c},{b,d},{a,b,d},{a,b,c,},{a,c,d},{b,c,d}}.

Since, {a,c} & {a,d} are $\pi g\alpha$-closed sets & {a,c} $\cap$ {a,d} = {a} is a $\pi g\alpha$- open set, hence $\pi g\alpha$-cl({a}) = {a,c} $\cap$ {a,d} $\cap$ {a,b,c} $\cap$ {a,b,d} $\cap$ {a,c,d} = {a} does not imply that {a} is a $\pi g\alpha$-closed set.

The following theorem exists as a result of definition (3.1)& example (3.1):

Theorem (3.1): If A is $\pi g\alpha$- closed in (X,τ) , then $\pi g\alpha - cl(A)$ = A, but the converse is not true.

Remark (3.1): $\pi g\alpha - cl(A)$of a subset A in a topological space (X,τ) can never be treated as the smallest $\pi g\alpha$-closed set containing A.

Theorem (3.2): If A and B are subsets of (X, τ), then

(a) $\pi g\alpha - cl(\varphi) = \varphi$ and $\pi g\alpha - cl(X) = X$.

(b) $A \subset B \Rightarrow \pi g\alpha - cl(A) \subset \pi g\alpha - cl(B)$.

(c) $\pi g\alpha - cl(\pi g\alpha - cl(A)) = \pi g\alpha - cl(A)$.

(d) $\pi g\alpha - cl(A \cup B) = \pi g\alpha - cl(A) \cup \pi g\alpha - cl(B)$ where $\tau_{\pi g\alpha}^C$ is closed under finite union.

(e) $\pi g\alpha - cl(A \cap B) = \pi g\alpha - cl(A) \cap \pi g\alpha - cl(B)$, where $\tau_{\pi g\alpha}^C$ is closed under finite intersection.

Proof : Straight forward & natural so omitted.

Definition(3.2): For a subset A of a space (X,τ),

(i) $gcl(A) = \cap\{F: F$ is g -closed & $A \subset F\}$.

(ii) $\alpha gcl(A) = \cap\{F: F$ is αg -closed & $A \subset F\}$.

(iii) $\pi gcl(A) = \cap\{F: F$ is πg -closed & $A \subset F\}$.

With the help of above closures, we coin the following notions which are useful to establish the following theorems:

Definition(3.3): For a topological space (X,τ),

(a) $\alpha\tau^* = \{U \subset X : \alpha gcl(U^C) = U^C\}$.

(b) $\pi g\tau^* = \{U \subset X : \pi gcl(U^C) = U^C\}$.

(c) $\pi g\alpha\tau^* = \{U \subset X : \pi g\alpha cl(U^C) = U^C\}$.

Theorem (3.3): If $\pi g\alpha o$(X,τ) is closed under the formation of finite unions, then $\pi g\alpha\tau^*$ is a topology on X.

Proof: Let (X,τ) be a topological space, then by the structure of $\pi g\alpha\tau^*$, we have $\pi g\alpha\tau^* = \{U \subset X : \pi g\alpha cl(U^C) = U^C\}$.

It is, here, required to prove that (X, $\pi g\alpha\tau^*$) is a topological space.

Now, (i) Since, φ & X are $\pi g\alpha$-closed sets so $\pi g\alpha cl(\varphi) = \varphi$.

& $\pi g\alpha cl(X) = X$. Also, $\varphi^C = X$ & $X^C = \varphi$. Consequently, φ & X$\in \pi g\alpha\tau^*$.

(ii) Let $\{A_\gamma\}_{\gamma\in\Delta}$ be an index family of members of $\pi g\alpha\tau^*$.

Then $A_\gamma \in \pi g\alpha\tau^*$ whenever $\pi g\alpha cl(A_\gamma^C) = A_\gamma^C, \forall \gamma \in \Delta$.

$$Next, \pi g\alpha cl\{(\bigcup_{\gamma\in\Delta} A_\gamma)^C\} = \pi g\alpha cl(\bigcap_{\gamma\in\Delta} A_\gamma^C),$$

$$\subset \bigcap_{\gamma\in\Delta} \{\pi g\alpha cl(A_\gamma^C)\},$$

$$= \bigcap_{\gamma\in\Delta} (A_\gamma^C) = (\bigcup_{\gamma\in\Delta} A_\gamma)^C$$

But $(\bigcup_{\gamma\in\Delta} A_\gamma)^C \subset \pi g\alpha cl\left\{\left(\bigcup_{\gamma\in\Delta} A_\gamma\right)^C\right\}$

$$\therefore \pi g\alpha cl(\bigcup_{\gamma\in\Delta} A_\gamma)^C = \left(\bigcup_{\gamma\in\Delta} A_\gamma\right)^C \Rightarrow \bigcup_{\gamma\in\Delta} A_\gamma \in \pi g\alpha\tau^*$$

Consequently, $\{A_\gamma\} \in \pi g\alpha\tau^* \Rightarrow \bigcup_{\gamma\in\Delta} A_\gamma \in \pi g\alpha\tau^*$

(iii)Let $\{A_r : r = 1,2,3,......n\}$ be a family of finite number of members of $\pi g\alpha\tau^*$.

Then $A_r \in \pi g\alpha\tau^*$ where $\pi g\alpha cl(A_r^C\} = A_r^C, \forall r = 1,2,......n.$

Next, $\pi g\alpha cl\{(\bigcap_{r=1}^{n} A_r)^C\} = \pi g\alpha cl\{\bigcup_{r=1}^{n} A_r^C\},$

$$= \bigcup_{r=1}^{n}\{\pi g\alpha cl(A_r^C)\},$$

$$= \bigcup_{r=1}^{n} A_r^C = \left(\bigcap_{r=1}^{n} A_r\right)^C$$

$$\Rightarrow \bigcap_{r=1}^{n} A_r \in \pi g\alpha\tau^*.$$

Consequently, $\{A_r : r = 1,2,3,......n\} \in \pi g\alpha\tau^* \Rightarrow \bigcap_{r=1}^{n} A_r \in \pi g\alpha\tau^*$.

Since, all the three axioms for being a topology are satisfied, hence $\pi g\alpha\tau^*$ is a topology on X. Hence, the theorem.

Theorem (3.4): for a subset A of topological space (X,τ), the following statements hold good:

(a) $A \subset \pi g\alpha cl(A) \subset \pi gcl(A).$

(b) $A \subset \pi g\alpha cl(A) \subset \alpha gcl(A).$

(c) $\pi g\tau^* \subset \pi g\alpha\tau^*.$

(d) $\alpha\tau^* \subset \pi g\alpha\tau^*$

Proof: Let (X,τ) be a topological space & $A \subseteq X$.

(a) Since, every πg-closed set is a $\pi g\alpha$-closed set ,hence,

$$A \subset \pi g\alpha cl(A) \subset \pi gcl(A).$$

(b) Also, every αg-closed set is a $\pi g\alpha$-closed set, so

$$A \subset \pi g\alpha cl(A) \subset \alpha gcl(A).$$

(c) We have, $A \in \pi g\tau^* \Rightarrow \pi gcl(A^C) = A^C$

$$\Rightarrow \pi g\alpha cl(A)^C \subset \pi gcl(A^C) = A^C \quad \text{[using(a)]}$$

$$\Rightarrow \pi g\alpha cl(A)^C \subset A^C$$

$$\Rightarrow \pi g\alpha cl(A)^C = A^C \quad [\Theta\ A^C \subset \pi g\alpha cl(A^C)]$$

$$\Rightarrow A \in \pi g\alpha\tau^*$$

i.e. $\pi g\tau^* \subset \pi g\alpha\tau^*.$

(d) By definition, $A \in \alpha\tau^* \Rightarrow \alpha gcl(A^C) = A^C$

$$\Rightarrow \pi\alpha gcl(A^C) \subset \alpha gcl(A^C) = A^C \ \text{[using(b)]}$$

$$\Rightarrow \pi g\alpha cl(A^C) \subset A^C$$

$$\Rightarrow \pi g\alpha cl(A^C) = A^C [\Theta\ A^C \subset \pi g\alpha cl(A^C)]$$

$$\Rightarrow A \in \pi g\alpha\tau^*$$

i.e. $\alpha\tau^* \subset \pi g\alpha\tau^*.$

Theorem (3.5): In a topological space (X,τ), the following hold good:

(a) Every $\pi g\alpha$-closed set is αg-closed $\Rightarrow \pi g\alpha\tau^* = \alpha\tau^*$

(b) Every $\pi g\alpha$-closed set is πg-closed $\Rightarrow \pi g\alpha\tau^* = \pi g\tau^*$

(c) Every $\pi g\alpha$-closed set is -closed $\Rightarrow \pi g\alpha\tau^* = \tau$

Proof: Suppose that (X,τ) is a topological space.

Given that every $\pi g\alpha$-closed set is αg-closed, so that $A \subset \alpha gcl(A) \subset \pi g\alpha - cl(A), \forall A \subseteq X$.

Let $A \in \pi g\alpha\tau^* \Rightarrow \pi g\alpha cl(A^C) = A^C$

$$\Rightarrow A^C \subset \alpha gcl(A^C) \subset \pi g\alpha cl(A^C) = A^C$$

$$\Rightarrow A^C \subset \alpha gcl(A^C) \subset A^C$$

$$\Rightarrow \alpha gcl(A^C) = A^C$$

$$\Rightarrow A \in \alpha\tau^*$$

$$\Rightarrow \pi g\alpha\tau^* \subset \alpha\tau^*$$

But $\alpha\tau^* \subset \pi g\alpha\tau^*$ [using(d) of th.(3.4)]

Hence, $\pi g\alpha\tau^* = \alpha\tau^*$

(a)Given that every $\pi g\alpha$-closed set is πg-closed, so that $A \subset \pi gcl(A) \subset \pi g\alpha - cl(A), \quad \forall A \subseteq X$.

Let $A \in \pi g\alpha\tau^* \Rightarrow \pi g\alpha cl(A^C) = A^C$

$$\Rightarrow A^C \subset \pi gcl(A^C) \subset \pi g\alpha cl(A^C) = A^C$$

$$\Rightarrow A^C \subset \pi gcl(A^C) \subset A^C$$

$$\Rightarrow \pi gcl(A^C) = A^C$$

$$\Rightarrow A \in \pi g\tau^*$$

$$\Rightarrow \pi g\alpha\tau^* \subset \pi g\tau^*$$

But $\pi g\tau^* \subset \pi g\alpha\tau^*$ [using(c) of th.(3.4)]

Hence, $\pi g\alpha\tau^* = \pi g\tau^*$

(b)Given that every $\pi g\alpha$-closed set is -closed, so that $A \subset cl(A) \subset \pi g\alpha - cl(A)$,

Let, $A \in \pi g\alpha\tau^* \Rightarrow \pi g\alpha cl(A^C) = A^C$

$$\Rightarrow A^C \subset cl(A^C) \subset \pi g\alpha cl(A^C) = A^C$$

$$\Rightarrow A^C \subset cl(A^C) \subset A^C$$

$$\Rightarrow cl(A^C) = A^C$$

$$\Rightarrow A \in \tau$$

$$\Rightarrow \pi g\alpha\tau^* \subset \tau$$

But $\tau \subset \pi g\alpha\tau^*$

Hence, always $\pi g\alpha\tau^* = \tau$

Hence, the theorem.

Theorem(3.6): A topological space (X,τ) is $\pi g\alpha - T_{1/2}$ space iff $\pi g\alpha\tau^* = \tau_\alpha$.

Proof: Necessity.

Let (X,τ) be a $\pi g\alpha - T_{1/2}$ space, then every $\pi g\alpha$-closed set is α-closed. This means that $A \subset \alpha cl(A) \subset \pi g\alpha cl(A)$ for every subset A of X.

Now,

$$A \in \pi g\alpha\tau^* \Rightarrow \pi g\alpha cl(A^C) = A^C$$

$$\Rightarrow A^C \subset \alpha cl(A^C) \subset \pi g\alpha cl(A^C) = A^C$$
$$\Rightarrow A^C \subset \alpha cl(A^C) \subset A^C$$
$$\Rightarrow \alpha cl(A^C) = A^C$$
$$\Rightarrow A \in \tau_\alpha$$
$$\Rightarrow \pi g\alpha\tau^* \subset \tau_\alpha$$

But $\tau_\alpha \subset \pi g\alpha\tau^*$

Hence, $\pi g\alpha\tau^* = \tau_\alpha$

Sufficiency:

Let (X,τ) be a topological space for which $\pi g\alpha\tau^* = \tau_\alpha$.

Now, any member $A \in \pi g\alpha\tau^*$ is an α-open set.

But $A \in \pi g\alpha\tau^* \Rightarrow \pi g\alpha cl(A^C) = A^C$

$\Rightarrow \pi g\alpha cl(A^C)$ is an α-closed set.

$\Rightarrow \pi g\alpha cl(A^C)$ is a $\pi g\alpha$-closed because every α-closed set is a $\pi g\alpha$-closed set.

$\Rightarrow A^C$ is a $\pi g\alpha$-closed set.

$\Rightarrow A$ is a $\pi g\alpha$-open set.

This means that every $\pi g\alpha$-open set is an α-open set .i.e. every $\pi g\alpha$ -closed set is an α-closed set which proves that (X,τ) is a $\pi g\alpha - T_{1/2}$ space. Hence the theorem.

As a consequence of definition (2.5),(2.6)&(2.7) and theorem(3.6)(c), the following theorem appears:

Theorem (3.7): a topological space (X,τ), the following are equivalent:

(a) $\pi g\alpha\tau^* = \tau$

(b) (X,τ) is α-maximal & $T_{\pi g\alpha}$ space.

(c) (X,τ) is $\alpha - T_{1/2}$ & $T_{\pi g\alpha}$ space.

§4. SEMI πgα-CONTINUOUS FUNCTIONS:

In this section, the notion of Semi $\pi g\alpha$-continuous function is defined and its relations with $\pi g\alpha$-continuous & $\pi g\alpha$-irresolute functions are established.

Definition (4.1): A function $f:(X,\tau) \to (Y,\sigma)$ is called Semi-$\pi g\alpha$-continuous if $f^{-1}(V)$ is $\pi g\alpha$-closed in X for every α-closed set V of Y.

A mapping $f:(X,\tau) \to (Y,\sigma)$ provides the following implications:

$\pi g\alpha$-irresolute $\Rightarrow$ semi $\pi g\alpha$- continuous $\Rightarrow$ $\pi g\alpha$- continuous.

However, the reverse implications are not in general true as illustrated by the following example:

Example(4.1):

Let X = { l,m,n,p},τ = {φ,{m},{n},{m,n},X}, τ^C = { φ,{l,p},{l,m,p},{l,n,p},X}

Then

τ_α^C ={φ,{l},{p},{l,p},{l,m,p},{l,n,p},X}

τ_π = { φ,{m},{n},{m,n},X}.

$\tau^C_{\pi g\alpha}$ = {φ,{l},{p},{l,m},{l,n},{l,p},{m,p},{n,p},{l,m,p},{l,m,n}, {l,n,p},{m,n,p},X}

= ℘(X) – {{m},{n},{m,n}}.

& let Y = { a,b,c,d} , σ = { φ,{a},{a,b},{c,d},{a,c,d}, Y},

σ^C ={ φ,{b},{a,b},{c,d},{b,c,d}, Y}, Then

σ^C_α = { φ,{b},{a,b},{c,d},{b,c,d},Y}= σ^C

σ_π = { φ,{a,b},{c,d},Y}

$\sigma^C_{\pi g\alpha}$ ={φ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},Y}

= ℘(Y).

Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(l) = b ,f(m) = a,f(n) = c, f(p) = d .

Then (i) f^{-1}(V) is $\pi g\alpha$-closed in X for every closed set V in Y. So f is semi $\pi g\alpha$-continuous.

But f is not $\pi g\alpha$-irresolute as f^{-1}({a}) = {m} which is not a $\pi g\alpha$-closed in X for the $\pi g\alpha$-closed set {a} in Y.

(ii)Also, f is semi $\pi g\alpha$–continuous as f^{-1}(V) is always a $\pi g\alpha$- closed set in X whenever V is a α-closed set in Y . But f is not a $\pi g\alpha$-irresolute mapping as f^{-1}({c}) = {n} which is not a $\pi g\alpha$-closed in X for a $\pi g\alpha$- closed set {c} in Y.

Example (4.2):

Let X={a,b,c,d,e},

τ = {φ,{a},{b},{a,b},{c,d},{a,c,d},{b,c,d},{a,b,c,d},{b,c,d,e},X}

τ^C = { φ,{a},{e},{a,e},{b,e},{a,b,e},{c,d,e},{a,c,d,e},{b,c,d,e},X}

Then, $\tau^C_\alpha = \tau^C$ & $\tau_\pi = \tau$

Now, $\tau^C_{\pi g\alpha}$ ={φ,{a},{e},{a,e},{b,e},{c,e},{d,e},{a,b,c},{a,b,e},{a,c,e},{c,d,e},{a,d,e},{b,d,e},

{a,b,c,e},{b,c,d,e},{a,b,d,e},{a,c,d,e},X}

Again, let Y = { l,m,n,p} , σ = { φ,{m},{n},{m,n}, Y},

σ^C ={ φ,{l,p}, {l,m,p},{l,n,p},Y},

Then

σ^C_α = { φ,{l},{p},{l,p},{l,m,p},{l,n,p},Y}

σ_π = { φ,{m},{n},{m,n},Y} = σ

& $\sigma^C_{\pi g\alpha}$ ={φ,{l}{p},{n},{l,m},{l,n},{l,p},{m,p},{n,p},{l,m,n},{l,m,p},{l,n,p},{m,n,p}Y}

= ℘(Y) –{{m},{n},{m,n}}.

Suppose that $g:(X,\tau)\rightarrow(Y,\sigma)$ is defined as g(a) = l =g(d),g(b) = m, g(c) = n, g(e) = p

(i) Since, g^{-1}(V) is always a $\pi g\alpha$-closed set in X when V is a closed set in Y, hence g is $\pi g\alpha$-continuous function.

(ii)Since, g^{-1}(V) =g^{-1}({l})= {a,d} which is not a $\pi g\alpha$-closed set in X, whenever V= {l} is a α-closed set in Y. Hence g^{-1}(V) is not a necessarily $\pi g\alpha$-closed set in X whenever V is α-closed in Y. i.e. g is not a semi- $\pi g\alpha$- continuous function.

Theorem(4.1): If $f:(X,\tau)\rightarrow(Y,\sigma)$ is $\pi g\alpha$-continuous and (Y,σ) is $\alpha - T_{1/2}$ as well as $\pi g\alpha - T_{1/2}$ space, then f is $\pi g\alpha$-irresolute.

Proof:
Let $f:(X,\tau)\to(Y,\sigma)$ be $\pi g\alpha$-continuous. Then f^{-1}(V) is $\pi g\alpha$-closed in X whenever V is closed in Y.
Since,(Y,σ) is $\alpha - T_{1/2}$ space , every α-closed set is closed and also for
(Y,σ) being $\pi g\alpha - T_{1/2}$ space, every $\pi g\alpha$-closed set is α-closed, hence, every $\pi g\alpha$-closed set in Y is closed.
Consequently, combining the above facts we observe that for every F to be $\pi g\alpha$ -closed in Y , f^{-1}(F) is $\pi g\alpha$-closed in X i.e. f is $\pi g\alpha$-irresolute.

Hence, the theorem.

Theorem(4.2): If $f:(X,\tau)\to(Y,\sigma)$ is bijective, regular open and semi $\pi g\alpha$-continuous , then f is $\pi g\alpha$-irresolute.

Proof: Let F be a $\pi g\alpha$-closed set in Y. Let f^{-1}(F) ⊂U….(1)
where U is π-open in X. Then F ⊂ f (U). Since, f is regular open & U is π-open in X, hence, f (U) is π-open in Y. Again, as f (U) is π-open and F is $\pi g\alpha$ -closed in Y, so αcl(F) ⊂ f (U) which gives that f^{-1}(αcl(F)) ⊂U.
Since, f is semi-$\pi g\alpha$-ccontinuous, hence, f^{-1}(αcl(F)) is $\pi g\alpha$-closed . Consequently, αcl [f^{-1}(αcl (F))] ⊂U as f^{-1}(αcl(F)) ⊂U.

Now, α cl[f^{-1}(αcl (F))] ⊂U.

⇒ αcl[(f^{-1}(F)] ⊂ αcl [f^{-1}(αcl(F))] ⊂U

⇒ αcl[(f^{-1}(F)] ⊂U ……. ..(2)

Combining (1)&(2), we have
αcl [f^{-1}(αcl (F))] ⊂U as f^{-1}(F) ⊂U & U is π-open. This means that f^{-1}(F) is also a $\pi g\alpha$- closed in X. Hence, f is $\pi g\alpha$- irresolute because the inverse image
f^{-1}(F) of every $\pi g\alpha$-closed set F in Y is also a $\pi g\alpha$- closed set in X.

Hence, the theorem.

Theorem (4.3): If $f:(X,\tau)\to(Y,\sigma)$ is $\pi g\alpha$-continuous from one space (X,τ) into another α-maximal space (Y,σ), then f is semi $\pi g\alpha$—continuous.

Proof: Let F be an α-closed set of (Y,σ). Given that (Y,σ) is α-maximal, so F is closed in (Y,σ). Since, f is $\pi g\alpha$-continuous, hence, f^{-1}(F) is $\pi g\alpha$-closed in(X,τ). i.e. f is semi $\pi g\alpha$ -continuous.

Hence , the theorem.

Theorem(4.4): If $f:(X,\tau)\to(Y,\sigma)$ is regular-open, semi-$\pi g\alpha$-continuous and pre-α-open bijection, then

[(X,τ) is $\pi g\alpha - T_{1/2}$ space] ⇒[(Y,σ) is $\pi g\alpha - T_{1/2}$ space].

Proof: Suppose that F is a $\pi g\alpha$-closed subset of (Y,σ). Theorem (4.3) provides that f^{-1}(F) is $\pi g\alpha$-closed in (X,τ).
Since, (X,τ) is $\pi g\alpha - T_{1/2}$ space, hence, f^{-1}(F) is α- closed in (X,τ).
Since, f is bijective and pre- α- open, F = f (f^{-1}(F)) is α-closed in (Y,σ).
Hence, every $\pi g\alpha$-closed set in (Y,σ) is an α-closed set in (Y,σ). Consequently, (Y,σ) is $\pi g\alpha - T_{1/2}$ space.

Hence, the theorem.

§5. $\pi g\alpha o$-COMPACT SPACES.

The concept to the classes of $\pi g\alpha o$-compact spaces and the effects of $\pi g\alpha$- continuous/ irresolute and semi $\pi g\alpha$-continuous mappings on $\pi g\alpha o$-compact spaces are lying under the literature & discussion of this section.

Definition (5.1): A topological space (X,τ)is called πg-compact if every cover of X by πg-open sets.(i.e. πgopen cover) has finite subcover.

Definition (5.2): A topological space (X,τ)is called $\pi g\alpha$-compact if every cover of X by $\pi g\alpha$-open sets(i.e. $\pi g\alpha$ -open cover) has finite subcover.

Definition (5.3): A subset A of a space (X,τ)is called $\pi g\alpha o$-compact if

(A, τ_A) is $\pi g\alpha o$ -compact where (A, τ_A) is a subspace of (X,τ).

Definition (5.4): A α -compact space is a topological space in which every cover by α -open sets has a finite subcover.

Observations:

(a) In $T_{\pi g\alpha}$ -spaces the concept of compactness and $\pi g\alpha o$ -compactness coincide.

(b) A $\pi g\alpha - T_{1/2}$ space is $\pi g\alpha o$- compact iff it is α - compact.

(c) $\pi g\alpha o$- compactness is a topological property.

Theorem (5.5): If (X,τ)is a topological space, then the following implications hold:

(a) $\pi g\alpha o$- compact $\Rightarrow$ (b) πg- compact $\Rightarrow$ (c) g-compact.

Proof: (a) $\Rightarrow$(b) :

Let (X,τ) be a $\pi g\alpha o$-compact space. Let C be a cover of πg- open sets.

Since , every πg-open set is a $\pi g\alpha$-open set, hence C is a cover of $\pi g\alpha$-open sets .By assumption C has a finite subcover , so (X,τ)is a πg-compact.

(b)$\Rightarrow$(c):

Let (X,τ) be a πg -compact space. Let C be a cover of g -open sets. Since, every g- open set is a πg- open , hence C is a cover of πg- open sets and by assumption C must have a finite subcover. Hence (X,τ) is a g -compact.

Hence, the theorem.

Theorem (5.6): If $f:(X,\tau)\rightarrow(Y,\sigma)$ is surjective strongly $\pi g\alpha$–continuous –mapping and (X,τ)is compact , then (Y,σ) is $\pi g\alpha o$-compact.

Proof: Suppose that $f:(X,\tau)\rightarrow(Y,\sigma)$ is a strongly $\pi g\alpha$-continuous surjective mapping and (X,τ)is a compact space.

Let C $=\{U_r\}_{r\in\Delta}$ be cover of $\pi g\alpha$-open sets of Y.

Since, f is strongly $\pi g\alpha$- continuous, $f^{-1}(U_\alpha)$ is open in X for each $\alpha\epsilon\Delta$. Since (X,τ) is compact, hence, the open cover $\{f^{-1}(U_r)\}_{r\in\Delta}$ for X has a finite subcover, say , $\{f^{-1}(U_r): r\in\Delta_0\subset\Delta \;\&\; \Delta_0$ is a finite set }.

Thus , $X \subseteq \bigcup\{f^{-1}(U_r): r\in\Delta_0\subset\Delta\}$.

$\Rightarrow$ $f(X) \subseteq f(\bigcup\{f^{-1}(U_r): r\in\Delta_0\subset\Delta\})$.

$\Rightarrow$ $Y = f(X) \subseteq \bigcup\{f(f^{-1}(U_r)) : r \in \Delta_0 \subset \Delta\}$. as f is surjective.

$\Rightarrow$ $Y \subseteq \bigcup\{U_r : r \in \Delta_0 \subset \Delta\}$.

$\Rightarrow$ (Y,σ) is a $\pi g\alpha$o-compact space.

Hence, the theorem.

Theorem (5.7): If $f:(X,\tau) \to (Y,\sigma)$ is surjective $\pi g\alpha$o-continuous and (X,τ)is a $\pi g\alpha$o-compact , then (Y,σ) is compact.

Proof : Let $f:(X,\tau) \to (Y,\sigma)$ is surjective $\pi g\alpha$-continuous mapping and (X,τ) is a $\pi g\alpha$o- compact space.

Let C = $\{U_r\}_{r\in\Delta}$ be an open cover of Y.

Since, f is $\pi g\alpha$-continuous, $f^{-1}(U_r)$ is $\pi g\alpha$ -open in X for each $\{r \in \Delta\}$. Since (X,τ) is $\pi g\alpha o$-compact, hence the cover $\{f^{-1}(U_r) : r \in \Delta\}$ has a finite subcover, say $\{f^{-1}(U_r) : r \in \Delta_0 \subset \Delta$ where Δ_0 is a finite set$\}$.

This means that

$$X \subseteq \bigcup\{f^{-1}(U_r) : r \in \Delta_0 \subset \Delta\}.$$

$\Rightarrow$ $f(X) \subseteq f(\bigcup\{f^{-1}(U_r) : r \in \Delta_0 \subset \Delta\})$.

$\Rightarrow$ $Y = f(X) \subseteq \bigcup\{f(f^{-1}(U_r)) : r \in \Delta_0 \subset \Delta\}$ as f is surjective.

$\Rightarrow$ $Y \subseteq \bigcup\{U_r : r \in \Delta_0 \subset \Delta\}$.

$\Rightarrow$ (Y,σ) is a compact space .

Hence, the theorem.

Note: 1. The surjective $\pi g\alpha$ -irresolute image of a $\pi g\alpha o$-compact space is $\pi g\alpha o$-compact.

2. The $\pi g\alpha$-irresolute image f(A)of a $\pi g\alpha o$-compact set A relative to (X,τ) is $\pi g\alpha o$-compact relative to (Y,σ) where $f : X \to Y$ is the irresolute mapping.

Theorem (5.8): The surjective semi $\pi g\alpha$ -continuous image of a $\pi g\alpha o$ -compact space is α -compact.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be a surjective semi $\pi g\alpha$ -continuous mapping from a $\pi g\alpha o$-compact space(X,τ) to another topological space(Y,σ).

Let C = $\{U_r\}_{r\in\Delta}$ be an α -open cover of Y.

Since, f is semi $\pi g\alpha$ -continuous, $f^{-1}(U_r)$ is $\pi g\alpha$ -open in X for each $\{r \in \Delta\}$.

Since, (X,τ) is $\pi g\alpha o$-compact, hence the cover $\{f^{-1}(U_r) : r \in \Delta\}$ has a finite subcover, say $\{f^{-1}(U_r) : r \in \Delta_0 \subset \Delta$ & Δ_0 is finite.$\}$

This means that

$X \subseteq \bigcup\{f^{-1}(U_r) : r \in \Delta_0 \subset \Delta\}$.

$\Rightarrow$ $f(X) \subseteq f(\bigcup\{f^{-1}(U_r) : r \in \Delta_0 \subset \Delta\})$.

$\Rightarrow$ $Y = f(X) \subseteq \bigcup\{f(f^{-1}(U_r)) : r \in \Delta_0 \subset \Delta\}$, f is surjective.

$\Rightarrow$ $Y \subseteq \bigcup\{U_r : r \in \Delta_0 \subset \Delta\}$.

$\Rightarrow$ (Y,σ) is α- compact space.

CONCLUSION :

In the present paper, the authors investigated new concept of - $\pi g\alpha o$-compact topological spaces in which the basic properties of $\pi g\alpha - T_{1/2}$ space & $T_{\pi g\alpha}$ space along with $\alpha - T_{1/2}$ & $\alpha -$ maximal spaces have been discovered.

This paper highlights the behavior of $\pi g\alpha o$-compactness under the effects of $\pi g\alpha$ -continuity & semi- $\pi g\alpha$-continuity.

Also, it includes the inter-relationship of $\alpha\tau^*$ & $\pi g\tau^*$ with $\pi g\alpha\tau^*$ as well as the relation in between $\pi g\alpha - T_{1/2}$ and $\pi g\alpha\tau^*$, τ_α.

The subject material has been treated systematically & presented in a coherent and lucid way. This paper includes self explanatory examples for the respective concepts and is helpful to the reader to follow the indicated paths to new ideas.

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