Maximum Throughput of cognitive Relay systems

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ABSTRACT
One of the greatest challenges in cognitive radio systems is power allocation of the secondary users for maximizing throughput. When the primary users give up the frequency bands, the secondary users utilize the spectrum holes. Furthermore, the secondary users can simultaneously occupy the spectrum with interference-aware. In this paper we consider approach of the spectrum interference to support the maximum throughput of the network. We investigate our method under one secondary user. To show the performance of our approach, we compare it with equal power allocation between the secondary user and relay.

Key words:- Cognitive radio, Relay, Energy detection, Spectrum sensing, Optimization

1. INTRODUCTION
Over the last decade, cognitive radio networks have grown rapidly to support more spectrum usage for unlicensed users and have obtained more popularity in wireless services. Two heuristic algorithms for maximizing spectrum efficiency are proposed in [1] called Edge colouring algorithm and Clique determination algorithm. It is shown the Edge colouring has better performance than the clique heuristic. For the dynamic channel allocation of cognitive radio systems, two methods of auction named first price and second price have been studied in [2], whereas these methods yield similar performance in terms of outcome and efficiency. There are a lot of optimal throughput trade-off approaches in various scenarios lead to maximize the achievable throughput and optimize the sensing slot duration under power constrains, that the highest throughput yields whereas the most detection probability is achieved [3], [4]. Higher detection probability and lower false alarm are resulted by increasing the sensing time, according to the detection theory As known, an increase of the sensing time leads to a decrease of the data transmission also a decrease of the achievable throughput for cognitive radio systems. According to [4], an improved throughput is achieved by implementing data transmission and spectrum sensing at the same time. In [5], a fixed sensing time is considered for optimizing the frame duration and the problem of throughput collision trade-off is investigated to maximize throughput. Trade-off between sensing time and throughput is addressed in [6] by a particle swarm optimization (PSO) algorithm. The usage of PSO scheme leads to fast convergence by performing stopping threshold subject to secondary user (SU) gain, sensing performance and detect probability. PSO approach is greater than other methods due to low computation complexity, energy efficiency and fast convergence time.
The problem of shadowing and path loss is one of the most problems of spectrum sensing in hard multipath fading and inside building and it causes the SU interference to the licensed system. Multiple cognitive SUs are applied in cooperative system in order to overcome the shadowing and path loss problem. It is shown an increase of cooperative users can improve the performance.

In [7], the optimal linear coefficient of cooperative spectrum sensing has been derived under data fusion technique. The optimal voting rule has been addressed in [8] for any detector of cooperative spectrum sensing, also an optimal detection threshold is derived and a fast spectrum sensing algorithm is proposed for a large network. A cooperative three node relay fading channel has been considered in [9] and it has been obtained the optimal allocation for spectrum in a decode and forward relay channel. Time and power distributions have been optimized between the direct transmission and relay phases. In [10], it has been investigated overhead throughput trade off with single stage and two stage cooperative sensing over Rayleigh fading channels. It has been derived an optimal sensing time and an optimal number of SUs for maximizing throughput.

The remainder of the paper is structured as follows. Section 2 describes the system model for spectrum sensing. Section 3 achieves the optimal power allocation for single SU to obtain the optimum throughput. In section 4, the simulation environment and results are described. Finally, the paper is concluded in section 5.

2. SYSTEM MODEL AND ANALYSIS

We consider a cognitive relay network where consists of a secondary transmitter, a secondary relay and a secondary receiver use the spectrum of the primary system (primary transmitter and primary receiver). We mark $H_{SR}$, $H_{KS}$, $H_{SS}$, $H_{PR}$, $H_{PP}$, $H_{PS1}$ and $H_{PS2}$ as the channel coefficients of SU-$R$, R-SU-$R$, SU-$R$-SU-$R$, PU-$R$, PU-$R$-SU-$R$, PU-$R$-SU-$R$ and PU-$R$-SU-$R$ links. The channels are supposed to be Rayleigh flat fading. The additive white noise is distributed as circularly symmetric complex Gaussian (CSCG) with zero mean and variance $\sigma_w^2$.

In cognitive radio networks, to access the spectrum, discover the spectrum holes and understand the channel is idle or active, the spectrum sensing mechanism is implemented. If the channel is idle the SU can opportunistically use the spectrum otherwise the SU should postpone transmissions until the spectrum sensing is done again. The interference temperature model is employed to solve this limitation. When the PU occupies the channel, the SU can transmit simultaneously without interference provided that the transmitter power of the SU doesn’t exceed a threshold. The interference temperature can be denoted as:

\[ I_c(f, W) = \frac{P_t(f, W)}{W} = \frac{1}{K} \int_{f-W/2}^{f+W/2} S(f) df \]  (1)

where $f_c$ is the central frequency, $W$ is the frequency bandwidth, $S(f)$ is the power spectrum density and $P_t(f, W)$ is the interference power of the SU. $K$ represents Boltzmann’s constant ($1.38 \times 10^{-23}$). The maximum interference transmission power of the SU is derived by

\[ P_{\text{threshold}}(f_c, W) = \frac{KW}{M} I_{\text{threshold}}(f_c) \]  (2)

where $I_{\text{threshold}}(f_c)$ is the interference temperature threshold and $M$ is the attenuation factor of multipath and fading. Let each frame structure of the spectrum consist of two time slots $(T_1 + T_2)$. Suppose amplify and forward (AF) relaying style for the cognitive Relay system. At the beginning of each frame, the spectrum is listened to identify the condition of the channel (idle or active) for duration $T_1 + T_2$.

If the PU is detected by the energy detector in the previous frame, the SU will transmit with low
power (i.e. $P_{SU} < P_{p, \text{threshold}}$) in the current frame, as discussed above, otherwise the SU will transmit with high power for the spectrum holes. Furthermore, the spectrum sensing and the data transmitting are implemented simultaneously in all frames except the first frame because the cognitive radio network doesn’t know about the spectrum conditions and only monitors the spectrum in this frame. 

To organize the system model, let the primary transmitted signal be $p(n)$ and the secondary transmitted signal is $s(n)$ with zero mean and variances $\mathbb{E}[|p(n)|^2]=1$ and $\mathbb{E}[|s(n)|^2]=1$, respectively. Consider $P_p$ is the transmission power of the PU and $P_s$ is the transmission power of the SU. For simplicity of calculation, assume the time slots of $T_1$ and $T_2$ are equal. Also, assume an amplifying forward factor of the relay is $\sqrt{\rho}$. 

As known, there are two hypotheses $H_0$ and $H_1$ for the cognitive radio systems. The hypothesis $H_0$ is used when the PU is absent and the hypothesis $H_1$ is applied when the PU is present. If $\varphi$ denotes the indicator of the PU, then the hypothesis $H_0$ occurs for $\varphi=0$ and the hypothesis $H_1$ occurs for $\varphi=1$. Let the channel gains as $G_{SR} = |H_{SR}|^2$, $G_{RS} = |H_{RS}|^2$, $G_{SS} = |H_{SS}|^2$, $G_{PR} = |H_{PR}|^2$, $G_{PP} = |H_{PP}|^2$, $G_{PS} = |H_{PS}|^2$, and $G_{PS2} = |H_{PS2}|^2$.

In the first time slot (i.e. $T_1$), when the secondary relay and the secondary receiver monitor the spectrum, the SU sends data to the secondary relay and the secondary receivers are given as follows:

$$x_{R1}(n) = \sqrt{\rho} P_p H_{SR} s_1(n) + \varphi \sqrt{\rho} P_p H_{PS} P_1(n) + w_{R1}(n)$$  \hspace{0.5cm} (3) 

$$x_{S1}(n) = \sqrt{\rho} P_p H_{SS} s(n) + \varphi \sqrt{\rho} P_p H_{PS} P_1(n) + w_{S1}(n)$$  \hspace{0.5cm} (4) 

Where $n = 1, 2, \ldots, N$ and $N = T_f f_s$, $f_s$ is the sampling frequency. The variances of noises $w_{R1}(n)$ and $w_{S1}(n)$ are $\mathbb{E}[w_{R1}(n)^2] = \sigma_w^2$, $\mathbb{E}[w_{S1}(n)^2] = \sigma_w^2$.

In the second time slot $T_2$, the secondary transmitter and the secondary receiver listen to the channel for the presence of the PU whereas the secondary relay amplifies and sends the combined received signal from the stage $T_1$ to the secondary transmitter. The received signals at the secondary transmitter and receiver are derived by

$$x_{T2}(n) = \sqrt{\rho} P_p H_{SR} x_{R1}(n) + \varphi \sqrt{\rho} P_p H_{PS} P_2(n)$$  \hspace{0.5cm} (5) 

$$+ w_{T2}(n) = \sqrt{\rho} \sqrt{\frac{P_p}{P_S}} H_{SR} H_{SR} s_1(n)$$  \hspace{0.5cm} (5) 

$$\varphi \sqrt{\rho} \sqrt{\frac{P_p}{P_S}} H_{PS} P_1(n) + H_{PS} P_1(n)$$  \hspace{0.5cm} (5) 

$$+ \sqrt{\rho} H_{SR} \varphi w_{R1}(n) + w_{T2}(n)$$  \hspace{0.5cm} (5) 

$$x_{S2}(n) = \sqrt{\rho} P_p H_{RS} x_{S1}(n) + \varphi \sqrt{\rho} P_p H_{PS} P_2(n)$$  \hspace{0.5cm} (6) 

$$+ w_{S2}(n) = \sqrt{\rho} \sqrt{\frac{P_p}{P_S}} H_{RS} H_{SR} s_1(n)$$  \hspace{0.5cm} (6) 

$$\varphi \sqrt{\rho} \sqrt{\frac{P_p}{P_S}} H_{PS} P_1(n) + H_{PS} P_1(n)$$  \hspace{0.5cm} (6) 

$$+ \sqrt{\rho} H_{RS} \varphi w_{R1}(n) + w_{S2}(n)$$  \hspace{0.5cm} (6) 

where $n = N, N+1, \ldots, 2N$ and the variance of noises $w_{T2}(n)$ is $\mathbb{E}[w_{T2}(n)^2] = \sigma_w^2$ and the variance of $w_{S2}(n)$ is $\mathbb{E}[w_{S2}(n)^2] = \sigma_w^2$. By eliminating self interference of $s_1(n)$ in (5), the received signal at the secondary transmitter can be expressed as

$$\tilde{x}_{T2}(n) = \varphi \sqrt{\rho} \sqrt{\frac{P_p}{P_S}} H_{SR} H_{RS} P_1(n) + H_{PS} P_1(n)$$  \hspace{0.5cm} (7) 

$$+ \sqrt{\rho} H_{SR} \varphi w_{R1}(n) + w_{T2}(n)$$

The energy detector at the secondary transmitter takes the samples of $\tilde{x}_{T2}(n)$, accumulates the powers of $\tilde{x}_{T2}(n)$ and gives the test statistics as follows:

$$T(\tilde{x}_{T2}) = \frac{1}{N} \sum_{n=N+1}^{2N} [\tilde{x}_{T2}(n)]^2$$  \hspace{0.5cm} (8) 

according to the central limit theorem, the test statistics can be modeled by Gaussian distribution for sufficiently large sample number ($N \geq 100$).
Under the likelihood ratio Neyman Pearson hypothesis, the best decision \( T(\tilde{x}_{T_2}) \) can be obtained in comparison with a threshold \( \delta \). Assume \( p_f \) and \( p_d \) are the false alarm probability and the detection probability, respectively. Let \( p_h(y) \) be the probability density function (PDF) of the test statistics with Chi-square distribution under hypothesis \( H_0 \) and \( p_i(y) \) is the PDF of \( T(\tilde{x}_{T_2}) \) under hypothesis \( H_1 \). So \( p_f \) and \( p_d \) can be written as
\[
p_f(\delta) = \Pr(T(\tilde{x}_{T_2}) > \delta | H_0) = \int_{\delta}^{\infty} p_h(y) dy \quad (9)
\]
\[
p_d(\delta) = \Pr(T(\tilde{x}_{T_2}) > \delta | H_1) = \int_{\delta}^{\infty} p_i(y) dy \quad (10)
\]
As known, the higher detection probability yields the better security for the PU and the lower false alarm probability results the more access to the frequency holes for the SU with high transmitted power. To this end, the low false alarm probability and the high detection probability should be derived as much as possible for a good performance.

For clarity, \( p_1(n) \) and \( p_s(n) \) are is distributed as CSCG and \( p_a(n) \), \( p_2(n) \), \( w_{R1}(n) \), \( w_{S1}(n) \), \( w_{S2}(n) \) and \( W_{R1}(n) \) are supposed independently from each other.

Under hypothesis \( H_0 \), the mean of the test statistics is \( E(T_0) = \mu_0 \) and the variance is \( \text{var}(T_0) = 1/N\mu_0^2 \), where \( \mu_0 = \rho G_{sr}\sigma_w^2 + \sigma_u^2 \).

Similarly Under hypothesis \( H_1 \), the mean of the test statistics is \( E(T_1) = \mu_1 \) and the variance of the test statistics is \( \text{var}(T_1) = 1/N\mu_1^2 \) where \( \mu_1 = G_{PS1}P + \rho G_{SR}(P_{pr} + \sigma_w^2) + \sigma_u^2 \).

The false alarm probability and the detection probability are given by
\[
p_f = Q\left(\frac{\delta - E(T_0)}{\sqrt{\text{var}(T_0)}}\right) \quad (11)
\]
\[
p_d = Q\left(\frac{\delta - E(T_1)}{\sqrt{\text{var}(T_1)}}\right) \quad (12)
\]
where \( Q(\cdot) \) denotes the complementary function of the Gaussian standard distribution, defined as
\[
Q(x) = \int_{x}^{\infty} \exp(-\frac{y^2}{2}) dy \quad (13)
\]

3. THE MAXIMUM THROUGHPUT

In this section, we express the maximum achievable throughput of the SU under four scenarios. The SU can transmit data according to the sensing result and the state of the PU. When the channel is idle and the SU doesn’t detect the presence of the PU, it will transmit with high power of \( P_{S0} \) and the rate of \( C_{00} \). If a false alarm occurs the SU should transmit with low power of \( P_{S1} \) and the rate of \( C_{01} \) since it imagines the frequency band is occupied by the PU.

When the channel is active and the PU is detected properly, the SU should transmit with low power of \( P_{S1} \) and the rate of \( C_{11} \) due to the presence of the interference. If a missed detection happens the transmission power and the rate of the SU should be \( P_{S0} \) and \( C_{10} \) respectively, since it supposes the PU is absent. We assume the transmission power of the PU is constant for total scenarios, denoted by \( P_p \). Also, as discussed in previous section, \( P_{S1} \) must be much lower than \( P_{S0} \) where \( P_{S0} \gg P_{S1} \) and \( P_{S1} \leq P_{\text{Threshold}} \).

From the viewpoint of the SU, the maximum achievable throughput should be derived. Whereas from the viewpoint of the PU, the minimum missed detection probability should be calculated to solve optimization problems. Since these two subjects cannot compare together, the target detection probability is supposed to be high and constant in our computations and simulations where \( p_d \geq 90\% \).

For the constant target detection probability \( p_d \), by substituting (12) in (11) and eliminating \( \delta \) in (12), the false alarm probability can be achieved by
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\[ p_f = Q \left( \frac{Q^{-1}(\overline{p}) \mu + (\mu - \mu_0)\sqrt{N}}{\mu_0} \right) \]

\[ = Q \left( Q^{-1}(\overline{p}) + \sqrt{N} \delta_p + Q^{-1}(\overline{p}) \right) \]

where

\[ \delta_p = \frac{\mu - \mu_0}{\mu_0} = \frac{G_{ps1}P_p + \rho G_{sr}G_{ps2}P_p}{\rho G_{sr}\sigma_n^2 + \sigma_w^2} \]

\[ = \frac{\delta_{ps1} + \rho G_{sr}\delta_{pr}}{1 + \rho G_{sr}} \]

where \( \delta_{ps1} = \frac{G_{ps1}P_p}{\sigma_n^2} \), \( \delta_{pr} = \frac{G_{ps2}P_p}{\sigma_n^2} \). In practice, \( \delta_{ps1} \) is the received SNR from the PU at the secondary transmitter and \( \delta_{pr} \) is the received SNR at the secondary receiver.

The probability of idling and acting the frequency band are denoted by \( p(H_0) \) and \( p(H_1) \) where \( p(H_0) + p(H_1) = 1 \). For four scenarios mentioned, the maximum achievable throughput of the SU is proposed by

\[ \max_{(p_{so},p_{si},p_{s})} R = 0.5 \left( p(H_1) \left( 1 - p_f \right) C_{f0} \right) + \left( p(H_0) \left( p_f \right) C_{i0} \right) + \left( p(H_1) \left( 1 - p_f \right) C_{f1} \right) + \left( p(H_0) \left( p_f \right) C_{i1} \right) \]

Subject to: \( P_{so} + P_f \leq P_{max} \), \( 0 \leq p_f \leq 1 \), where

\[ C_{f0} = \log_2 \left( 1 + \frac{G_{so}P_{so}}{\sigma_n^2} + \frac{\rho G_{sr}G_{ps1}P_{so}}{\rho G_{sr}\sigma_n^2 + \sigma_w^2} \right) \]

\[ = \log_2 \left( 1 + \chi_{o0}P_{so} \right) \]

\[ C_{i0} = \log_2 \left( 1 + \frac{G_{si}P_{si}}{\sigma_n^2} + \frac{\rho G_{sr}G_{ps2}P_{si}}{\rho G_{sr}\sigma_n^2 + \sigma_w^2} \right) \]

\[ = \log_2 \left( 1 + \chi_{i0}P_{si} \right) \]

\[ C_{f0} = \log_2 \left( 1 + \frac{G_{so}P_{so}}{\sigma_n^2} + \frac{\rho G_{sr}G_{ps1}P_{so}}{\rho G_{sr}\sigma_n^2 + \sigma_w^2} \right) \]

\[ + \frac{\rho G_{sr}G_{ps2}P_{so} + \rho G_{sr}G_{ps1}P_{so}}{\rho G_{sr}\sigma_n^2 + \sigma_w^2} \]

\[ = \log_2 \left( 1 + \chi_{f0}P_{so} \right) \]

\[ C_{i1} = \log_2 \left( 1 + \frac{G_{si}P_{si}}{\sigma_n^2} + \frac{\rho G_{sr}G_{ps1}P_{si}}{\rho G_{sr}\sigma_n^2 + \sigma_w^2} \right) \]

\[ + \frac{\rho G_{sr}G_{ps2}P_{si} + \rho G_{sr}G_{ps1}P_{si}}{\rho G_{sr}\sigma_n^2 + \sigma_w^2} \]

\[ = \log_2 \left( 1 + \chi_{i1}P_{si} \right) \]

4. SIMULATION AND RESULT

In this section, we illustrate the simulation results for our proposed cognitive relay network. Let the probability of idling and acting the frequency band be \( p(H_0) = 0.6 \) and \( p(H_1) = 0.4 \). The number of samples is \( N = 100 \). We assume the channel gains as \( G_{sr} = -4dB \), \( G_{so} = -5dB \). \( G_{ps1} = -8dB \), \( G_{ps2} = -10dB \). The target detection probability is considered to be \( \overline{p} = 90\% \). In order to simplifying, we only focus on the transmit power of the SU with high power (i.e. \( P_{so} \)) and suppose the transmit power of the SU with low power is fixed in total simulation whereas \( P_{so} = 1dB \). As mentioned, the channels are considered to be Rayleigh flat fading. The variance of the circularly symmetric complex Gaussian (CSCG) noise is \( \sigma_n^2 = 0dB \). The transmission power of the PU is \( P_f = 5dB \).

In Fig (1), the optimum achievable throughput of the SU is shown versus the amplifying forward factor of the relay \( \rho \) for different values of the maximum transmit power constraint. It can be deduced, as the maximum transmit power constraint increases the achievable throughput of the SU obtains the higher value. This results can be justified by the fact that the transmit power of the SU \( P_{so} \) is increased by the growth of the \( P_{max} \). Furthermore, the optimum amplifying forward factor of the relay is derived between 2 and 3, as
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called $\rho_{\text{max}}$. As seen in Fig (1), when the $\rho$ is lower than $\rho_{\text{max}}$, the transmit power parameters of the SU are amplified more than the noise and interference powers, so the achievable throughput is improved. For the values larger than $\rho_{\text{max}}$, the achievable throughput decreases because the parameters of the noise and interference powers are amplified more than the transmit power of the SU.

Fig (1): The achievable throughput of the SU versus the amplifying forward factor of the relay

![Graph](image1)

As seen in Fig (1), the $\rho$ is lower than $\rho_{\text{max}}$, the transmit power parameters of the SU are amplified more than the noise and interference powers, so the achievable throughput is improved. For the values larger than $\rho_{\text{max}}$, the achievable throughput decreases because the parameters of the noise and interference powers are amplified more than the transmit power of the SU.

Fig (2): The achievable throughput of the SU versus the maximum transmit power constraints per dB for two different power allocation methods.

![Graph](image2)

Fig (2) illustrates the achievable throughput of the SU versus the maximum transmit power constraints for two different power allocation methods: equal power allocation and our proposed power allocation. Equal power allocation associates the transmit power of the relay to be equal with the transmit power of the secondary transmitter under four defined scenarios. As seen, the proposed power allocation scheme outperforms the equal power allocation scheme.

5. CONCLUSION

In this paper, we proposed a new cognitive relay system that maximizes the achievable throughput of the SU with optimal power allocation. We supposed four scenarios in our calculation. Simulation results demonstrate the better performance of the optimal power allocation than the equal power allocation. Finally, in our future research, we decide to generalize our proposed cognitive relay network for cooperative spectrum sensing.

6. REFERENCES

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