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ON ANALYZING THE MELODIC STRUCTURE OF A NORTH INDIAN RAGA THROUGH AN ARIMA MODEL

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ABSTRACT

Time series is a series of observations in chronological order. The argument or predictor is time which may or may not be the time of clock, i.e. , it could well be simply the instance represented by indices 1, 2, 3…which is certainly true for a sequence of notes in a raga depicting its melodic structure with some musical sense of completeness. The dependent variable is the pitch characterizing the note. The present study shows how an ARIMA model can effectively analyze the melodic structure of a North Indian raga**.** While the model itself is reserved for prediction, the lag of unity in the model also suggests a Markov chain of order one which can then be used to simulate the raga sequence on a computer. In this way our study supports both prediction and composition**.**

Keywords: Time series; ARIMA model, raga; melodic structure; Markov chain; transition probability matrix

1. INTRODUCTION

A raga is a melodic structure with fixed notes and a set of rules characterizing a certain mood conveyed by performance. However, we can also statistically analyze the melodic structure of the raga without going into performance by considering a sequence of notes that capture the emotion of the raga in some sense of completeness as given by Indian music theory. The advantage of this study is that it is fairly general and does not depend on any specific artist. Thus our conclusion would also be fairly general giving an overview of the raga from a theoretical angle with special emphasis on modeling. Another advantage is that for performance data, the argument would yield unequal spaced onsets (actual point of arrival) of notes complicating the analysis. Before describing the modeling, it is worth taking a look at some of the basic features of the North Indian raga Multani, which we shall analyze here**.**

- **Raga: Multani [Dutta(2006)]**

Thaat: Todi (Thaat is a kind of grouping of ragas according to the specific notes used)
Aroh (ascent): *N* S, g m P, N **S**
Awaroh(descent)**: S** N d Pm g r S
Jati(another grouping of raga reflecting no. of distinct notes used in ascent-descent): Aurabh-

Sampoorna (5 distinct notes used in ascent, 7 in descent)
Vadi Swar (most important note): P
Samvadi Swar (second most important note): S
Prakriti (nature): restful
Pakad (catch): *N* S, m g, P g, r S
Nyas swars (Stay notes): S g P
Time of rendition: 3 PM to 6 PM

ABBREVIATIONS:-

The letters S, R, G, M, P, D and N stand for Sa (always Sudh), Sudh Re, Sudh Ga, Sudh Ma, Pa (always Sudh), Sudh Dha and Sudh Ni respectively. The letters r, g, m, d, n represent Komal Re, Komal Ga, Tibra Ma, Komal Dha and Komal Ni respectively.

A note in Normal type indicates that it belongs to middle octave; if in italics it is implied that the note belongs to the octave just lower than the middle octave while a bold type indicates it belongs to the octave just higher than the middle octave. Sa is the tonic in Indian music.

Here is Rajan P. Parrikar's description of this raga, reproduced with permission except for the notation for octave which is ours (http://www.parrikar.org/raga-central/multani). Parrikar is a recognized expert in Indian classical music.

"Multani is among the 'big' Ragas, highly regarded by the afficionado of vocal music for its weighty mien and wide melodic compass. Although its basic swaric material is drawn from the Todi thAT - **S r g m P d N** - it carries no hint or trace of the Todi Raganga. Multani has a highly evolved and independent swaroopa all its own.

Let us examine the Raga lakshaNAs. To recap the notation convention: a swara enclosed in brackets represents a kaNa (grace) to the swara immediately following it.

S**,** *N* S g (S)r(*N*)S

Both r and d are dropped in Arohi prayogas; the avaroha is sampoorNa. The peculiar ucchAraNa (intonation) of r mediated by a kaNa (grace) of S is vital to Multani. Recall the vastly different behavior of Todi in this region, with its deergha r and an intimate coupling with g**.** An inopportune nyAsa on r spells the kiss of death for Multani. Further divergence between Todi and Multani in matters concerning g is suggested in the next tonal strip.

N S (m)g m P**,** m P (m)g**,** m g (S)r(*N*)S

Characteristic of Multani is the Arohi ucchAraNa of g: it is tugged with m as in (m)g (m)g m P**.** Since g is approached from m, it has the effect of raising the shruti of g to a location above its nominal komal value. This in turn elevates the shruti of r. These microtonal nuances are later demonstrated tellingly by Pandit Ramashreya Jha "Ramrang." The teevra madhyam in Multani is very close to the pancham, in the latter's penumbra, as it were.

P**,** (m)g P**,** P (P)d(m)P**,** P (m)g**,** m g m g (S)r(*N*)S

The treatment of d is congruent to that accorded r. The purNAvritti (repetition) of m g in avarohi prayogas is a point of note. As is the langhan of m, occasionally from g to P and more often through a meeND-laden avarohi P to g. The importance of a powerful pancham to Multani should be evident by now.

(m)g m P N**,** N**,** **S, S g (S)r**(N)**S**

The uttarAnga launch proceeds thus, with a deergha N. The sharp m P N curve presents a source of discomfort to many a Khayal singer especially in the faster passages; the tendency to instead detour through m d N must be checked.

S, N **S** N d P**,** m P (m)g**,** m g m g (S)r(*N*)S

This sentence completes the overall avarohi picture."

- See also Jairazbhoy (1971). Readers knowing Western music but new to Indian music are referred to a useful website by C.S. Jones (http://cnx.org/content/m12459/1.6/)

We assume that the reader has a sound knowledge of statistics and is familiar with an ARIMA model. So we move straight to the statistical analysis. Details of the working of an ARIMA model have been, however, briefed in appendix B. We have also done a melody analysis of the structure of raga Multani using statistics and the interested reader is referred to Chakraborty et. al. (2009).

2. Statistical Analysis

Taking the tonic Sa at C conventionally, the twelve notes in the middle octave can be represented by the numbers 0 to 11 respectively.

Sa of the next higher octave will be assigned the number 12 etc while Ni (sudh) of the lower octave (before middle) is assigned the number -1 etc.

C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B	
S	*r*	*R*	*g*	*G*	*M*	*m*	*P*	*d*	*D*	*n*	*N*	(lower octave)
-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	

S	r	R	g	G	M	m	P	d	D	n	N	(middle octave)
0	1	2	3	4	5	6	7	8	9	10	11	

S	**r**	**R**	**g**	**G**	**M**	**m**	**P**	**d**	**D**	**n**	**N**	(higher octave)
12	13	14	15	16	17	18	19	20	21	22	23	

The sequence of notes in raga Multani depicting its melodic structure is taken from a standard text [Dutta (2006)]. This was coded with the help of above and is detailed in the appendix A. At the identification stage the correlogram of realization shows that acf in fig. 1(A) decrease very sharply revealed the presence of stationarity so there is no need to transform the data. However seasonality to some extent appeared in fig. 1 (A) but no statistical significance is there to having it under the consideration. Fig. 1 (B) indicates that pacf cut-off at first lag and damping towards zero. Now the all model will be tentative model for the value p = 1, d = 0 and q =1. The tentative model will be ARIMA (1,0,0), ARIMA (0,0,1) and ARIMA (1,0,1). Table-1 Contains all the fitted tentative ARIMA model with its selection criterion.

Among the Fitted ARIMA models ARIMA (1, 0, 1) model has its AR (Autoregressive) coefficient significant and MA (Moving average) coefficient non-significant with lower value of MAE (Mean Absolute Error), whereas ARIMA (0, 0, 1) model have its MA coefficient significant but failed in Ljung-Box test of residuals independence. It also have the lower value of R^2 and greater value of MAE and MaxAPE (Maximum Absolute Percentage error). In the context of ARIMA (1, 0, 0), this model have its AR coefficient significant, higher value of R^2 (58.3%) and lower value of RMSE (Root Mean Square Error), MAE, MaxAPE with successfully passing of residual assumption of independence. Finally it is observed that ARIMA (1, 0, 0) model is the best fitted model among the algebraic family of ARIMA because it fulfills all the selection criteria with independence of residuals.

Identification

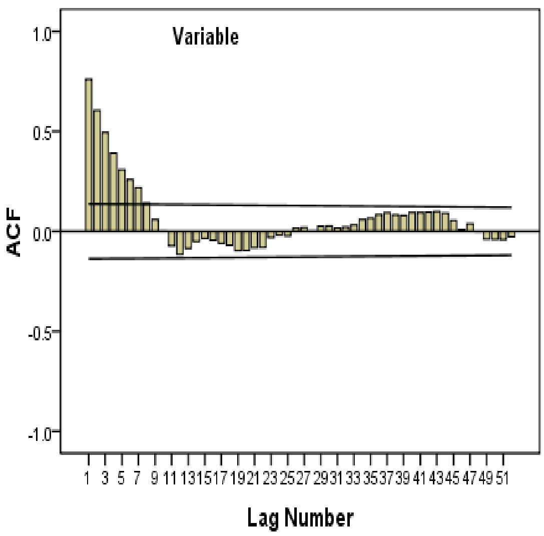

Fig.-1(A) **ACF**

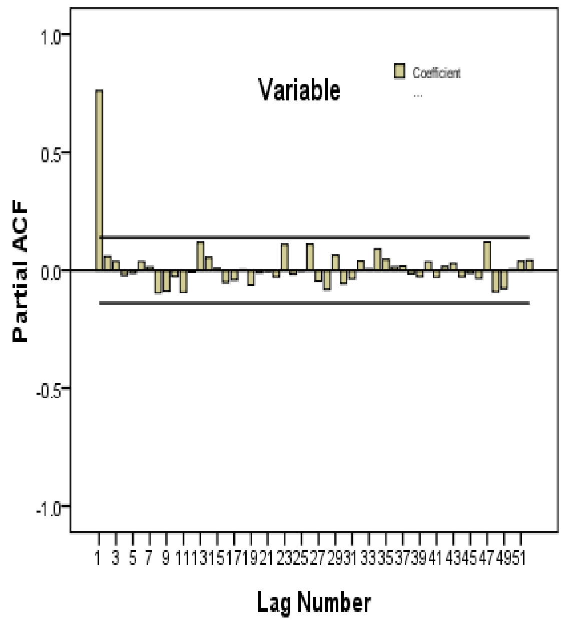

Fig.-1(B) PACF

Fitting of Autoregressive Integrated model moving average Model

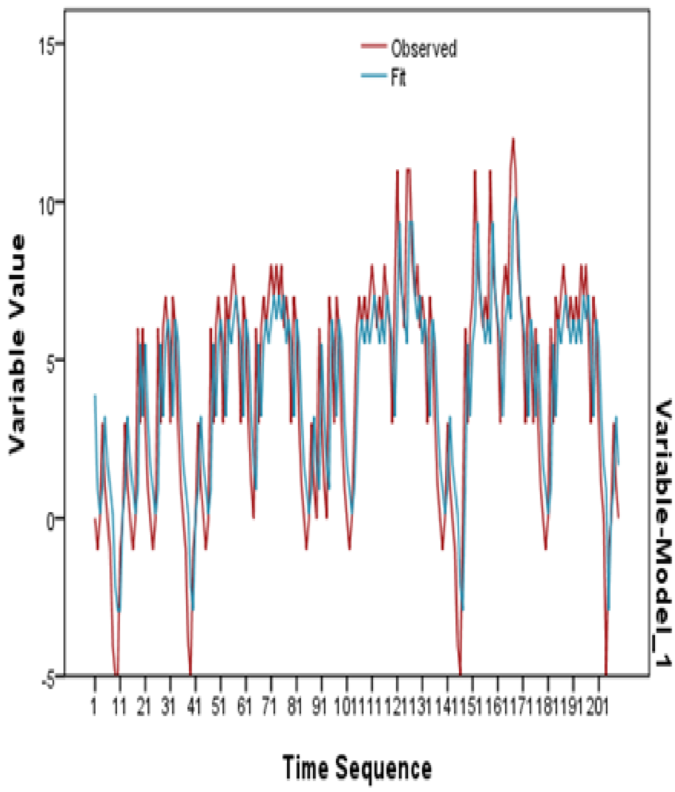

Fig.-2(A)

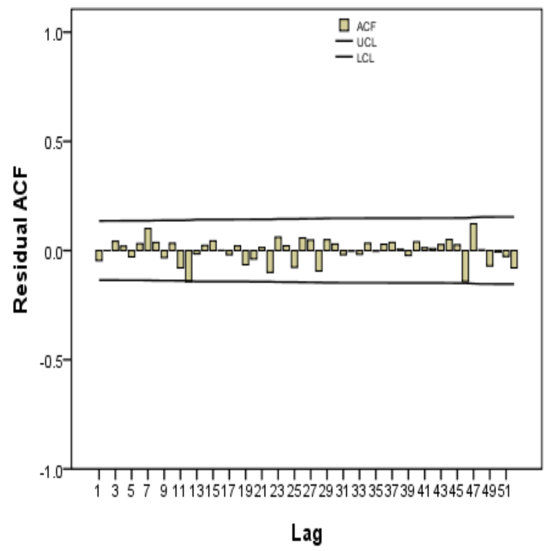

Fig.-2(B)

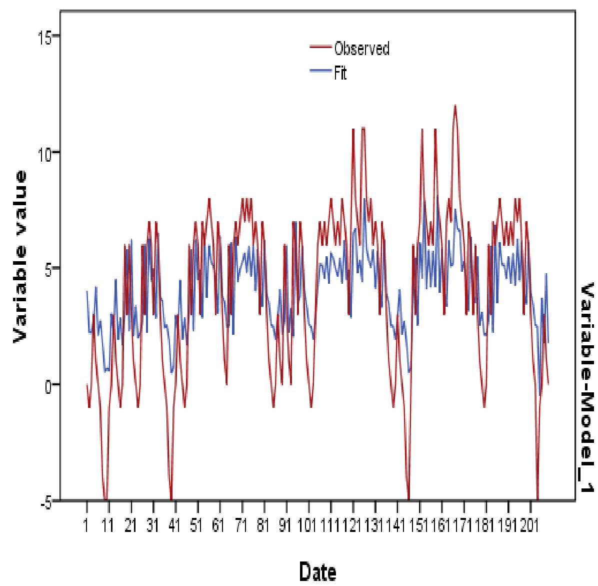

Fig.-3(A)

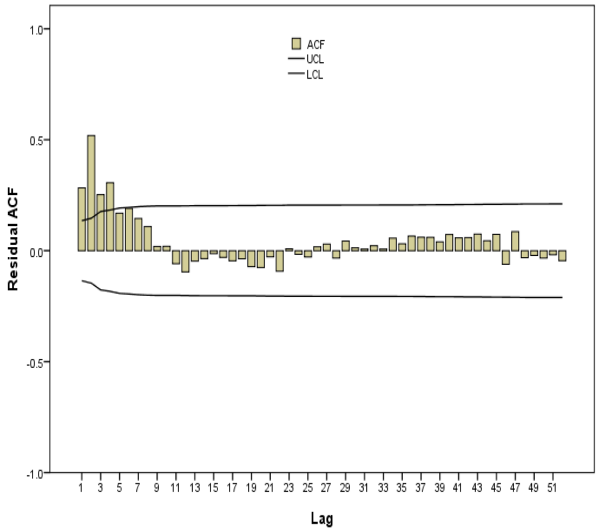

Fig.-3(B)

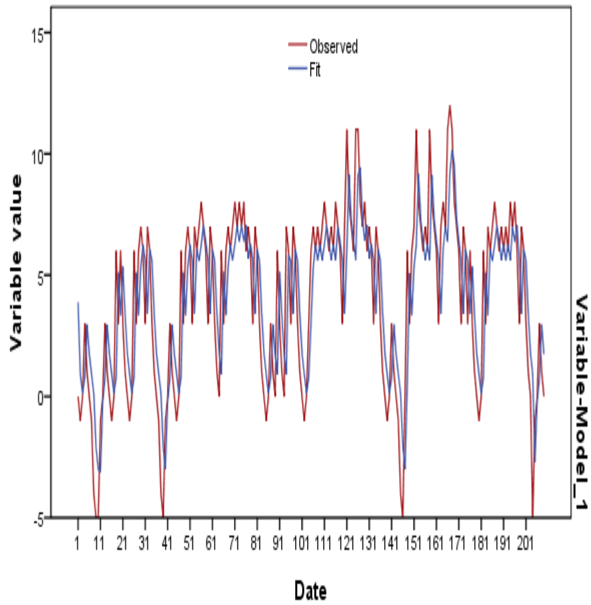

Fig.-4(A)

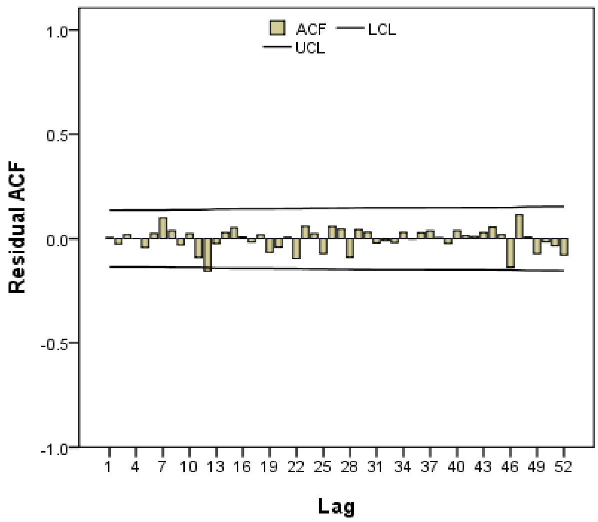

Fig.-4(B)

3. Utility of our study

The fitted model is

Table:1 Fitted ARIMA models

Model	AR	MA	CONST	R^2	RMSE	MAE	MaxAPE	Ljunj-Box
ARIMA(1,0,0)	0.767*	--	3.911*	58.3	2.365	1.907	221.246	11.042 ^{NS}
ARIMA(0,0,1)	--	-0.600*	4.021*	41.0	2.815	2.355	387.183	134.168*
ARIMA(1,0,1)	0.805*	0.092 ^{NS}	3.885*	58.5	2.365	1.901	241.197	11.828 ^{NS}

$$\mathbf{Y_t = 3.911^* + 0.767^*Y_{t-1} + \varepsilon_i}$$

This equation satisfies all the criteria regarding the best fitted model. *Since the lag is 1, we are motivated to compute the transition probability matrix of a first order Markov chain*

[Wikipedia]. Deciding the order in a Markov chain is crucial as higher order need to be beneficial (Nierhaus, 2008). This means we are taking that the probability of the next note depends on what the present note is. Thus the probability P(g/r) gives the probability that the next note would be g given that the present note is r. This is calculated by the number of times r has been followed by g divided by the number of times r has occurred in the entire sequence. However, in cases where the present note is S, since this is the last note in the sequence, the probability P(r/S) for example would be calculated by the number of times S has been followed by r divided by one subtracted from the number of times S has occurred. This is because for the last occurrence of S, there is no information of the next transition, so the possibilities of a transition are one less than the number of occurrences of S. Table 3 gives the transition probability matrix of Multani sequence. Table 2 gives the overall note distribution. We emphasize here that the ARIMA model itself be reserved only for prediction, while the transition probability matrix for the Markov chain of order one (validated by the lag of unity in the ARIMA model) be used to simulate compose a Multani sequence by running simulations on a computer. In this way our study supports both prediction and composition. The following algorithm is proposed to compose a Multani note sequence.

Step 1: Without any loss in generality, for example, take the note at instance 1 to be the tonic Sa.

Step 2: Using the transition probabilities at Sa (see row one of table 3), one simulates the next note. The strategy is to generate a uniform variate X in the range
[0, 1] and, depending on whether X falls in [0, 7/28], [7/28, 14/28], [14,28, 16/28] or [16/28, 28/28], the note at instance 2 is obtained as either g, m, P or N with respective probabilities 7/28, 7/28, 2/28 and 12/28. If the note at instance 1 is other than Sa, use the corresponding transition probability row as given in table 3.

Step 3: Using the note simulated at instance 2, and using the corresponding transition probability row of this note, the next note at instance 3 is simulated etc.

Caution: The octaves of the notes should be decided using Indian music theory. See also the next section.

Table 2: Overall Note distribution in RAGA MULTANI

Notes	**No. of Occurrences**
S	29
r	15
g	33
m	46
P	46
d	19
N	21

Table 3: The Transition Probability Matrix of Raga Multani:

	S	r	g	m	P	d	N
S	0	0	7/28	7/28	7/28	0	12/28
r	1	0	0	0	0	0	0
g	0	15/33	0	7/33	11/33	0	0
m	0	0	26/46	0	16/46	2/46	2/46
P	1/46	0	0	29/46	1/46	9/46	6/46
d	0	0	0	3/19	16/19	0	0
N	12/21	0	0	0	0	8/21	1/21

4. CONCLUSION

Critics often blame statisticians that their analysis only supports music prediction but not music composition. For example, in a blog titled *Limitations of Statistical Methods for*

Analyzing Music [Dorrell (2005)], it is quite clearly stated "…In fact, it is safe to conclude that anyone who claims to have a good *predictive* algorithm, but who does not have a corresponding *generative* algorithm (i.e. an algorithm to *compose* music), probably has *not* discovered the secret of what music is." The use of a predictive ARIMA model of lag one validating a Markov chain of first order which in turn can effectively compose a raga sequence is thus valuable both from a statistical and a musical perspective. One can certainly play such a simulated sequence in some instrument such as a piano and thereby select other features like duration and loudness of the notes subjectively. The second part, like selection of octaves of the notes, should again be done using music theory. The result is a "semi-natural" composition that can arguably capture the raga emotion better than an artificial composition created exclusively by the computer.

As a final comment, our semi-natural composition may not yield a pure Multani sequence, but will it not yield a tune which a good composer can easily fit, after suitable modifications, in a play or a movie for example? Countless number of songs in Indian movies, for example, are based on ragas and although, barring a few, they do not maintain the raga rules strictly, yet their role in promoting classical music among the laymen cannot be denied. Even a second grade composer who lacks original ideas can perhaps do better. We strongly advise these guys to compose raga based semi-natural tunes as suggested here which would be better than indiscriminately borrowing tunes from here and there and claiming them to be "original" at worst or even "inspirational" at best. To the question whether the results are true for the text only [4] or the raga we clarify that as we are analyzing a musical structure, and not a performance, and as the text [4] is a very standard text and not just any text,it is the raga that is represented. To the question why we have applied ARIMA model to discrete data, we quote a few lines(p.21-22) of [1]:-

"A time series is a set of observations generated sequentially in time. If the set is continuous, the time series is said to be continous. If the set is discrete, the time series is said to be discrete.....in this book we consider only discrete time series where observations are made at a fixed interval".on page 2, they write "For example, in a sales forecasting problem....." indicating the response can also be discrete!

ARIMA madels are described on p.89 onwards in [1]. The three fundamental assumptions of ARIMA madels do not include the fact that t and Y(t) have to be continuous. The assumptions only say (1) there must be at least 50 data points (2)the series is stationary and (3) the series is homoscedastic. And these can be achieved both for discrete and continuous data. For these three fundamental assumptions, we refer the reader to chap. 4 of the web book of regional science by Garrett and Leatherman (http://www.rri.wvu.edu/WebBook/Garrett/chapterfour.htm)

[Concluded]

APPENDIX A: Notes occurring sequentially in Multani (index t & corresponding pitch Y_t)

Table 4: RAGA MULTANI NOTE SEQUENCE

t	Y_t	t	Y_t		Y_t	t	Yt	t	Y_t
1	0	44	0	87	3	130	6	173	7
2	-1	45	-1	88	1	131	7	174	6
3	0	46	0	89	0	132	6	175	3
4	3	47	6	90	6	133	3	176	6
5	1	48	3	91	3	134	7	177	3
6	0	49	6	92	1	135	6	178	1
7	-1	50	7	93	0	136	3	179	0
8	-4	51	6	94	7	137	1	180	-1
9	-5	52	3	95	6	138	0	181	0
10	-5	53	7	96	3	139	-1	182	6
11	-1	54	6	97	7	140	0	183	3
12	0	55	7	98	6	141	3	184	7
13	3	56	8	99	3	142	1	185	6
14	1	57	7	100	1	143	0	186	7
15	0	58	6	101	0	144	-1	187	8
16	-1	59	3	102	-1	145	-4	188	7
17	0	60	7	103	0	146	-5	189	6
18	6	61	6	104	3	147	0	190	7
19	3	62	3	105	6	148	6	191	6
20	6	63	1	106	7	149	3	192	7
21	3	64	0	107	6	150	6	193	6

22	1	65	6	108	7	151	7	194	8
23	0	66	3	109	6	152	11	195	7
24	-1	67	6	110	7	153	8	196	8
25	0	68	7	111	8	154	7	197	6
26	6	69	6	112	7	155	6	198	3
27	3	70	7	113	6	156	7	199	7
28	6	71	8	114	7	157	6	200	6
t	**Y_t**	**t**	**Y_t**	**t**	**Y_t**	**t**	**Yt**	**t**	**Y_t**
29	7	72	7	115	6	158	11	201	3
30	6	73	8	116	8	159	8	202	1
31	3	74	7	117	7	160	7	203	0
32	7	75	8	118	6	161	6	204	-5
33	6	76	6	119	3	162	3	205	-1
34	3	77	7	120	7	163	7	206	0
35	1	78	6	121	11	164	8	207	3
36	0	79	3	122	8	165	7	208	1
37	-1	80	7	123	7	166	11	209	0
38	-4	81	6	124	6	167	12		
39	-5	82	3	125	11	168	11		
40	-1	83	1	126	11	169	8		
41	0	84	0	127	8	170	7		
42	3	85	-1	128	7	171	6		
43	1	86	0	129	8	172	3		

APPENDIX B

Modeling Through Univariate Box-Jenkins ARIMA (Autoregressive Integrated Moving Average Model):

ARIMA is one of the popular linear models in time series forecasting during the past three decades. Time series forecasting is an important area of forecasting in which past observations of the same variable are collected and analyzed to develop a model describing the underlying relationship. The modeling approach is particularly useful when little knowledge is available on the underlying data generating process or when there is no satisfactory explanatory model that relates the prediction variable to other explanatory variables. Much effort has been devoted over the past several decades to the development and improvement of time series forecasting model. The popularity of ARIMA model is due to its statistical properties as well as the well known Box-Jenkins methodology in model building process. In addition, various exponential smoothing model can be implemented by ARIMA models. Although ARIMA models are quite flexible in that they can represent several different types of time series, i.e. pure autoregressive (AR), pure moving average (MA) and combined AR and MA (ARMA) series their major limitation is the pre-assumed linear form of the model. [See Box and Jenkins (1970) and McKenzie (1984)]

There are three stages in fitting of ARIMA model are illustrated below:-

Stage 1- We use the estimated acf (autocorrelation function) and pacf (partial autocorrelation function) as guides to choosing one or more ARIMA models that seem appropriate. The basic idea is this: every ARIMA model has a theoretical acf and pacf associated with it. At the identification stage we compare the estimated acf and pacf calculated from the available data with various theoretical acf's and pacf's then tentatively choose the model whose theoretical acf and pacf most closely resemble the estimated acf and pacf of the data series.

Stage 2- Estimation- At this stage we get precise estimate of the coefficients of the model chosen at the identification stage. We fit this model to the available data series to get estimates. This stage provides some warning signals about the adequacy of the model. In particular, if the estimated coefficients don't satisfy certain mathematical inequality conditions, that model is rejected.

Stage 3-Diagnostic test- Box and Jenkins (1970) suggested some diagnostic checks to help to determine if an estimated model is statistically adequate. A model that fails this diagnostic test is rejected.

Autoregressive Integrated Moving Average Model

A time series is a set of values of a continuous variable Y (Y_1, Y_2… Y_n), ordered according to a discrete index variable t (1,2,..,n). However, it must be clearly stated that this direct reference to time is not required; a different meaning can be attributed to the index variable, provided that it is able to order the Y values. **With this understanding, it is certainly possible to model the structure of a North Indian raga where the notes come in a sequence**. In general, in a given time series the following can be recognizing and separated:

- A regular, long-term component of variability, termed trend that represent the whole evolution pattern of the series.
- Stationarity is a critical assumption of time series analysis, stipulating that statistical descriptors of the time series are invariant for different ranges of the series. Weak stationarity assumes only that the mean and variance are invariant.
- A regular short term component whose shape occurs periodically at intervals of s lags of the index variable, currently known as seasonality, because this term is also derived by application in economics.
- An AR (p) i.e. autoregressive component of order p which relates each value Z_t = Y_t (trend and seasonality) to the p previous Z values, according to the following linear relationship

$$Z_t = \varphi_1 Z_{t-1} + \varphi_2 Z_{t-2} + \text{............} + \varphi_p Z_{t-p} + \varepsilon_i$$

……..(1)

Where φ_i (i= 1,……,p) are parameters to be estimated and ε_i is the residual terms.

A MA (q) i.e. moving average component of q order, which relates each Z_t values to the q residual of the q previous Z estimates

$$Z_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta 2 \varepsilon_{t-2} \text{............} - \theta_q \varepsilon_{t-q}$$

……….(2)

Where θ_i (i= 1,……,p) are parameters to be estimated. According to Box-Jenkins a highly useful operator in time series theory is lag or backward linear operator (B) defined by $BZ_t = Z_{t-1}$

Consider the result of applying the lag operator twice to a series:

$B(BZ_t) = BZ_{t-1} = Z_{t-2}$

Such a double indication is indicated by B^2 and in general for any integer k, it can be written

$B^k Z_t = Z_{t-k}$

By using the backward operator, equation (1) can be rewritten as

$$Z_t - \varphi_1 Z_{t-1} - \varphi_2 Z_{t-2} - \text{..........} \text{..} - \varphi_p Z_{t-p} - \varepsilon_t = \varphi(B)$$

……(3)

Where φ (B) is the autoregressive operator of order p defined by

$$\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \text{..........} \text{.......} - \varphi_p B^p$$

Similarly, equation (2) can be written as

$$Z_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta 2 \varepsilon_{t-2} \text{...........} - \theta_q \varepsilon_{t-q}$$

= θ(B)……….(4)

Where θ(B) indicates the moving average operator of q order defined by

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \text{.........} - \theta_q B^q$$

The Autoregressive (AR) and Moving average (MA) component can be combined in an autoregressive moving average ARMA (p, q) model

$$Z_t = \varphi_1 Z_{t-1} + \varphi_2 Z_{t-2} + \dots + \varphi_p Z_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q}$$

Or in lag operator form

$$(1-\varphi_1 B - \varphi_2 B^2 - \dots - \varphi_p B^p) Z_t = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) \varepsilon_t$$

Finally,

$\Phi (B) Zt = \theta (B)\varepsilon$………..(5)

The value of AR (φ) term always should be keep in the equation with positive sign and negative sign attached to MA (θ_1) which is merely a convention. It makes no difference whether we use a negative or positive sign. There is one more condition about the coefficients of model that they should not exceed unity, which is known as invertibility condition.

From the tentative ARIMA models the best models were selected which has significant coefficients with lower values of Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Maximum Absolute Percentage error (Max APE) and residual should be independent in nature which is tested by Ljung-Box test.

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