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An application of the Cesaro summability to the wavelet approximation

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ABSTRACT:

In this paper we have been proved the rapid rate of convergence of the Cesaro summability method and we obtain the relation which is valid for avoiding the Gibb's phenomenon in intermediate levels of wavelet approximation. Finally some compression between the results obtained by the Norlund means and the Cesaro summability methods reveals a slight improvement concerning the reduction of excessive oscillations.

KEY WORDS: Summability method, Fourier series, approximation theory, wavelet frames.

INTRODUCTION:

Let $f \epsilon L^2([0,1]); \int f^2(t)dt < \infty$

A wavelet representation of 'f' is a series of the from

$$f = C_0 + \sum_{j\geq 0}^{\infty} \sum_{k=1}^{2^j} < f, \psi_{j,k} > \psi_{j,k}$$

Where C_0 is a constant, $\left(C_0 = \int_0^1 f(t)\,dt\right)$ and

$\theta_{j,k} \equiv < f, \quad \psi_{j,k} >= \int_0^1 f(t)\psi_{j,k}(t)dt$

and

The basic functions $\psi_{j,k}$ are orthonormal, oscillatory singnals, each with an associated scale 2^{-j} and position $k \times 2^{-j}$. $\psi_{j,k}$ is called the wavelet at scale 2^{-j} and position $k2^{-j}$.

Also we have, the Cesaro sum (c, r) of a series is extended to the copy of an infinite integral $\int_a^\infty f(a)\,da$ by taking in place of the integral $\lim_{n\to\infty} \int_a^n (1 - a/n)^r f(a)\,da$

This limiting value when it exists is called the sum (c, r) of the integral.

Thus instead of Fourier's repeated integrals ,

$$\frac{1}{\pi}\int_0^\infty da \int_{-\infty}^\infty f(\beta)\cos a(\beta - x)\,d\beta,$$

We have the sum (c,r)

$$\lim_{n\to\infty} I_n(r) = \lim_{n\to\infty} \frac{1}{\pi}\int_a^{\pi} (1-a/n)^r da \times$$

$$\int_{-\infty}^{\infty} f(\beta)\cos a(\beta - x)\, d\beta$$

The function $I_n(r)$ has the same relation to the infinite integral as the function $c_n(r)$ to the series, with the usual notation of Cesaro summability.
Therefore, it is shown that the Gibb's phenomenon occurs in Fourier's integral when $r = 0$, and does not occurs when $r = 1$. Also that the question of its occurrence in the sum (c,r) , for
$r > 0$, depends as in the case of Fourier series on the behavior of the function

$$\int_a^{\pi} (1-a/n)^r \frac{\sin a}{a} da$$

KNOWN RESULTS:

1. We know by Haar wavelets, the "first " wavelet basis that was developed.

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}(1\left\{t \in \left[2^{-j}(k-1), 2^{-j}\left(k-\frac{1}{2}\right)\right]\right\}$$

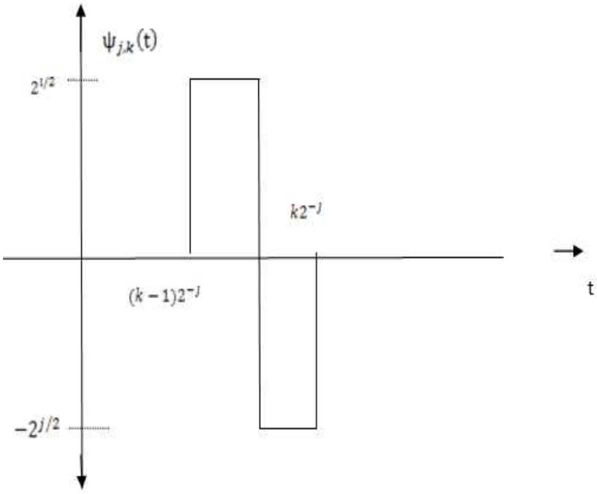
$$-1\left\{t \in \left[2^{-j}\left(k-\frac{1}{2}\right), 2^{-j}k\right]\right\})$$

$$\int_0^1 \psi_{j,k}(t)\, dt = 0, \quad \text{so that}$$

$$\int_0^1 \psi^2{}_{j,k}(t)\, dt = \int_{(k-1)2^{-j}}^{k^{2-j}} 2^j\, dt = 1$$

$\int_0^1 \psi_{j,k}(t)\psi_{l,m}(t)\, dt = 0,$ unless $j = l, k = m$ and f is a constant on $[2^{-j}(k-1), 2^{-j}k]$, then $\int f\, \psi_{j,k}(t)dt = 0$

because of this and the fact that each wavelet basis function is supported an a small region means that wavelet are blind to constant patches. By graphical we have,



2. If $\{\lambda_n\}$ is a non-negative and non-increasing sequence such that $\sum p_n\ \lambda_n < \infty$ where $\{p_n\}$ is a sequence of positive numbers such that $P_n \to \infty\ as\ n \to \infty$

and $\sum_{v=1}^{n} P_v\ A_v(t) = 0(P_n)$. Then the factored Fourier series $\sum A_n(t)\ P_n\ \lambda_n$ is summable $|N, p_n|_k$, $k \geq 1$.

3. If $\{\lambda_n\}$ is a non-negative and non-increasing sequence such that $\sum p_n\ \lambda_n < \infty$, where $\{p_n\}$ is a sequence of positive numbers such that $P_n \to \infty\ as\ n \to \infty$ then $P_n\ \lambda_n = 0(1)\ as\ n \to \infty$ and $\sum P_n\ \Delta\lambda_n < \infty$.

MAIN RESULT :-

Let $\{p_n\}$ is a sequence of positive numbers such that $P_n = p_1 + p_2 + ... + p_n \to \infty$ as $n \to \infty$ and $\{\lambda_n\}$ is a non-negative, non-increasing sequence such that $\sum p_n \lambda_n < \infty$. If

(i). $\sum_{v=1}^{n} P_v \; A_v(t) = 0(P_n)$

(ii). $\sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \left(\frac{p_{n-v-1}}{P_{n-1}}\right) = 0\left(\frac{p_v}{P_v}\right), \; as \; m \to \infty$

and

(iii). $P_{n-v-1} \; \Delta\lambda_v = 0(p_{n-v} \; \lambda_v)$, then the series $\sum A_n(t) P_n \; \lambda_n$ is summable $|N, p_n|_k$, $k \geq 1$.

By considering the above results in this paper we have been prove the main theorem.

PROOF :-

Let $t_n(x)$ be the n-th (N, p_n) mean of the series $\sum_{n=1}^{\infty} A_n(x) \; P_n \; \lambda_n$, then by definition we have

$$t_n(x) = \frac{1}{P_n} \sum_{v=0}^{n} p_{n-v} \sum_{r=0}^{v} A_r(x) \; P_r \; \lambda_r$$

$$= \frac{1}{P_n} \sum_{r=0}^{n} A_r(x) \; P_r \lambda_r \sum_{v=r}^{n} p_{n-v}$$

$$= \frac{1}{P_n} \sum_{r=0}^{n} A_r(x) \; P_r \; P_{n-r} \lambda_r \; .$$

Then

$$t_n(x) - t_{n-1}(x) = \frac{1}{P_n} \sum_{r=0}^{n} P_{n-r} \; P_r \lambda_r \; A_r(x) - \frac{1}{P_{n-1}} \sum_{r=0}^{n-1} P_{n-r-1} \; P_r \lambda_r A_r(x)$$

$$= \sum_{r=1}^{n} \left(\frac{P_{n-r}}{P_n} - \frac{P_{n-r-1}}{P_{n-1}} \right) P_r \; \lambda_r \; A_r(x)$$

$$= \frac{1}{P_n P_{n-1}} \sum_{r=1}^{n-1} \left(P_{n-v} \; P_{n-1} - P_{n-r-1} \; P_n \right) P_r \; \lambda_r A_r(x)$$

$$= \frac{1}{P_n P_{n-1}} \left[\sum_{r=1}^{n-1} \Delta\{(P_{n-r} \; P_{n-1} - P_{n-r-1} \; P_n) \; \lambda_r\} \left(\sum_{v=1}^{r} P_v \; A_v(x) \right) \right]$$

, using partial summation formula with $p_o = 0$

$$= \frac{1}{P_n P_{n-1}} \left[\sum_{r=1}^{n-1} \left(p_{n-r} \; P_{n-1} - p_{n-r-1} \; P_n \right) \lambda_r P_r \right.$$
$$\left. + \sum_{r=1}^{n-1} \left(p_{n-r-1} \; P_{n-1} - P_{n-r-2} \; p_n \right) P_r \Delta\lambda_r \right],$$

$= T_{n,1} + T_{n,2} + T_{n,3} + T_{n,4}$, say.

In order to complete the proof of the theorem, using Minkowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,i}|^k < \infty, \; for \; i = 1,2,3,4.$$

Now, we have

$$\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,1}|^k = \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \frac{1}{P_n^k} \left(\sum_{v=1}^{n-1} p_{n-v} \; P_v \; \lambda_v \right)^k$$

$$\leq \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \frac{1}{P_n} \left(\sum_{v=1}^{n-1} p_{n-v} \; P_v^k \; \lambda_v^k \right) \left(\frac{1}{P_n} \sum_{v=1}^{n-1} p_{n-v} \right)^{k-1}$$

,

Using Holder's inequality

$$= O(1) \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \frac{1}{P_n} \sum_{v=1}^{n-1} p_{n-v} \; P_v \; \lambda_v \left(P_v \; \lambda_v \right)^{k-1}$$

$$= O(1) \sum_{v=1}^{m} P_v \; \lambda_v \sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \frac{p_{n-v}}{P_n}$$

$$= O(1) \sum_{v=1}^{m} p_v \; \lambda_v$$

$$= O(1), \; as \; m \to \infty \; .$$

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|T_{n,2}|^k=\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_{n-1}^k}\left(\sum_{v=1}^{n-1}p_{n-v-1}\,P_v\;\lambda_v\right)^k$$

$$\leq\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_{n-1}}\left(\sum_{v=1}^{n-1}p_{n-v-1}\,P_v^k\;\lambda_v^k\right)\left(\frac{1}{P_{n-1}}\sum_{v=1}^{n-1}p_{n-v-1}\right)^{k-1}$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-n}\frac{1}{P_{n-1}}\sum_{v=1}^{n-1}p_{n-v-1}\;P_v\;\lambda_v\,(P_v\,\lambda_v)^{k-1}$$

$$=O(1)\sum_{v=1}^{m}P_v\;\lambda_v\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left(\frac{p_{n-v-1}}{P_{n-1}}\right),$$

$$=0(1)\sum_{v=1}^{m}p_v\;\lambda_v,$$

$$=0(1)\;,\;as\;m\to\infty.$$

Now,

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|T_{n,3}|^k=\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_n^k}\left(\sum_{v=1}^{n-1}p_{n-v-1}\,P_v\;\Delta\lambda_v\right)^k$$

$$\leq\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_n}\left(\sum_{v=1}^{n-1}p_{n-v-1}\,(P_v\,\Delta\lambda_v)^k\right)\left(\frac{1}{P_n}\sum_{v=1}^{n-1}p_{n-v-1}\right)^{k-1}$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_n}\sum_{v=1}^{n-1}p_{n-v-1}\;P_v\;\Delta\lambda_v(P_v\,\Delta\lambda_v)^{k-1}$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_n}\sum_{v=1}^{n-1}p_{n-v-1}\;P_v\;\Delta\lambda_v\;,$$

$$=O(1)\sum_{v=1}^{m}P_v\;\Delta\lambda_v\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{n-1}\left(\frac{p_{n-v-1}}{P_n}\right)$$

$$=0(1)\sum_{v=1}^{m}p_v\;\Delta\lambda_v\;,$$

$$=0(1)\;,\;m\to\infty.$$

Finally,

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|T_{n,4}|^k=\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{p_n^k}{P_n^kP_{n-1}^k}\left(\sum_{v=1}^{n-1}P_v\;P_{n-v-2}\;\Delta\lambda_v\right)^k$$

$$\leq 0(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_{n-1}^k}\left(\sum_{v=1}^{n-1}p_{n-v-1}\;P_v\;\lambda_v\right)^k$$

,

$$\leq 0(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_{n-1}}\sum_{v=1}^{n-1}p_{n-v-1}\;(P_v\;\lambda_v)^k\left(\frac{1}{P_{n-1}}\sum_{v=1}^{n-1}p_{n-v-1}\right)^{k-1}$$

$$=0(1)\sum_{n=2}^{n+1}\left(\frac{P_n}{p_n}\right)^{k-1}\frac{1}{P_{n-1}}\sum_{v=1}^{n-1}p_{n-v-1}\;P_v\;\lambda_v$$

$=0(1),\;as\;m\to\infty$, as above

This completes the proof of the theorem.

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