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On Two-Graded Manpower Model with Non-Homogeneous Poisson Recruitments

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ABSTRACT

Manpower Planning is the prerequisite for efficient management of any organization. The stability of the graded structure in corporate and government offices plays a dominant role for efficient planning and control of human resources. The constituent components of a manpower planning model are recruitment process, promotion process and leaving process. This paper addresses a two-graded manpower model with non-homogeneous compound Poisson recruitment processes. In the present day competitive environment the recruitment in some organizations is time dependent. The time dependent nature is induced in the model by characterizing the recruitment process with a non-homogeneous Poissonprocess. Further, in several organizations the recruitment is done in bulk, that is, the people are employed in batches of random size. If in many organizations the recruitment is done in batches. Therefore, the recruitment process is to be characterized withnon-homogeneous compound Poisson process. Assigning that the promotion and leaving processes follow Poisson processes, the transient behavior of the model is analyzed. Using differential equations the performance measures of the model, such as average number of employees in each grade, average duration of stay of an employee in each grade, the variances of the grade sizes and covariance between the two grade sizes are derived explicitly. The sensitivity of the model with respect to the parameters is analyzed through numerical illustrations. It is observed that the time dependent recruitment rate has significant influence on the grade size distribution. This model is useful for formulating the recruitment and promotion policies of the organizations.

Key Words: Graded manpower system; Compound Poisson Process; Joint probability generating function; Bulk recruitment.

1. INTRODUCTION:

Due to the utility of manpower models in allocation of resources, formulating the policies for human resource development, recently much emphasis is given for modeling the manpower systems. Even though, the organizations consider 3-M (Men, Machine, and Material) as the prime components of the resources of any organization,men is most important, (Srinivasa Rao, K. etal (2010)). Planning the manpower is an essential requirement of any organization, (Asis Kumar Chatopadhyay et al (2007)). For efficient implementation of best methodology and best practices, the characteristics of the system such as average number of employees in each

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grade, the mean duration of stay of an employee in any organization are needed.

Seal (1945), has pioneered the mathematical modeling of Manpower systems. Silock (1954) has analyzed the phenomenon of labor turnover as an analogy to demography. Bartholomew (1963, 1971) has studied the Manpower models utilizing the concepts of complete length of service of an employee in the organization is random and follows a probability distribution. Wang (2005), has reviewed then Manpower models with respect to the different approaches adopted for model building and analysis. Ugwuowo and Mc Clean (2000) have reviewed the heterogeneity in the Manpower models. Kannan Nilakantan (2014), studied the evaluation of staffing policies in Markov manpower systems and their extension to organizations with outsource personnel. Jeeva and Geetha (2013) have studied the Manpower model in fuzzy environment.Osagiede and Ekhosuehi (2015), have studied Manpower models under continuous-time Markov chain via sparse stochastic matrices. Lalitha, Devi, and Srinivasan (2014) have studied the problem of time to recruitment is studied for a single grade manpower system with attrition, generated by a geometric process of inter-policy decision times, using a univariate policy of recruitment based on shock model approach. Srinivasa Rao, K. etal (2003,2006) , Kondababuetal(2013,2014), Govinda Rao etal(2013,2014) have studied the graded manpower models assuming that the recruitment process follows a Poisson process. The major assumptions of Poisson process are “Independence, Regularity, and Heterogeneity in time”. Hence they assumed that the recruitment process is time independent. But in many practical situations arising at places like corporate offices, public sector organizations, where skilled manpower is employed, the recruitment process is time dependent and seldom obey the assumption of the homogeneity in time of the Poisson process. So to model these situations one has to consider the non-homogeneous Poisson process. In several organizations the recruitment is done in bulk, that is, the people are employed in batches of random size. If in any Homogeneous Poisson Process the recruitment is in batches then the recruitment process is to be characterized withnon-homogeneous compound Poisson process.

Hence, in this paper for efficient and effective modeling of the manpower situation we characterize the recruitment process with non-homogeneous compound Poisson process. Very little work has been reported in the area of manpower models non-homogeneous Poisson process having time dependent recruitment rates. The non-homogeneous compound Poisson process will also include the Poisson process as a particular case.

Using the difference-differential equations the joint probability generating function of the number of employees in both the grades is derived. The system characteristics of the model such as the average number of employees in each grade, the mean duration of stay of an employee in each grade, the variances of the grade sizes and covariance between the two grade sizes. The sensitivity of the model with respect to the changes in the parameters is studied through numerical illustrations. A comparative study of the model with that of homogeneous Poisson recruitment is also studied. This model is useful for scheduling the manpower systems in corporate and government offices.

2.MANPOWER MODEL AND TRANSIENT SOLUTION

In this section, consider a two graded manpower model in which the organization is having two grades namely, grade 1 and grade 2. At every recruitment a group of employees are recruited into grade 1. Let us assume that the actual number of employees in modules (batch)ofrandomsize(X) with probability density function C_x. If $\lambda_x(t)$ is the recruitment rate of batches of sizes x, then the composite recruitment rate $\lambda(t)$ equals to $\sum_x \lambda_x(t)$, where the composite recruitment process follows a non-homogeneous compound Poisson process. It is also assumed that once an employee is recruited in grade-1, after spending a random duration of time in grade-1, he may be promoted to grade-2 with promotion rate β or may leave the organization with a leaving rate α. It is also assumed that the employee after spending a random duration of time in grade-2, employee leaves the organization with a leaving rate γ.

It is further assumed that the promotion and the leaving processes follow non-homogeneous Poisson processes.With these assumptions the schematic diagram representing the manpower system in the organization is shown in Fig. 1.

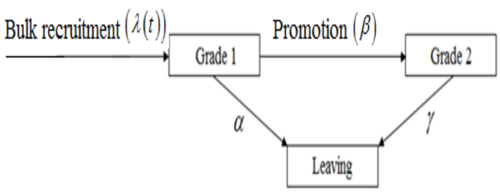

Fig.1

Let $P_{n,m}(t)$ be the probability that there are n employees in grade 1 and m employees in grade 2 at time t in the organization. The difference-differential equations governing the model are

$$\frac{\partial}{\partial t}P_{n,m}(t) = [-((\lambda_1+\lambda_2 t)+n\alpha+n\beta+m\gamma)]P_{n,m}(t)$$
$$+(n+1)\alpha P_{n+1,m}(t)+(n+1)\beta P_{n+1,m-1}(t)$$
$$+(m+1)\gamma P_{n,m+1}(t)+(\lambda_1+\lambda_2 t)$$
$$\left[\sum_{i=1}^{n} P_{n-i,m}(t)C_i\right] \text{for } m,n\geq 1 \qquad \cdots(1)$$

$$\frac{\partial}{\partial t}P_{n,0}(t) = [-((\lambda_1+\lambda_2 t)+n\alpha+n\beta)]P_{n,0}(t)$$
$$+(n+1)\alpha P_{n+1,0}(t)+\gamma P_{n,1}(t)$$
$$+(\lambda_1+\lambda_2 t)\left[\sum_{i=1}^{n} P_{n-i,0}(t)C_i\right],$$
$$\text{for } n\geq 1\cdots(2)$$

$$\frac{\partial}{\partial t}P_{0,m}(t) = \left[-((\lambda_1+\lambda_2 t)+m\gamma)h\right]P_{0,m}(t)$$
$$+\alpha P_{1,m}(t)+\beta P_{1,m-1}(t)$$
$$+(m+1)\gamma P_{0,m+1}(t),$$
$$\text{for } m\geq 1\cdots(3)$$

$$\frac{\partial}{\partial t}P_{0,0}(t) = \left[-(\lambda_1+\lambda_2 t)\right]P_{0,0}(t)$$
$$+\alpha P_{1,0}(t)+\gamma P_{0,1}(t) \qquad \cdots(4)$$

Let $G(Z_1,Z_2;t)=\sum_n\sum_m Z_1^n Z_2^m P_{n,m}(t)$

and $C(Z_1)=\sum_x C_x Z_1^x \quad \cdots(5)$

Multiplying equations (2.1) to (2.4) with corresponding $Z_1^n Z_2^m$ and summing over all $n=0,1,2,3,.....;m=0,1,2,3,.....$, we get

$$\frac{\partial}{\partial t}G(Z_1,Z_2;t) = -\lambda_1 G(Z_1,Z_2;t)-\alpha Z_1\frac{\partial}{\partial Z_1}G(Z_1,Z_2;t)$$
$$-\beta Z_1\frac{\partial}{\partial Z_1}G(Z_1,Z_2;t)-\gamma Z_2\frac{\partial}{\partial Z_2}G(Z_1,Z_2;t)$$
$$-\lambda_2 t.G(Z_1,Z_2;t)+\alpha\frac{\partial}{\partial Z_1}G(Z_1,Z_2;t)$$
$$+\beta Z_2\frac{\partial}{\partial Z_1}G(Z_1,Z_2;t)+\gamma\frac{\partial}{\partial Z_2}G(Z_1,Z_2;t)$$
$$+\lambda_1.C(Z_1).G(Z_1,Z_2;t)$$
$$+\lambda_2 t.C(Z_1).G(Z_1,Z_2;t) \qquad \cdots(6)$$

After simplification, we have

$$\frac{\partial}{\partial t}G(Z_1,Z_2;t) = \{\lambda_1[C(Z_1)-1]+\lambda_2 t[C(Z_1)-1]\}$$
$$G(Z_1,Z_2;t)+[-(\alpha+\beta)Z_1+\alpha+\beta Z_2]$$
$$\frac{\partial}{\partial Z_1}G(Z_1,Z_2;t)+\gamma(1-Z_2)$$
$$\frac{\partial}{\partial Z_2}G(Z_1,Z_2;t) \qquad \cdots(7)$$

Solving the equation (2.7) by Lagrange's method, the auxiliary equations are

$$\frac{dt}{1} = \frac{-dZ_1}{[\alpha(1-Z_1)+\beta(Z_2-Z_1)]} = \frac{-dZ_2}{\gamma(1-Z_2)}$$
$$= \frac{dG(Z_1,Z_2;t)}{\{\lambda_1[C(Z_1)-1]+\lambda_2 t[C(Z_1)-1]\}\,G(Z_1,Z_2;t)} \cdots (8)$$

The initial conditions of the system are

$$P_{N_0,M_0}(0)=1,\ P_{N_0,M_0}(t)=0 \text{ for } t>0 \qquad \dots(9)$$

That is initially the organization is having N_0 employees in grade 1 and M_0 employees in grade 2. Solving the equation (2.8), we get

$$A = (Z_2-1)e^{-\gamma t}$$

$$B = \left[(Z_1-1)+\frac{\beta}{\gamma-(\alpha+\beta)}(Z_2-1)\right]e^{-(\alpha+\beta)t}$$

$$C = G(Z_1,Z_2;t)\exp\left\{-(\lambda_1+\lambda_2 t)\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}C_x \binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s A^s B^{(r-s)} \frac{e^{[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]} + \lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s} C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s A^s B^{(r-s)}\frac{e^{[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\} \qquad \dots(10)$$

where $A, B,$ and C are arbitrary constants. The general solution of the equation (2.10) gives the joint probability generating function of the number of employees in both the grades as

$$G(Z_1,Z_2;t) = C.\exp\left\{(\lambda_1+\lambda_2 t)\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}C_x \binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s A^s B^{(r-s)} \frac{e^{[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]} - \lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s} C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s A^s B^{(r-s)}\frac{e^{[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\} \qquad \dots(11)$$

Using the initial conditions and substituting the value of C in equation (11), we get the joint probability generating function of $P_{n,m}(t)$ as

$$G(Z_1,Z_2;t) = \exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s (Z_2-1)^s \left[(Z_1-1)+\frac{\beta}{\gamma-(\alpha+\beta)}(Z_2-1)\right]^{(r-s)} \frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]} + \lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s} C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s (Z_2-1)^s \left[(Z_1-1)+\frac{\beta}{\gamma-(\alpha+\beta)}(Z_2-1)\right]^{(r-s)} \frac{1}{[\gamma s+(\alpha+\beta)(r-s)]} - \lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s (Z_2-1)^s \left[(Z_1-1)+\frac{\beta}{\gamma-(\alpha+\beta)}(Z_2-1)\right]^{(r-s)} \frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$
$$\left[1-(1-Z_1)e^{-(\alpha+\beta)t} - \frac{\beta}{(\alpha+\beta)-\gamma}(1-z_2)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0} \left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0};\ |Z_1|<1, |Z_2|<1 \quad \dots(12)$$

3: CHARACTERISTICS OF THE MODEL

The characteristics of the model are obtained by using the equation (12).Expanding $G(Z_1,Z_2;t)$ and collecting the constant terms, we get the probability that there is no employee in the organization as

$$G_{0,0}(t)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}C_x\binom{x}{r}\binom{r}{s}\frac{\beta^s(\alpha-\gamma)^{(r-s)}}{[\gamma-(\alpha+\beta)]^r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right.$$
$$+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}C_x\binom{x}{r}\binom{r}{s}\frac{\beta^s(\alpha-\gamma)^{(r-s)}}{[\gamma-(\alpha+\beta)]^r}\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}C_x\binom{x}{r}\binom{r}{s}\frac{\beta^s(\alpha-\gamma)^{(r-s)}}{[\gamma-(\alpha+\beta)]^r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$
$$\left[1-e^{-(\alpha+\beta)t}-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0}\quad\ldots(13)$$

Taking $Z_2=1$ equation (12), we get the probability generating function of the number of employees in grade 1 in the organization as

$$G(Z_1;t)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(Z_1-1)^r\frac{1-e^{-[(\alpha+\beta)r t]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}\frac{(Z_1-1)^r}{(\alpha+\beta)r}\right.$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(Z_1-1)^r\frac{1-e^{-[(\alpha+\beta)r t]}}{[(\alpha+\beta)r]^2}\right\}$$
$$\left[1-(1-Z_1)e^{-(\alpha+\beta)t}\right]^{N_0};|Z_1|<1\ldots(14)$$

expanding $G(Z_1;t)$ and collecting the constant terms, we get the probability that there is no grade 1 employee in the organization as

$$G_{0\cdot}(t)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)r t]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}\right.$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)r t]}}{[(\alpha+\beta)r]^2}\right\}$$
$$\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}\quad\ldots(15)$$

Similarly, taking $Z_1=1$ equation (12), we get the probability generating function of the number of employees in grade 2 in the organization as

$$G(Z_2;t)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r(Z_2-1)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right.$$
$$+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{(Z_2-1)^r}{[\gamma s+(\alpha+\beta)(r-s)]}$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}C_x\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\binom{x}{r}\binom{r}{s}(Z_2-1)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$
$$\left[1-\frac{\beta(1-Z_2)}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0};|Z_2|<1\quad\ldots(16)$$

Expanding $G(Z_2;t)$ and collecting the constant terms, we get the probability that there is no grade 2 employee in the organization as

$$G_{0}(t)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right.$$
$$+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$
$$\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0}\qquad\ldots(17)$$

The mean number of employees in grade 1 of the organization is

$$L_1=\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right]+N_0e^{-(\alpha+\beta)t}\quad\ldots(18)$$

The probability that there is at least one employee in grade 1 is

$$U_1=1-G_{0}(t)$$
$$=1-\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}$$
$$\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}\qquad\ldots(19)$$

The mean number of employees in grade 2 of the organization is

$$L_2=\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]$$
$$+\frac{\lambda_2\beta t}{\gamma(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty}xC_x\right)\left[\frac{\left(1-e^{-\gamma t}\right)(\alpha+\beta+\gamma)}{\gamma^2}+\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\gamma-\alpha-\beta}\right)\right]$$
$$+N_0\left(\frac{\beta}{\alpha+\beta-\gamma}\right)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+M_0e^{-\gamma t}\quad\ldots(20)$$

The probability that there is at least one employee in grade 2 is

$$U_2=1-G_{0}(t)$$
$$=1-\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right.$$
$$+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$
$$\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0}\qquad\ldots(21)$$

The mean number of employees in the organization is $L=L_1+L_2$.

Substituting the values of L_1 and L_2 from the equations (18) and (20), we get

$$L=\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x$$
$$-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right]+\mathrm{N}_0e^{-(\alpha+\beta)t}$$
$$+\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]\mathrm{T}$$
$$+\frac{\lambda_2\beta t}{\gamma(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty}xC_x\right)$$
$$\left[\frac{\left(1-e^{-\gamma t}\right)(\alpha+\beta+\gamma)}{\gamma^2}+\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\gamma-\alpha-\beta}\right)\right]$$
$$\mathrm{N}_0\left(\frac{\beta}{\alpha+\beta-\gamma}\right)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+\mathrm{M}_0e^{-\gamma t}\ldots(22)$$

he average duration of stay of an employee in grade 1 is

$$W_1=\frac{L_1}{(\alpha+\beta)(1-G_{0}(t))}$$
$$=\frac{\left\{\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right]+\mathrm{N}_0e^{-(\alpha+\beta)t}\right\}}{(\alpha+\beta)\left(1-\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}\right)}\ldots(23)$$

The average duration of stay of an employee in grade 2 is

$$W_2=\frac{L_2}{\gamma(1-G_{0}(t))}$$
$$=\frac{\left\{\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]+\frac{\lambda_2\beta t}{\gamma(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty}xC_x\right)\left[\frac{\left(1-e^{-\gamma t}\right)(\alpha+\beta+\gamma)}{\gamma^2}+\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\gamma-\alpha-\beta}\right)\right]+\mathrm{N}_0\left(\frac{\beta}{\alpha+\beta-\gamma}\right)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+\mathrm{M}_0e^{-\gamma t}\right\}}{\gamma\left(1-\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0}\right)}\ldots(24)$$

The variance of the number of employees in grade 1 is

$$V_1 = \frac{\lambda_1}{2(\alpha+\beta)}\sum_{x=1}^{\infty} x\,(x-1)C_x\left[1-e^{-2(\alpha+\beta)t}\right]$$
$$+\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty} xC_x\left[1-e^{-(\alpha+\beta)t}\right]$$
$$+\frac{\lambda_2 t}{2(\alpha+\beta)}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)$$
$$-\frac{\lambda_2}{[2(\alpha+\beta)]^2}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[1-e^{-2(\alpha+\beta)t}\right]$$
$$+\frac{\lambda_2 t}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right)$$
$$-\frac{\lambda_2}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty} xC_x\right)\left[1-e^{-(\alpha+\beta)t}\right]$$
$$+N_0\left[e^{-(\alpha+\beta)t}-e^{-2(\alpha+\beta)t}\right] \qquad \ldots(25)$$

T he variance of the number of employees in grade 2 is

$$V_2 = \frac{\lambda_1\beta^2}{(\gamma-(\alpha+\beta))^2}\left(\sum_{x=1}^{\infty} x\,(x-1)C_x\right)$$
$$\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)-2\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{(\alpha+\beta+\gamma)}\right)+\left(\frac{1-e^{-2\gamma t}}{2\gamma}\right)\right]$$
$$+\frac{\lambda_2 t\beta^2}{(\gamma-(\alpha+\beta))^2}\left(\sum_{x=1}^{\infty} x\,(x-1)C_x\right)$$
$$\left[\left(\frac{1}{2(\alpha+\beta)}\right)-\left(\frac{2}{(\alpha+\beta+\gamma)}\right)+\left(\frac{1}{2\gamma}\right)\right]$$
$$-\frac{\lambda_2\beta^2}{(\gamma-(\alpha+\beta))^2}\left(\sum_{x=1}^{\infty} x\,(x-1)C_x\right)$$
$$\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{(2(\alpha+\beta))^2}\right)-2\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{(\alpha+\beta+\gamma)^2}\right)\right.$$
$$\left.+\left(\frac{1-e^{-2\gamma t}}{(2\gamma)^2}\right)\right]+\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right)$$
$$\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-(\alpha+\beta)t}-e^{-\gamma t}}{\gamma-(\alpha+\beta)}\right)\right]$$
$$+\frac{\lambda_2 t\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right)\left(\frac{1}{\gamma}\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty} xC_x\right)$$
$$\left[\frac{(\alpha+\beta+\gamma)(1-e^{-\gamma t})}{\gamma^2}-\left(\frac{e^{-(\alpha+\beta)t}-e^{-\gamma t}}{\gamma-(\alpha+\beta)}\right)\right]$$
$$+\frac{N_0}{(\alpha+\beta)-\gamma}\left[e^{-\gamma t}-e^{-(\alpha+\beta)t}\right]$$
$$\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]$$
$$+M_0\left(e^{-\gamma t}-e^{-2\gamma t}\right) \qquad \ldots(26)$$

The coefficient of variation of the number of employees in grade 1 is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} \qquad \ldots(27)$$

where L_1 and V_1 are as given in equations (18) and (25).

The coefficient of variation of the number of employees in grade 2 is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \qquad \ldots(28)$$

where L_2 and V_2 are as given in equations (20) and (26).

The co-variance between grade1 and grade 2 sizes in the organization is

$$Cov(N,M) = \frac{\lambda_1\beta}{(\alpha+\beta-\gamma)}\left(\sum_{x=1}^{\infty} x\,(x-1)C_x\right)$$
$$\left[\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{\alpha+\beta+\gamma}\right)-\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)\right]$$
$$+\frac{N_0\beta}{(\alpha+\beta)-\gamma}\left[e^{-2(\alpha+\beta)t}-e^{-(\alpha+\beta+\gamma)t}\right]$$
$$+\frac{\lambda_2 t\beta}{2(\alpha+\beta)(\alpha+\beta+\gamma)}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)$$

$$+\frac{\lambda_1\lambda_2 t}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty}xC_x\right)^2\left(1-e^{-(\alpha+\beta)t}\right)$$
$$\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)+\left(\frac{\lambda_2 t}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)\right)^2$$
$$\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)-\frac{\lambda_2^2 t}{(\alpha+\beta)^3}\left(\sum_{x=1}^{\infty}xC_x\right)^2$$
$$\left(1-e^{-(\alpha+\beta)t}\right)\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)$$
$$+\frac{\lambda_2 tN_0}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}xC_x\right)e^{-(\alpha+\beta)t}\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)$$
$$+\frac{\lambda_2\beta}{\gamma-(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x(x-1)C_x\right)$$
$$\left[\frac{1-e^{-(\alpha+\beta+\gamma)t}}{(\alpha+\beta+\gamma)^2}-\frac{1-e^{-2(\alpha+\beta)t}}{(2(\alpha+\beta))^2}\right]\ldots(29)$$

4: CHARACTERISTICS OF THE MODEL WITH UNIFORM BATCH SIZE DISTRIBUTION

To study the performance of the model one has to specify the batch size distribution of the bulk recruitment. That is the number of employees recruited at a time is a random variable and follows a specific distribution. Let us assume that the number of employees in a batch of recruitment follows uniform distribution with parameters a and b. The probability mass function of the batch size distribution is

$$C_x=\frac{1}{b-a+1};x=a,a+1,........,b.$$

The mean number of employees in each batch is $\frac{a+b}{2}$.

The variance of the batch size is $\frac{1}{12}\left[(b-a+1)^2-1\right]$.

Substituting the value of C_x in (12), we get the joint probability generating function of the number of employees in grade 1 and grade 2 is obtained as

$$G(Z_1,Z_2;t)=\exp\Bigg\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}$$
$$\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s(Z_2-1)^s$$
$$\left[(Z_1-1)+\frac{\beta}{\gamma-(\alpha+\beta)}(Z_2-1)\right]^{(r-s)}$$
$$\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}$$
$$\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s(Z_2-1)^s$$
$$\left[(Z_1-1)+\frac{\beta}{\gamma-(\alpha+\beta)}(Z_2-1)\right]^{(r-s)}$$
$$\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}\left(\frac{1}{b-a+1}\right)$$
$$\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^s(Z_2-1)^s$$
$$\left[(Z_1-1)+\frac{\beta}{\gamma-(\alpha+\beta)}(Z_2-1)\right]^{(r-s)}$$
$$\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\Bigg\}\Big[1-(1-Z_1)$$
$$e^{-(\alpha+\beta)t}-\frac{\beta}{(\alpha+\beta)-\gamma}(1-Z_2)$$
$$\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\Big]^{N_0}$$
$$\left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0};|Z_1|<1,|Z_2|<1\ldots(30)$$

expanding $G(Z_1,Z_2;t)$ and collecting the constant terms, we get the probability that there is no employee in the organization as

$$G_{0,0}(t) = \exp\left\{\lambda_1 \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\frac{\beta^s(\alpha-\gamma)^{(r-s)}}{[\gamma-(\alpha+\beta)]^r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right.$$
$$+\lambda_2 t \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\frac{\beta^s(\alpha-\gamma)^{(r-s)}}{[\gamma-(\alpha+\beta)]^r}\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}$$
$$\left.-\lambda_2 \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\frac{\beta^s(\alpha-\gamma)^{(r-s)}}{[\gamma-(\alpha+\beta)]^r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$
$$\left[1-e^{-(\alpha+\beta)t}-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0} \quad \ldots(31)$$

Taking $Z_2 = 1$ equation (30), we get the probability generating function of the number of employees in grade 1 in the organization as

$$G(Z_1;t)=\exp\left\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{(Z_1-1)^r}{(\alpha+\beta)r}\right.$$
$$\left.-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}\left[1-(1-Z_1)e^{-(\alpha+\beta)t}\right]^{N_0};|Z_1|<1 \ldots(32)$$

Expanding $G(Z_1;t)$ and collecting the constant terms, we get the probability that there is no grade 1 employee in the organization as

$$G_{0\cdot}(t)=\exp\left\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}\right.$$
$$\left.-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0} \quad \ldots(33)$$

Similarly, taking $Z_1 = 1$ equation (30), we get the probability generating function of the number of employees in grade 2 in the organization as

$$G(Z_2;t)=\exp\left\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}(Z_2-1)^r\right.$$
$$+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)(-1)^{2r-s}\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r\frac{(Z_2-1)^r}{[\gamma s+(\alpha+\beta)(r-s)]}$$
$$\left.-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^r(Z_2-1)^r\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$
$$\left[1-\frac{\beta(1-Z_2)}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0};|Z_2|<1 \ldots(34)$$

Expanding $G(Z_2;t)$ and collecting the constant terms, we get the probability that there is no grade 2 employee in the organization as

$$G_{0}(t)=\exp\left\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right.$$

$$+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}$$

$$\left.-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^2}\right\}$$

$$\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0} \quad ...(35)$$

The mean number of employees in grade 1 of the organization is

$$L_1=\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]+N_0e^{-(\alpha+\beta)t} \quad ...(36)$$

The probability that there is at least one employee in grade 1 is

$$U_1=1-G_{0}(t)$$

$$=1-\exp\left\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}(-1)^r+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}\right. \qquad \text{T}$$

$$\left.-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0} \quad ...(37)$$

he mean number of employees in grade 2 of the organization is

$$L_2=\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]+\frac{\lambda_2\beta t}{\gamma(\alpha+\beta)}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\text{T}$$

$$\left[\frac{\left(1-e^{-\gamma t}\right)(\alpha+\beta+\gamma)}{\gamma^2}+\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\gamma-\alpha-\beta}\right)\right]+N_0\left(\frac{\beta}{\alpha+\beta-\gamma}\right)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+M_0e^{-\gamma t} ...(38)$$

he probability that there is at least one employee in grade 2 is

$$U_2=1-G_{0}(t)$$

$$=1-\exp\left\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right.$$

$$+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}$$

$$-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}$$

$$\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^{2}}\Bigg\}$$
$$\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}$$
$$\left[1-e^{-\gamma t}\right]^{M_0} \quad \ldots(39)$$

The mean number of employees in the organization is $L = L_1 + L_2$.

Substituting the values of L_1 and L_2 from the equations (36) and (38), we get

$$L=\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]$$
$$+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)+N_0 e^{-(\alpha+\beta)t}$$
$$-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]$$
$$+\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)$$
$$\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]+\frac{\lambda_2\beta t}{\gamma(\alpha+\beta)}$$
$$\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}$$
$$\left[\frac{\left(1-e^{-\gamma t}\right)(\alpha+\beta+\gamma)}{\gamma^2}+\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\gamma-\alpha-\beta}\right)\right]$$
$$\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)+N_0\left(\frac{\beta}{\alpha+\beta-\gamma}\right)$$
$$\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+M_0 e^{-\gamma t} \quad \ldots(40)$$

T

he average duration of stay of an employee in grade 1 is

$$W_1=\frac{L_1}{(\alpha+\beta)\left(1-G_{0}(t)\right)}$$
$$=\frac{\left\{\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)+N_0e^{-(\alpha+\beta)t}-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]\right\}}{(\alpha+\beta)\left(1-\exp\left\{\lambda_1\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r\left[\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right]\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}\right)} \quad \ldots(41)$$

The average duration of stay of an employee in grade 2 is

$$W_2=\frac{L_2}{\gamma\left(1-G_{0}(t)\right)}$$
$$\left\{\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\right.$$
$$\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]$$
$$+\frac{\lambda_2\beta t}{\gamma(\alpha+\beta)}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)$$
$$-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)$$

$$\left[\frac{\left(1-e^{-\gamma t}\right)\left(\alpha+\beta+\gamma\right)}{\gamma^{2}}+\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\gamma-\alpha-\beta}\right)\right]$$

$$=\frac{+\mathrm{N}_0\left(\dfrac{\beta}{\alpha+\beta-\gamma}\right)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+\mathrm{M}_0 e^{-\gamma t}\Big\}}{\gamma\left(1-\exp\left\{\lambda_1\displaystyle\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\right.\right.}$$

$$\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\left(\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]}\right)$$

$$+\lambda_2 t\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)\binom{x}{r}$$

$$\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1}{[\gamma s+(\alpha+\beta)(r-s)]}$$

$$-\lambda_2\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{3r-s}\left(\frac{1}{b-a+1}\right)$$

$$\left.\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{\gamma-(\alpha+\beta)}\right)^{r}\frac{1-e^{-[\gamma s+(\alpha+\beta)(r-s)]t}}{[\gamma s+(\alpha+\beta)(r-s)]^{2}}\right\}$$

$$\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}$$

$$\left.\left[1-e^{-\gamma t}\right]^{M_0}\right) \qquad \ldots(42)$$

The variance of the number of employees in grade 1 is

$$V_1=\frac{\lambda_1}{2(\alpha+\beta)}\sum_{x=a}^{b}x\,(x-1)\left(\frac{1}{b-a+1}\right)\left[1-e^{-2(\alpha+\beta)t}\right]$$

$$+\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]$$

$$+\frac{\lambda_2 t}{2(\alpha+\beta)}\left(\sum_{x=a}^{b}x(x-1)\left(\frac{1}{b-a+1}\right)\right)$$

$$-\frac{\lambda_2}{\left[2(\alpha+\beta)\right]^{2}}\left(\sum_{x=a}^{b}x(x-1)\left(\frac{1}{b-a+1}\right)\right)$$

$$\left[1-e^{-2(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)$$

$$-\frac{\lambda_2}{(\alpha+\beta)^{2}}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\left[1-e^{-(\alpha+\beta)t}\right]$$

$$+N_0\left[e^{-(\alpha+\beta)t}-e^{-2(\alpha+\beta)t}\right] \qquad \ldots(43)$$

The variance of the number of employees in grade 2 is

$$V_2=\frac{\lambda_1\beta^{2}}{\left(\gamma-(\alpha+\beta)\right)^{2}}\left(\sum_{x=a}^{b}x\,(x-1)\left(\frac{1}{b-a+1}\right)\right)$$

$$\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)-2\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{(\alpha+\beta+\gamma)}\right)+\left(\frac{1-e^{-2\gamma t}}{2\gamma}\right)\right]$$

$$+\frac{\lambda_2 t\beta^{2}}{\left(\gamma-(\alpha+\beta)\right)^{2}}\left(\sum_{x=a}^{b}x\,(x-1)\left(\frac{1}{b-a+1}\right)\right)$$

$$\left[\left(\frac{1}{2(\alpha+\beta)}\right)-\left(\frac{2}{(\alpha+\beta+\gamma)}\right)+\left(\frac{1}{2\gamma}\right)\right]$$

$$-\frac{\lambda_2\beta^{2}}{\left(\gamma-(\alpha+\beta)\right)^{2}}\left(\sum_{x=a}^{b}x\,(x-1)\left(\frac{1}{b-a+1}\right)\right)$$

$$\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{\left(2(\alpha+\beta)\right)^{2}}\right)-2\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{(\alpha+\beta+\gamma)^{2}}\right)+\left(\frac{1-e^{-2\gamma t}}{(2\gamma)^{2}}\right)\right]$$

$$+\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)$$

$$\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-(\alpha+\beta)t}-e^{-\gamma t}}{\gamma-(\alpha+\beta)}\right)\right]+\frac{\lambda_2 t\beta}{(\alpha+\beta)}$$

$$\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\left(\frac{1}{\gamma}\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^{2}}$$

$$\left[\frac{(\alpha+\beta+\gamma)\left(1-e^{-\gamma t}\right)}{\gamma^{2}}-\left(\frac{e^{-(\alpha+\beta)t}-e^{-\gamma t}}{\gamma-(\alpha+\beta)}\right)\right]$$

$$\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)+\frac{N_0}{(\alpha+\beta)-\gamma}$$

$$\left[1-\frac{\beta}{(\alpha+\beta)-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]$$

$$\left[e^{-\gamma t}-e^{-(\alpha+\beta)t}\right]+\mathrm{M}_0\left(e^{-\gamma t}-e^{-2\gamma t}\right) \qquad \ldots(44)$$

The coefficient of variation of the number of employees in grade 1 is

$$CV_1=\frac{\sqrt{V_1}}{L_1} \qquad \ldots(45)$$

where L_1 and V_1 are as given in equations (36) and (43).

The coefficient of variation of the number of employees in grade 2 is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \qquad \ldots(46)$$

Where L_2 and V_2 are as given in equations (38) and (44).

The co-variance between grade1 and grade 2 sizes in the organization is

$$Cov(N,M) = \frac{\lambda_1 \beta}{(\alpha+\beta-\gamma)}\left(\sum_{x=a}^{b} x\,(x-1)\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{\alpha+\beta+\gamma}\right)-\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)\right]$$
$$+\frac{N_0\beta}{(\alpha+\beta)-\gamma}\left[e^{-2(\alpha+\beta)t}-e^{-(\alpha+\beta+\gamma)t}\right]$$
$$+\frac{\lambda_2 t\beta}{2(\alpha+\beta)(\alpha+\beta+\gamma)}\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)$$
$$+\frac{\lambda_1\lambda_2 t}{(\alpha+\beta)^2}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)^2\left(1-e^{-(\alpha+\beta)t}\right)\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)$$
$$+\left(\frac{\lambda_2 t}{(\alpha+\beta)}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\right)^2\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)-\frac{\lambda_2^2 t}{(\alpha+\beta)^3}$$
$$\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)^2\left(1-e^{-(\alpha+\beta)t}\right)\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)+\frac{\lambda_2 t N_0}{(\alpha+\beta)}$$
$$\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left(\frac{\gamma-(\alpha+2\beta)}{\gamma-(\alpha+\beta)}\right)e^{-(\alpha+\beta)t}$$
$$+\frac{\lambda_2\beta}{\gamma-(\alpha+\beta)}\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)$$
$$\left[\frac{1-e^{-(\alpha+\beta+\gamma)t}}{(\alpha+\beta+\gamma)^2}-\frac{1-e^{-2(\alpha+\beta)t}}{(2(\alpha+\beta))^2}\right] \qquad \ldots(47)$$

5 NUMERICAL ILLUSTRATION AND RESULTS

In this section, the behavior of the model is discussed through a numerical illustration. Different values of the parameters are considered for recruitment, promotion rate and leaving rates of the system. Since the performance characteristics of the manpower model are highly sensitive with respect to the time, the transient behavior of the model is studied through computing the performance measures with the following set of values for the model parameters.

$t = 0.4, 0.5, 0.6, 0.7, 0.8;\ \lambda_1 = 2, 3, 4, 5, 6;$
$\lambda_2 = 1, 2, 3, 4, 5;\ \alpha = 2, 3, 4\ 5, 6;$
$\beta = 3, 4, 5, 6, 7;\ \gamma = 4, 5, 6, 7, 8;$
$a = 5, 6, 7, 8, 9;\ b = 25, 30, 35, 40, 45;$
$N_0 = 300, 400, 500, 600, 700;$
$M_0 = 100, 300, 500, 700, 900;$

Using the equations (36) and (38) the average number of employees in grade 1 and in grade 2 are computed and presented in **Table 1**. The relationship between the change in parameters and the average number of employees in each grade are shown in **figure 2**.

It is observed that the average number of employees in the grade 1 and grade 2 in the organization is highly sensitive with respect to changes in time. As time (t) varies from 0.4 to 0.8 units, the average number of employees in the grade 1 decreasing from46.470 to 13.196, when other parameters are fixed at (2,1,2,3,4,5,25,300,100) for $(\lambda_1, \lambda_2, \alpha, \beta, \gamma, a, b, N_0, M_0)$. Similarly, the averagenumber of employees in grade 2 in the organization decreases from 82.556 to 29.011for given values of other parameters. The average number of employees in grade 1 and grade 2 are decreasing with respect to time.As the leaving rate of grade 1 employee (α) varies from 2 to 6 units, the average number of employees in grade 1 reduces from 32.220 to 14.810 when other parameters are fixed at (0.8,6,5,3,4,5,25,300, and 100)for $(t, \lambda_1, \lambda_2, \beta, \gamma, a, b, N_0, M_0)$.

Similarly, the average number of employees in grade 2 in the organization reduces from 40.119 to 20.412 for given values of the other parameters. The average number of employees in grade 1 and grade 2 are simultaneously decreasing with respect to the leaving rate of grade 1 employee (α).The recruitment parameter λ_1 varies from 2 to 6 units; the average number of employees in grade 1 is increasing from 6.517 to 7.617, when other parameters are fixed at (0.8,1,2,3,4,5,25,300, and 100) for $(t, \lambda_2, \alpha, \beta, \gamma, a, b, N_0, M_0)$. Similarly, the average number of employees in grade 2 in the organization increases from 13.196 to 24.976 for given values of the other parameters. The average number of employees in grade 1 and grade 2 are simultaneously increasing with respect to the recruitment parameter λ_1 increases.The recruitment parameter λ_2 varies from 1 to 5 units; the average number of employees in grade 1 is increasing from 24.97600 to 32.21996, when other parameters are fixed at (0.8,6,2,3,4,5,25,300, and 100) for $(t, \lambda_2, \alpha, \beta, \gamma, a, b, N_0, M_0)$. Similarly, the average number of employees in grade 2 in the organization increases from 36.836 to 40.119 for given values of the other parameters. The average number of employees in grade 1 and grade 2 are simultaneously increasing with respect to the recruitment parameter λ_2 increases.Similarly, the average number of employees in grade 2 in the organization reduces from 40.119 to 20.412 for given values of the other parameters. The average number of employees in grade 1 and grade 2 are simultaneously decreasing with respect to the leaving rate of grade 1 employee (α).

The recruitment parameter λ_1 varies from 2 to 6 units; the average number of employees in grade 1 is increasing from 6.517 to 7.617, when other parameters are fixed at (0.8,1,2,3,4,5,25,300, and 100) for

Table 1: The values of L_1, L_2,W_1 andW_2 for different values of parameters

t	λ_1	λ_2	α	β	□	a	b	N_0	M_0	L_1	L_2	W_1	W_2
0.4	2	1	2	3	4	5	25	300	100	46.470	82.556	9.294	20.639
0.5	2	1	2	3	4	5	25	300	100	31.082	64.622	6.216	16.155
0.6	2	1	2	3	4	5	25	300	100	21.867	49.679	4.373	12.420
0.7	2	1	2	3	4	5	25	300	100	16.396	37.921	3.279	9.480
0.8	2	1	2	3	4	5	25	300	100	13.196	29.011	2.639	7.253
0.8	3	1	2	3	4	5	25	300	100	16.141	30.968	3.228	7.742
0.8	4	1	2	3	4	5	25	300	100	19.086	32.924	3.817	8.231
0.8	5	1	2	3	4	5	25	300	100	22.031	34.880	4.406	8.720
0.8	6	1	2	3	4	5	25	300	100	24.976	36.836	4.995	9.209
0.8	6	2	2	3	4	5	25	300	100	26.787	37.657	5.357	9.414
0.8	6	3	2	3	4	5	25	300	100	28.598	38.478	5.720	9.619
0.8	6	4	2	3	4	5	25	300	100	30.409	39.298	6.082	9.814
0.8	6	5	2	3	4	5	25	300	100	32.220	40.119	6.444	10.030
0.8	6	5	3	3	4	5	25	300	100	23.692	31.737	4.213	8.119
0.8	6	5	4	3	4	5	25	300	100	19.556	26.644	2.995	6.828
0.8	6	5	5	3	4	5	25	300	100	16.794	23.059	2.257	5.916
0.8	6	5	6	3	4	5	25	300	100	14.810	20.412	1.773	5.239
0.8	6	5	6	4	4	5	25	300	100	13.298	23.289	1.435	5.994
0.8	6	5	6	5	4	5	25	300	100	12.094	25.543	1.187	6.585
0.8	6	5	6	6	4	5	25	300	100	11.103	27.371	0.999	7.066
0.8	6	5	6	7	4	5	25	300	100	10.269	28.891	0.854	7.467
0.8	6	5	6	7	5	5	25	300	100	10.269	19.440	0.854	4.058
0.8	6	5	6	7	6	5	25	300	100	10.269	14.240	0.854	2.499
0.8	6	5	6	7	7	5	25	300	100	10.269	11.202	0.854	1.696
0.8	6	5	6	7	8	5	25	300	100	10.269	9.299	0.854	1.238
0.8	6	5	6	7	8	6	25	300	100	10.611	9.582	0.883	1.276
0.8	6	5	6	7	8	7	25	300	100	10.953	9.862	0.911	1.313
0.8	6	5	6	7	8	8	25	300	100	11.295	10.144	0.939	1.351
0.8	6	5	6	7	8	9	25	300	100	11.637	10.426	0.968	1.389
0.8	6	5	6	7	8	9	30	300	100	13.347	11.834	1.110	1.577
0.8	6	5	6	7	8	9	35	300	100	15.057	13.242	1.252	1.766
0.8	6	5	6	7	8	9	40	300	100	16.767	14.650	1.394	1.954
0.8	6	5	6	7	8	9	45	300	100	18.477	16.058	1.537	2.143
0.8	6	5	6	7	8	9	45	400	100	18.480	16.286	1.537	2.172
0.8	6	5	6	7	8	9	45	500	100	18.483	16.514	1.537	2.200
0.8	6	5	6	7	8	9	45	600	100	18.486	16.743	1.538	2.229
0.8	6	5	6	7	8	9	45	700	100	18.489	16.971	1.538	2.257
0.8	6	5	6	7	8	9	45	700	300	18.489	17.303	1.538	2.299
0.8	6	5	6	7	8	9	45	700	500	18.489	17.636	1.538	2.340
0.8	6	5	6	7	8	9	45	700	700	18.489	17.968	1.538	2.382
0.8	6	5	6	7	8	9	45	700	900	18.489	18.300	1.538	2.423

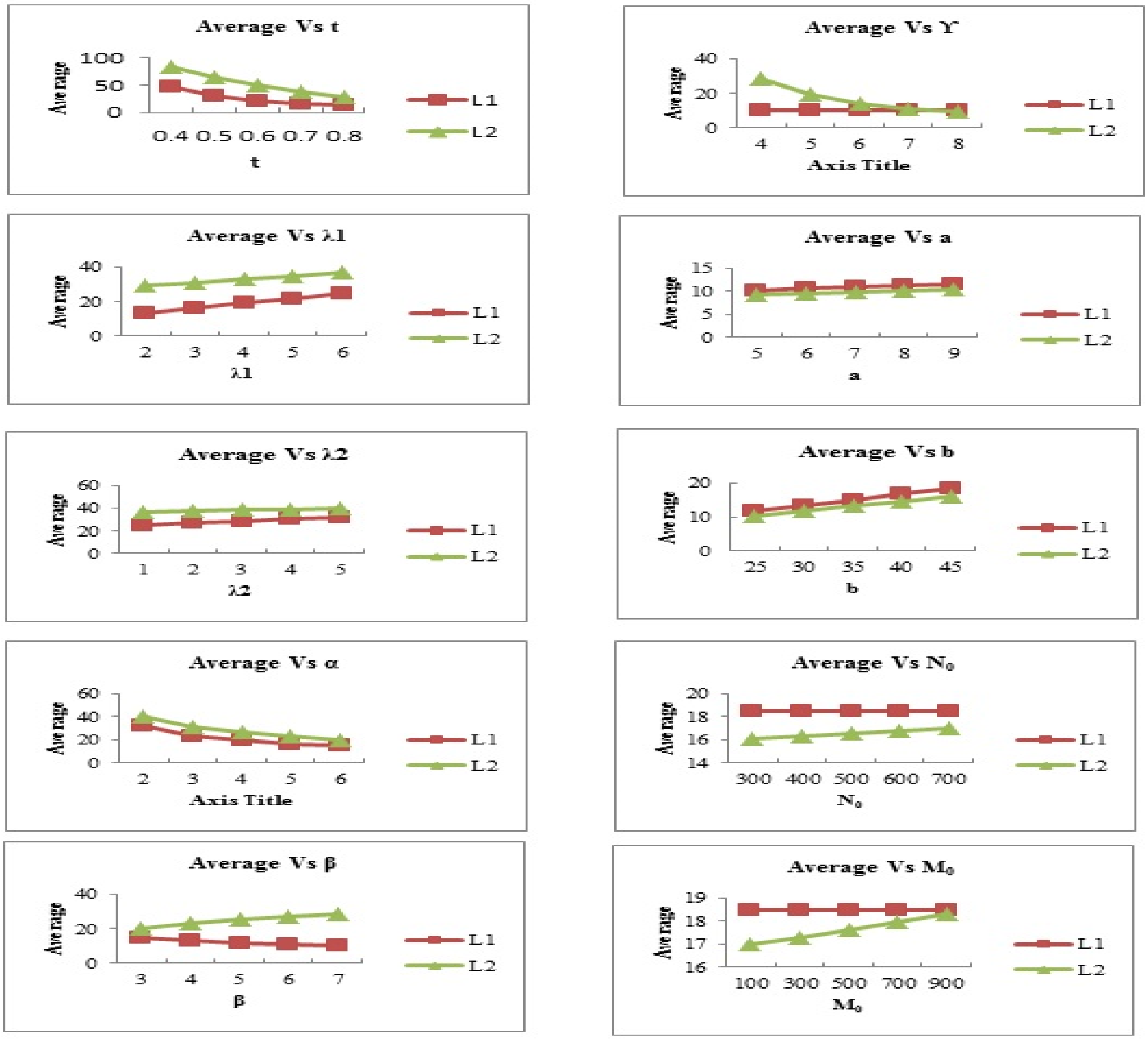

Figure 2: Relationship between Average and $t, \lambda_1, \lambda_2, \alpha, \beta, \gamma, a, b, N_0$ and M_0.

$(t, \lambda_2, \alpha, \beta, \gamma, a, b, N_0, M_0)$. Similarly, the average number of employees in grade 2 in the organization increases from 13.196 to 24.976 for given values of the other parameters. The average number of employees in grade 1 and grade 2 are simultaneously increasing with respect to the recruitment parameter λ_1 increases. The recruitment parameter λ_2 varies from 1 to 5 units; the average number of employees in grade 1 is increasing from 24.976 to 32.220, when other parameters are fixed at (0.8,6,2,3,4,5,25,300, and 100)for $(t, \lambda_2, \alpha, \beta, \gamma, a, b, N_0, M_0)$. Similarly, the average number of employees in grade 2 in the organization increases from 36.836 to 40.119 for given values of the other parameters. The average number of employees in grade 1 and grade 2 are simultaneously increasing with respect to the recruitment parameter λ_2 increases.When the promotion rate from grade 1 to grade $2(\beta)$ varies from 3 to 7, the average number of employees in grade 1 reduces from 14.810 to 10.269, when other parameters are fixed at (0.8,6,5,6,4,5,25,300, and 100) for $(t, \lambda_1, \lambda_2, \alpha, \gamma, a, b, N_0, M_0)$. Similarly, the average number of employees in grade 2 in the organization increases from20.412 to28.891for given values of the other parameters.

As the leaving rate of grade 2 employee (γ) varies from 4 to 8 units, the average number of employees in grade 1 is stable (10.269) when other parameters are fixed at (0.8,6,5,6,7,5,25,300, and 100)for $(t,\lambda_1,\lambda_2,\alpha,\beta,a,b,N_0,M_0)$. Similarly, the average number of employees in grade 2 in the organization reduces from 28.891 to 9.299 for given values of the other parameters. It is further observed that the initial number of employees in grade 1 (N_0) varies from 300 to 700 units, the average number of employees in grade 1 increasing from 18.477 to 18.490 when other parameters are fixed at (0.8,6,5,6,7,8,9,45, and 100)for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,M_0)$. Similarly, the average number of employees in grade 2 in the organization increasing from387.373 to 387.567 for given values of the other parameters.

As the initial number of employees in grade 2 (M_0) increases from 100 to 900, the average number of employees in grade 1 in the organization have no influence, when the other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 9, 45, and 700)for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,N_0)$., the initial number of employees in grade 2 increases from 100 to 900, the average number of employees in grade 2 increasing from 16.971 to 18.300 for given values of the other parameters.As the uniform batch size distribution parameter (a) varies from 5 to 9 units the average number of employees in the grade 1 is increasing from 10.269 to 11.673, when other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 25, 300 and 100) for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,b,N_0,M_0)$. Similarly, the average number of employees in grade 2 in the organization is increasing from 9.299 to 10.426 for given values of the other parameters. It is further observed that the batch size distribution parameter (b) varies from 25 to 45 units the average number of employees in the grade 1 is increasing from 11.637 to 18.477, when other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 9, 300 and 100)for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,N_0,M_0)$. Similarly, the average number of employees in grade 2 in the organization is increasing from 10.426 to 16.058 for given values of the other parameters.Using the equations (41) and (42) the average duration of stay of an employee in grade 1 and in grade 2 in the organization at different values of the parameters are computed and presented in **Table 1**. The relationship between the change in parameters and the average duration of stay of an employee in each grade are shown in **figure 3**.

It is observed that the average duration of stay of an employee in grade 1 and grade 2 in the organization are highly sensitive with respect to changes in time. As time varies from 0.4 to 0.8 units, the average duration of stay of an employee in the grade 1 decreasing from 9.294 to 2.639, when other parameters are fixed at (2,1,2,3,4,5,25,300,100) for $(\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,N_0,M_0)$. Similarly, the average duration of stay of an employee in the grade 2 in the organization decreasing from 20.639 to 7.253for given values of other parameters. The increase in the average duration of stay of an employee in grade 1 is moderate, when compared to that of grade 2 employees. The effect of leaving rate of grade 1 employees (α) in the organization is also studied. As the leaving rate of grade 1 employee (α) varies from 2 to 6, the average duration of stay of an employee in the grade 1 reduces from 6.444 to 1.773, when other parameters are fixed at (0.8,6,5,3,4,5,25,300, and 100)for $(t,\lambda_1,\lambda_2,\beta,\gamma,a,b,N_0,M_0)$. Similarly, the average duration of stay of an employee in grade 2 in the organization reduces from10.030 to5.239 for given values of the other parameters.

The recruitment parameter λ_1 varies from 2 to 6 units, the average duration of stay an employee in grade 1 is increasing from 2.639 to 4.995, when other parameters are fixed at (0.8,1,2,3,4,5,25,300, and 100) for $(t,\lambda_2,\alpha,\beta,\gamma,a,b,N_0,M_0)$. Similarly, the average duration of stay of an employee in grade 2 in the organization increases from7.253to 9.209for given values of the other parameters. The increase in average duration of stay of an employee in grade 1 is moderate, when compared to that of grade 2 employees.

other parameters are fixed at (0.8,6,2,3,4,5,25,300, and 100) for $(t,\lambda_1,\alpha,\beta,\gamma,a,b,N_0,M_0)$. Similarly, the average duration of stay of an employee in grade 2 in the organization increasing from9.209 to 10.030for given values of the other parameters. The increase in average number of employees in grade 2 is moderate, when compared to that of grade 1 employees.
When the promotion rate from grade 1 to grade $2(\beta)$ varies from 3 to 7, the average duration of stay of an employee in grade 1 reduces from 1.773

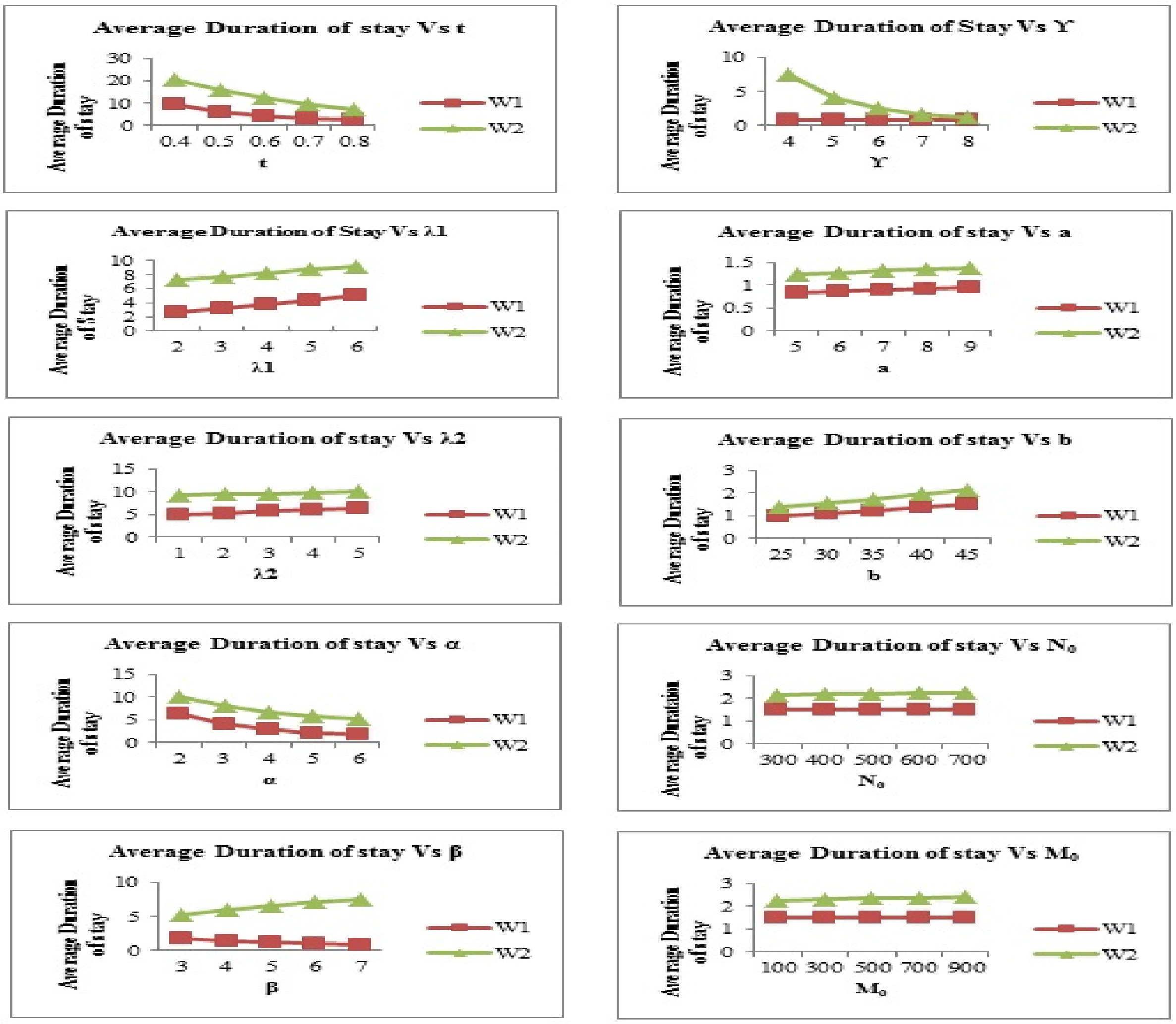

Figure 3: Relationship between Average Duration of ***stay and*** $t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,N_0$ and M_0.

The recruitment parameter λ_2 varies from 1 to 5 units; the average duration of stay of an employee in grade 1 is increasing from 4.995 to 6.444, when to 0.854, when other parameters are fixed at (0.8,6,5,6,4,5,25,300, and 100)for $\left(t,\lambda_1,\lambda_2,\alpha,\gamma,a,b,N_0,M_0\right)$.

Similarly, the average duration of stay of an employee in grade 2 in the organization increasing from 5.239 to 7.467for given values of the other parameters.As the leaving rate of grade 2 employees (γ) varies from 4 to 8 units, the average duration of stay of an employee in grade 1 remains constant (0.854), when other parameters are fixed at (0.8,6,5,6,7,5,25,300, and 100)for $\left(t,\lambda_1,\lambda_2,\alpha,\beta,a,b,N_0,M_0\right)$.

Similarly, the average number of employees in grade 2 in the organization reduces from7.467 to 1.238for given values of the other parameters.

As the uniform batch size distribution parameter (a) varies from 5 to 9 units the average duration of stay of an employee in the grade 1 is decreasing from 0.854 to 0.968, when other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 25, 300 and 100) for $\left(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,b,N_0,M_0\right)$.

Similarly, the average duration of stay of an employee in grade 2 in the organization is increasing from 1.238 to 1.389 for given values of the other parameters. It is further observed that the batch size distribution parameter (b) varies from 25 to 45 units the average duration of stay of an employee in the grade 1 is increasing from 0.968 to 1.537, when other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 9, 300 and 100) for $\left(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,b,N_0,M_0\right)$.

Similarly, the average duration of stay of an employee in grade 2 in the organization is increasing from 1.389 to 2.143 for given values of the other parameters.

It is further observed that the initial number of employees in grade 1 (N_0) varies from 300 to 700 units, the average duration of stay of an employee in grade 1 increasing from 1.537 to 1.538 when other parameters are fixed at (0.8,6,5,6,7,8,9,45, and 100)for $\left(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,M_0\right)$.

Similarly, the average duration of stay of an employee in grade 2 in the organization increasingfrom 2.143 to 2.257 for given values of the other parameters.

As the initial number of employees in grade 2 (M_0) increases from 100 to 900, the average duration of stay of an employee in grade 1 in the organization have no influence, when the other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 9, 45, and 700)for $\left(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,N_0\right)$, the initial number of employees in grade 2 increases from 100 to 900, the average duration of stay of an employee in grade 2 increasing from 2.257 to 2.423 for given values of the other parameters.

Using the equations (43) and (44) the variance of the number of employees in grade 1 and in grade 2 in the organization at different values of the parameters are computed and presented in **Table 2**. The relationship between the change in parameters and the variance of the number of employeesin each grade are shown in **figure 4**.

It is observed that thevariance of the number of employees in grade 1 and grade 2 in the organization are highly sensitive with respect to changes in time. As time varies from 0.4 to 0.8 units, the variance of the number of employeesin grade 1 decreasing from 96.850 to 79.679, when other parameters are fixed at (2,1,2,3,4,5,25,300,100) for $\left(\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,N_0,M_0\right)$.

Similarly, the variance of the number of employees in grade 2 in the organization decreasing from 75.916 to 42.417for given values of other parameters. The decrease in the variance of the number of employees in grade 2 is moderate, when compared to that of grade 1employees.The effect of leaving rate of grade 1 employees (α) in the organization is also studied. As the leaving rate of grade 1 employee (α) varies from 2 to 6, the variance of the number of employees in grade 1 reduces from 266.407 to 137.838, when other parameters are fixed at (0.8,6,5,3,4,5,25,300, and 100) for $(t,\lambda_1,\lambda_2,\beta,\gamma,a,b,N_0,M_0)$. Similarly, the variance of the number of employees in grade 2 in the organization reduces from 89.256 to 39.458 for given values of the other parameters. The decrease in average number of employees in grade 2 is moderate, when compared to that of grade 1.

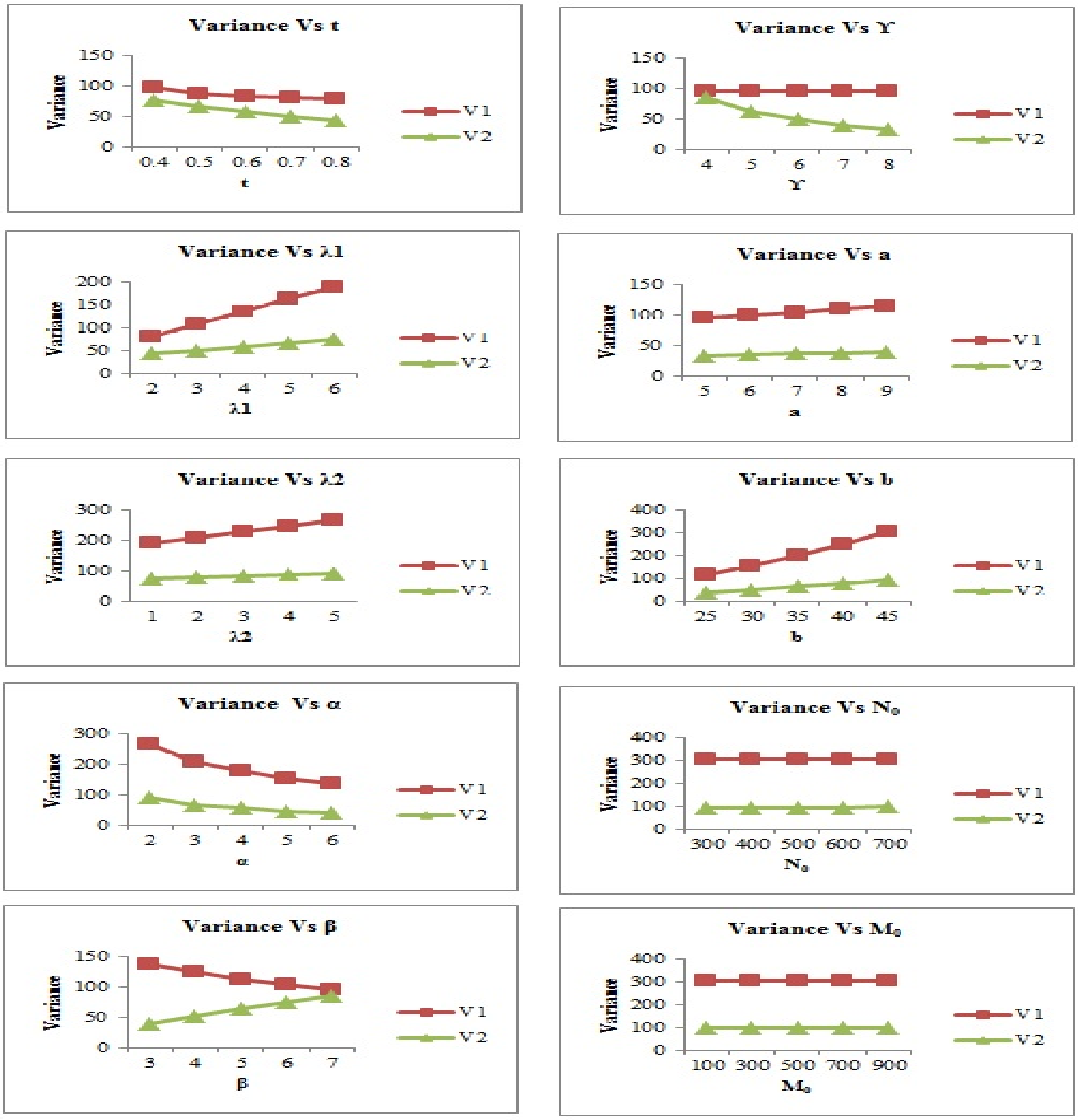

Figure 4: Relationship between Variance and $t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,N_0$ and M_0

Table 2: The values of L_1, L2, V_1, V_2, CV_1, CV_2 and Cov (N, M) for different values of parameters

t	λ_1	λ_2	α	β	□	a	b	N_0	M_0	V_1	V_2	CV_1	CV_2	Cov(N,M)
0.4	2	1	2	3	4	5	25	300	100	96.850	75.916	0.212	0.105	231.277
0.5	2	1	2	3	4	5	25	300	100	87.945	66.573	0.302	0.126	200.601
0.6	2	1	2	3	4	5	25	300	100	82.674	57.263	0.416	0.152	174.899
0.7	2	1	2	3	4	5	25	300	100	80.213	49.067	0.546	0.185	157.208
0.8	2	1	2	3	4	5	25	300	100	79.679	42.417	0.676	0.224	147.527
0.8	3	1	2	3	4	5	25	300	100	107.283	50.373	0.642	0.229	183.985
0.8	4	1	2	3	4	5	25	300	100	134.886	58.328	0.609	0.232	220.444
0.8	5	1	2	3	4	5	25	300	100	162.490	66.284	0.579	0.233	256.902
0.8	6	1	2	3	4	5	25	300	100	190.093	74.240	0.552	0.234	293.260
0.8	6	2	2	3	4	5	25	300	100	209.172	77.994	0.540	0.235	572.747
0.8	6	3	2	3	4	5	25	300	100	228.250	81.748	0.528	0.235	886.905
0.8	6	4	2	3	4	5	25	300	100	247.329	85.502	0.517	0.235	1235.83
0.8	6	5	2	3	4	5	25	300	100	266.407	89.256	0.507	0.235	1619.53
0.8	6	5	3	3	4	5	25	300	100	205.924	67.381	0.606	0.259	525.884
0.8	6	5	4	3	4	5	25	300	100	176.612	54.925	0.680	0.278	309.227
0.8	6	5	5	3	4	5	25	300	100	154.772	46.030	0.741	0.294	209.525
0.8	6	5	6	3	4	5	25	300	100	137.838	39.459	0.793	0.308	153.787
0.8	6	5	6	4	4	5	25	300	100	124.298	51.791	0.838	0.309	137.086
0.8	6	5	6	5	4	5	25	300	100	113.207	63.578	0.880	0.312	123.173
0.8	6	5	6	6	4	5	25	300	100	103.946	74.694	0.918	0.316	111.574
0.8	6	5	6	7	4	5	25	300	100	96.092	85.101	0.955	0.319	101.828
0.8	6	5	6	7	5	5	25	300	100	96.092	62.820	0.955	0.408	103.650
0.8	6	5	6	7	6	5	25	300	100	96.092	48.882	0.955	0.491	106.718
0.8	6	5	6	7	7	5	25	300	100	96.092	39.621	0.955	0.562	111.528
0.8	6	5	6	7	8	5	25	300	100	96.092	33.111	0.955	0.619	119.007
0.8	6	5	6	7	8	6	25	300	100	100.377	34.487	0.944	0.613	126.459
0.8	6	5	6	7	8	7	25	300	100	104.894	35.927	0.935	0.608	134.188
0.8	6	5	6	7	8	8	25	300	100	109.643	37.431	0.927	0.603	142.195
0.8	6	5	6	7	8	9	25	300	100	114.624	39.000	0.520	0.599	150.480
0.8	6	5	6	7	8	9	30	300	100	152.867	50.544	0.926	0.601	119.285
0.8	6	5	6	7	8	9	35	300	100	196.908	63.698	0.932	0.603	255.040
0.8	6	5	6	7	8	9	40	300	100	246.748	78.460	0.937	0.605	317.735
0.8	6	5	6	7	8	9	45	300	100	302.387	94.832	0.941	0.606	387.373
0.8	6	5	6	7	8	9	45	400	100	302.391	95.059	0.941	0.599	387.421
0.8	6	5	6	7	8	9	45	500	100	302.391	95.287	0.941	0.591	387.470
0.8	6	5	6	7	8	9	45	600	100	302.397	95.515	0.941	0.584	387.518
0.8	6	5	6	7	8	9	45	700	100	302.400	95.743	0.941	0.577	387.567
0.8	6	5	6	7	8	9	45	700	300	302.400	96.075	0.941	0.566	387.567
0.8	6	5	6	7	8	9	45	700	500	302.400	96.406	0.941	0.557	387.567
0.8	6	5	6	7	8	9	45	700	700	302.400	96.738	0.941	0.547	387.567
0.8	6	5	6	7	8	9	45	700	900	302.400	97.070	0.941	0.538	387.567

The recruitment parameter λ_1 varies from 2 to 6 units, the variance of the number of employees in grade 1 is increasing from 79.679 to 190.093, when other parameters are fixed at (0.8,1,2,3,4,5,25,300, and 100) for $(t,\lambda_2,\alpha,\beta,\gamma,a,b,N_0,M_0)$. Similarly, the variance of the number of employees in grade 2 in the organization increasing from 42.417 to 74.240 for given values of the other parameters.

The recruitmentparameter λ_2 varies from 1 to 5 units; the variance of the number of employees in grade 1 is increasing from 190.093 to 266.407, when other parameters are fixed at (0.8,6,2,3,4,5,25,300, and 100) for $(t,\lambda_2,\alpha,\beta,\gamma,a,b,N_0,M_0)$. Similarly, the variance of the number of employeesin grade 2 in the organization increases from74.240 to 89.256 for given values of the other parameters. The increase in the variance of the number of employees in grade 2 is moderate, when compared to that of grade 1 employees.

When the promotion rate from grade 1 to grade 2(β) varies from 3 to 7, the variance of the number of employees in grade 1 reduces from 137.838 to 96.092, when other parameters are fixed at (0.8,6,5,6,4,5,25,300, and 100) for $(t,\lambda_1,\lambda_2,\alpha,\gamma,a,b,N_0,M_0)$. Similarly, the variance of the number of employees in grade 2 in the organization increasing from39.458 to 85.101 for given values of the other parameters.

As the leaving rate of grade 2 employee (γ) varies from 4 to 8 units, the variance of the number of employees in grade 1 is 96.092 remains unchanged, when other parameters are fixed at (0.8,6,5,6,7,5,25,300, and 100)for $(t,\lambda_1,\lambda_2,\alpha,\beta,a,b,N_0,M_0)$. Similarly, the variance of the number of employees in grade 2 in the organization reduces from85.101 to 33.111 for given values of the other parameters.

As the uniform batch size distribution parameter (a) varies from 5 to 9 units the variance of the number of employeesin the grade 1 is increasing from 96.092 to 114.624, when other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 25, 300 and 100)for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,b,N_0,M_0)$. Similarly, the variance of the number of employeesin grade 2 in the organization is increasing from 33.111 to 38.999 for given values of the other parameters. It is further observed that the batch size distribution parameter (b) varies from 25 to 45 units the variance of the number of employeesin grade 1 is increasing from 114.624 to 302.387, when other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 9, 300 and 100) for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,b,N_0,M_0)$. Similarly, the variance of the number of employees in grade 2 in the organization is increasing from 38.999 to 94.831 for given values of the other parameters.

It is further observed that the initial number of employees in grade 1 (N_0) varies from 300 to 700 units, the variance of the number of employeesin grade 1 increasing from 302.387 to 302.399 when other parameters are fixed at (0.8,6,5,6,7,8,9,45, and 100)for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,M_0)$. Similarly, the variance of the number of employeesin grade 2 in the organization increasing from 94.831 to 95.743 for given values of the other parameters.As the initial number of employees in grade 2 (M_0) increases from 100 to 900, the variance of the number of employeesin grade 1 in the organization have no influence, when the other parameters are fixed at (0.8, 6, 5, 6, 7, 8, 9, 45, and 700)for $(t,\lambda_1,\lambda_2,\alpha,\beta,\gamma,a,b,N_0)$, the initial number of employees in grade 2 increases from 100 to 900, the variance of the number of employeesin grade 2 increasing from 95.743 to 97.070 for given values of the other parameters.

6. COMPARATIVE STUDY

A comparative study of the developed model with that of homogeneous compound Poisson

t	Characteristics Measured	Non-homogeneous bulk recruitment	Homogeneous bulk recruitment	Difference	Percentage of Variation
0.4	L_1	46.46977	45.78857	0.68120	1.48771
	L_2	82.55608	82.48813	0.06795	0.08238
	W_1	9.29395	9.15771	0.13624	1.48771
	W_2	20.63902	20.62203	0.01699	0.08239
	V_1	96.85002	88.72364	8.12638	9.15920
	V_2	75.91592	74.98740	0.92852	1.23823
0.5	L_1	31.08224	30.13290	0.94934	3.15051
	L_2	64.62163	64.39127	0.23036	0.35775
	W_1	6.21645	6.02600	0.19045	3.16047
	W_2	16.15541	16.09782	0.05759	0.35775
	V_1	87.94507	77.11253	10.83254	14.04770
	V_2	66.57298	65.04099	1.53199	2.35542
0.6	L_1	21.86727	20.63740	1.22987	5.95942
	L_2	49.67922	49.26461	0.41461	0.84160
	W_1	4.37345	4.12748	0.24597	5.95933
	W_2	12.41981	12.31615	0.10366	0.84166
	V_1	82.67414	69.10482	13.56932	19.63585
	V_2	57.26310	55.03984	2.22326	4.03936
0.7	L_1	16.39615	14.87803	1.51812	10.20377
	L_2	37.92079	37.30774	0.61305	1.64322
	W_1	3.27923	2.97561	0.30362	10.20362
	W_2	9.48020	9.32694	0.15326	1.64320
	V_1	80.21318	63.89281	16.32037	25.54336
	V_2	49.06667	46.09565	2.97102	6.44534
0.8	L_1	13.19589	11.38480	1.81109	15.90797
	L_2	29.01135	28.19066	0.82069	2.91121
	W_1	2.63916	2.27696	0.36220	15.90717
	W_2	7.25284	7.04767	0.20517	2.91117
	V_1	79.67943	60.60094	19.07849	31.48217
	V_2	42.41695	38.66297	3.75398	9.70950

Table 3: Comparative study of model with non- homogeneous and homogeneous compound bulk recruitment models

bulkarrivals iscarried by taking $\lambda_2 = 0$ in the model and different values of t. Table 4 shows the points study of havingmodels with homogeneous and non-homogeneous compound Poisson bulk arrivals.From the table 3, it can also be observed that as time increases, the percentage variation of the performancemeasures between the models also increases. The model with non-homogeneous compound Poisson bulk arrivals has higher utilisation than the model with homogeneous compound Poisson bulk arrivals. It

can also be observedthat the assumption of non-homogenous compound Poisson arrivals has a significant influence on all theperformance measures of the model. Time also has a significant effect on the system performancemeasures, and this model can predict the performance measures more accurately. This model also includessome of the earlier models as particular cases.

7. CONCLUSION:

This paper addresses and analyses a two-graded manpower model with non-homogeneous bulk recruitment. It is assumed that the manpower system consisting of two grades and the recruitment is done in the first grade with groups of random size depending on time. The recruitment process is characterized by non-homogeneous compound Poisson process. The explicit expressions of the system characteristics such as average number of employees in each grade, the mean duration of the grade size distribution, the variance of the grade size distribution, and the covariance between the number of employees in both the grades are derived. The sensitivity analysis of the model revealed that the bulk size recruitment parameters have significant influence on the system performance measures. The performance measures can be predicted more accurately and realistically using the developing model when the recruitment is done in bulk and time dependent. This model also includes several of the earlier existing manpower models as particular cases for specific values of particular cases. This model can also be extended by considering cost aspects and deriving the optimal values of the model parameters which will be pursued elsewhere.

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