International Journal of Advanced Computer and Mathematical Sciences.
Vol 1, Issue 1, Dec, 2010, pp 52-57. http://bipublication.com

STOCHASTIC MODELLING OF DAILY RAINFALL AT ADUTHURAI

SAMUEL SELVARAJ R.[1], TAMIL SELVI S. [2]
PRESIDENCY COLLEGE, DEPARTMENT OF PHYSICS, CHENNAI.[1]
DHANALAKSHMI SRINIVASAN COLLEGE OF ENGINEERING & TECHNOLOGY,
MAMALLAPURAM, CHENNAI.[2]

ABSTRACT
An application of stochastic process for describing and analysing the daily rainfall pattern at Aduthurai is presented. A model based on the first-order Markov chain was developed. The model used in this study consists of rainfall occurrence model and rainfall magnitude model. Results of the study suggests that first order Markov chaining with two parameters gamma distributions were found to be adequate to generate daily rainfall sequences at Aduthurai.

Keywords: daily rainfall, first order Markov chain, rainfall occurrence model, rainfall magnitude model

[I] INTRODUCTION

The natural systems are so complex that no exact laws have yet been developed that can explain completely and precisely the natural hydrological phenomena [3]. Rainfall affects weather variables affecting the growth and development of crops, and the spread of diseases and pests. Hence rainfall forms the principal input to all agronomic models. The future probability of occurrence of rainfall can be used for crop planning and management and water management decisions, as the risk due to weather uncertainty can be reduced.

The Markov models are frequently proposed to quickly obtain forecasts of the weather "states" at some future time using information given by the current state. One of the applications of the Markov chain models is the daily precipitation occurrence forecast.

One of the statistical techniques is the Markov chain used to predict precipitation on short term, at meteorological stations. The Markov chain models have two advantages:
(1) the forecasts are available immediately after the observations are done because the use as predictors only the local information on the weather and (2) they need minimal computation after the climatological data have been processed.

A first-order Markov chain is one in which knowing one variable (like cloudiness, precipitation amount, temperature, fog, frost, wind) at time t is sufficient to forecast it at some later time [6].
Aduthurai is a small town situated in Thanjavur district of Tamil Nadu. Aduthurai (11° N, 79°5' E) situated near Kumbakonam on the southern side of river Veerachozhan and Cauvery of Tamil Nadu has an average annual rainfall of 950mm. It receives rainfall mostly during the north east monsoon (October - December). The objective of this study is to simulate daily rainfall sequences for Aduthurai to use as inputs to crop, hydrologic and water resources models.

[II] REVIEW OF LITERATURE

Markov chains specify the state of each day as ‘wet’ or ‘dry’ and develop a relation between the state of the current day and the states of the preceding days. The order of the Markov chain is the number of preceding days taken into account. Most Markov chain models referred in the literature are first order [8]. Many authors have used Markov chains to model the daily occurrence of precipitation. Gabriel and Neumann (1962) [4] analyzed the occurrence of rain by fitting a two-state, first-order Markov chain. Carey and Haan (1978) [2] used multi-state Markov chain models to generate daily rainfall depths. The low order chains are mostly preferable for two reasons. The number of parameters to be estimated is kept to be a minimum, so that better estimates are obtained. Second, the subsequent use of the fitted model to calculate other quantities, such as the probabilities of long dry spells, is simpler. The distribution of the amounts of rainfall on wet days is usually modelled by gamma distributions [1].

[III] MODEL STRUCTURE

The model consists of (i) rainfall occurrence model and (ii) rainfall magnitude model.

3.1. Rainfall occurrence model

A Russian mathematician, Markov, introduced the concept of a process (later named after him ‘a Markov process’) in which a sequence or chain of discrete states in time for which the probability of transition from one state to any given state in the next step in the chain depends on the condition during the previous step [9]. Daily rainfall includes the occurrence of rain and the amount of rain. Markov chain process is used to find out rainfall occurrence . Once rainfall occurrence has been specified , rainfall amount is then generated using a Gamma or mixed Exponential distribution [7].

A first order Markov chain is a stochastic process having the property that the value of the process at time t, X_t , depends only on its value at time t-1, X_{t-1}, and not on the sequence of values that the process passed through in arriving at X_{t-1}.

A ‘C state’ Markov chain requires that C(C-1) transition probabilities be estimated and the remaining C P_{ij} can be determined using the relation

$$\sum_{j=1} p\text{ij} = 1 \qquad (1)$$

The C^2 transition probabilities are given by the stochastic matrix P.

$$P = \begin{pmatrix} p11 & p12 & \dots. & p1c \\ p21 & p22 & \dots. & p2c \\ \dots & \dots & \dots & \dots \\ pc1 & pc2 & \dots. & pcc \end{pmatrix} \quad (2)$$

Once P is known, all that is required to determine the probabilistic behavior of the Markov Chain is the initial state of the chain. In the following, $p_j^{(n)}$ denotes the probability that the chain is in state j at step or time n. The 1xC vector $p^{(n)}$ has elements $p_j^{(n)}$. Thus

$p^{(n)} = [\ p_1^{(n)}, p_2^{(n)}, \ldots, p_c^{(n)}\ ]$ and $p^{(1)} = p^{(0)} p$ (3)

where, $p^{(0)}$ is the initial probability vector.

In general, $p^{(n+c)} = p^{(c)}\ p^{(n)}$ (4)

Where, p^n is the n th power of p.

3.2. Parameter estimation

The parameters for the occurrence model are transition probabilities, p_{ij} s, which forms the transition matrix P. The estimate for p_{ij} is given by

$$P_{ij} = \frac{n_{ij}}{\sum_{j=1}^{c} nij} \qquad (5)$$

where, n_{ij} is the number of times the observed data went from state i to state j.

For finding the daily transition probabilities, n_{ij} s for each day of the year are counted for the entire period of record and for monthly p_{ij} s, n_{ij} s are counted for each month of the year throughout the entire record length. Eleven years (1985 - 1995) of daily rainfall data at Aduthurai were used to estimate the parameters.

3.3. Model for rainfall magnitudes on wet days

The rainfall amounts on wet days are modeled by a two parameter gamma distribution with the density function is given by

$$P_x(x) = \frac{\lambda^{\eta}\ x^{\eta-1}\ e^{-\lambda x}}{\Gamma_{(\eta)}} \qquad (6)$$

where, $\Gamma_{(\eta)} = (\eta - 1)!$ for $\eta = 1,2,3\ldots\ldots$ (7)

and $\eta = t^{\eta-1}\ e^{-1}$ (8)

in which η and λ are the shape and scale parameters respectively.

3.4. Parameter estimation

The parameters of the gamma distribution η and λ were estimated using Greenwood and Durand (1960) [5] method as given below

$$\eta^* = \frac{(0.5000876 + 0.1648852\, y - 0.0544274\, y^2)}{y}$$

for $0 \leq y \leq 0.5772$ (9)

$$\eta^* = \frac{8.898919 + 9.05995\, y - 0.09775373\, y^2}{y(17.79728 + 11.968477\, y + y^2)}$$

for $0.5772 \leq y \leq 17.0$ (10)

where, $y = \ln \bar{u} - \overline{\ln u}$

in which, u is the rainfall amount on wet days and ‾ denotes 'the arithmetic mean of'.

The estimate η^* was corrected for small-sample bias using Bowman and Shenton equation,

$$\eta = \frac{(n-3)\eta^*}{n} \qquad (12)$$

where, n is the sample size

the estimate for λ is

$\lambda = \frac{\eta}{\bar{u}}$ (13)

3.5. MODEL CALCULATION : NOVEMBER 1989 – ONE MONTH

[TABLE- 1] RAINFALL DATA AND STATES

Rainfall	States
7.4	3
0.6	2
0.0	1
11.4	3
0.0	1
0.0	1
0.0	1
0.0	1
0.0	1
9.4	3
4.6	3
57.2	4
74.2	4
17.0	3
0.8	2
0.0	1
0.0	1
19.4	3
59.4	4
16.8	3
0.0	1
0.0	1
0.0	1
0.0	1
0.0	1
0.0	1
0.0	1
0.0	1
0.6	2
34.0	4

TRANSITION PROBABILITY MATRIX

$$\mathbf{n_{ij}} = \begin{pmatrix} n11 & n12 & n13 & n14 \\ n21 & n22 & n23 & n24 \\ n31 & n32 & n33 & n34 \\ n41 & n42 & n43 & n44 \end{pmatrix},$$

$$\mathbf{n_{ij}} = \begin{pmatrix} 6 & 0 & 3 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

$$p_{ij} = \frac{n_{ij}}{\sum_{j=1}^{c} nij}$$

$$p_{ij} = \begin{pmatrix} \frac{6}{9} & 0 & \frac{3}{9} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

$$p_{ij} = \begin{pmatrix} 0.6666 & 0 & 0.3333 & 0 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

NOVEMBER - 1989

STATE - 3

RAINFALL	STATE
0.0	1
11.4	3
0.0	1
9.4	3
0.0	1
19.4	3

u Range = 40.2

ū Range = 13.4

n =14

$\ln u = \frac{\ln u}{n} = 0.26384$

$\ln \bar{u} = 2.59525$

$$y = \frac{\ln \bar{u}}{\overline{\ln u}} = 9.83647$$

$$\eta^* = \frac{8.898919+9.05995\,(9.83647)-0.09775373(9.83647)^2}{9.83647(17.79728+11.968477\,(9.83647)+(9.83647)^2)} =$$

$$\eta^* = 0.03876$$

$$\eta = \frac{(30-3)\eta*}{30} = 0.034884$$

$$\lambda = \frac{\eta}{\bar{u}} = 0.002603$$

[IV] RESULTS AND DISCUSSION

Markov chains indicated that first order Markov chains can adequately represent the rainfall occurrences in all the months. A number of states are used for representing rainfall in a wet day as a more good fit can be obtained for the distribution representing the rainfall amount in each class. The rainfall data are given in [Table -1] for the model calculation. The states and their boundary limits are given in [Table -2]. The monthly transition probabilities are given in [Table-3]. The two parameters, η and λ of the gamma distributions are given in [Table- 4].

[V] CONCLUSION

First order Markov chains with two parameters gamma distributions were found to be adequate to model daily rainfall sequences at Aduthurai. The model consists of 9 Markov chains with 144 parameters representing the rainfall occurrence process in each month and 26 gamma distributions with 52 parameters to model the amount of precipitation corresponding to each state in each month.

[TABLE- 2]. States and their boundaries

State	Limits (mm)		
1	0.0	-	0.0
2	0.1	-	3.9
3	4.0	-	27.9
4	28.0	-	∞

[TABLE- 3]. Monthly transition probabilities

April

0.9771	0.0000	0.0150	0.0000
0.5000	0.0000	0.5000	0.0000
1.0000	0.0000	0.0000	0.0000
0.5000	0.0000	0.0000	0.5000

May

0.9271	0.0405	0.0324	0.0000
0.0000	0.5000	0.5000	0.0000
0.4000	0.6000	0.0000	0.0000
0.1000	0.0000	0.0000	0.0000

June

0.9320	0.0350	0.0260	0.0000
0.6330	0.1000	0.2660	0.0000
0.5000	0.1250	0.3750	0.0000
0.0000	0.0000	0.5000	0.5000

July

0.8484	0.0617	0.0707	0.0191
0.7500	0.1875	0.0625	0.0000
0.6250	0.0000	0.3750	0.0000
0.5000	0.5000	0.0000	0.0000

August

0.7962	0.0584	0.0520	0.0932
0.3330	0.2272	0.3939	0.0454
0.7045	0.2045	0.0681	0.0227
0.5000	0.2500	0.0000	0.2500

September

0.7708	0.1081	0.0721	0.0489
0.6250	0.0000	0.3125	0.0625
0.5890	0.2072	0.2035	0.0000
0.5000	0.0000	0.2500	0.2500

October

0.6910	0.1532	0.1178	0.0378
0.2603	0.4114	0.3282	0.0000
0.3259	0.2592	0.3542	0.0606
0.8000	0.2000	0.0000	0.0000

November

0.8605	0.0212	0.1181	0.0000
0.3565	0.1885	0.2196	0.2351
0.1514	0.3030	0.2878	0.2576
0.0000	0.0625	0.4374	0.5000

December

0.7617	0.1566	0.0566	0.0250
0.6915	0.2667	0.0416	0.0000
0.2581	0.3167	0.2334	0.1916
0.2221	0.2221	0.2778	0.2778

[TABLE- 4] Parameters η and λ for Gamma distribution

Months	η			λ		
	State 2	State 3	State 4	State 2	State 3	State 4
April	0.07134	0.29697	0.07134	0.02460	0.05723	0.00085
May	0.15204	0.25202	----------	0.08944	0.01390	----------
June	0.06450	0.22060	0.04760	0.04864	0.03207	0.00087
July	0.11027	0.15131	0.12623	0.10268	0.01710	0.00409
August	0.42702	0.08356	0.05995	0.40384	0.01141	0.00146
September	0.20862	0.06117	0.05020	0.16365	0.00735	0.00106
October	0.05160	0.04488	0.02293	0.04360	0.00478	0.00042
November	0.02696	0.02768	----------	0.10281	0.00241	----------
December	0.06317	0.04720	0.02242	0.06274	0.00464	0.00026

[VI] REFERENCE

1. Bruhn, J.A., Fry, W.E. and Fick, G.W., [1980], *Simulation of daily weather data using theoretical probability distributions*, J.Appl. Meterol., 19(9), pp. 1029 – 1036.
2. Carey, D.I. and Haan, C.T., [1978], *Markov process for simulating daily point rainfall, J.Irrig. and Drain Div.*, 104, pp. 111 – 125.
3. Coe, R. and Stern, R.D., [1982], *Fitting models to daily rainfall data*, J. Appl. Meterol., 21, pp. 1024 – 1031.
4. Gabriel,K.R. and Neumann, J., [1962], *A Markov chain model for daily rainfall occurrences at Tel Aviv*, Quar.J. Roy. Meteor.Soc., 88, pp. 90 – 95.
5. Greenwood, J. and Durand., [1960], *Aids for fitting the Gamma distribution by maximum likelihood*, Technometrics., 2, pp. 55 – 65.
6. Liana Cazacioc and Elena Corina Cipu., [2004], *Evaluation of the transition probabilities for daily precipitation time series using a Markov chain model*, Proceedings of the 3rd International Colloquium “Mathematics in Engineering and Numerical Physics” October 7-9, Bucharest, Romania, pp. 82 – 92.
7. Srikanthan, R., [2005], *Stochastic generation of daily rainfall using a nested transition probability model*, 29th Hydrology and Water Resources Symposium, Engineers Australia, 20 -23 February, 2005, Canberra.
8. Srikanthan. R. and McMahon, T.A., [2001], *Stochastic generation of annual, monthly and daily climate data*, Hydrology and Earth System Sciences, 5 (4), pp. 653 – 670.
9. Zohadie Bardaie, M. and Ahmad Che Abdul Salam., [1981], *A Stochastic Model of Daily Rainfall for Universiti Pertanian Malaysia*, Serdang, Pertanika, 4(1), pp. 1 - 9.